Wall Street and Silicon Valley: A Delicate Interaction*

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Abstract

Financial markets look at data on aggregate investment for clues about underlying profitability. At the same time, firms' investment depends on expected equity prices. This generates a two-way feedback between financial market prices and investment. In this paper we study the positive and normative implications of this interaction during episodes of intense technological change, when information about new investment opportunities is highly dispersed. Because high aggregate investment is "good news" for profitability, asset prices increase with aggregate investment. Because firms' incentives to invest in turn increase with asset prices, an endogenous complementarity emerges in investment decisions—a complementarity that is due purely to informational reasons. We show that this complementarity dampens the impact of fundamentals (shifts in underlying profitability) and amplifies the impact of noise (correlated errors in individual assessments of profitability). We next show that these effects are symptoms of inefficiency: equilibrium investment reacts too little to fundamentals and too much to noise. We finally discuss policies that improve efficiency without requiring any informational advantage on the government's side.

Keywords: heterogeneous information, complementarity, volatility, inefficiency, beauty contests.

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1 Introduction

Financial markets follow closely the release of macroeconomic and sectoral data, looking for signals about underlying economic fundamentals. In particular, high current levels of activity tend to forecast high future profitability, leading to an increase in asset prices. At the same time, financial prices affect the real economy by changing the incentive to invest for individual firms. For a start-up company, higher asset prices raise the value of a potential IPO and facilitate financing from venture capitalists. For firms already quoted on the stock market, higher asset prices raise the value of equity issues and the market valuation of further investments within the firm.

Both directions of causation—from real activity to financial prices and from financial prices to real investment—have been widely explored in existing theoretical and empirical work. The literature on the impact of macroeconomic news on asset prices goes back to Chen, Roll and Ross (1986) and Cutler, Poterba, and Summers (1989); the literature on the impact of asset prices on real investment goes back to Brainard and Tobin (1968), Tobin (1969), Bosworth (1975), Abel and Blanchard (1986), Barro (1990), and Morck, Shleifer and Vishny (1990).

In this paper, we document novel positive and normative implications stemming from the interaction of these two channels when agents do not share the same information. We first show that this interaction generates a feedback mechanism between the real and the financial sector of the economy: high investment drives up aggregate activity; financial markets interpret this as a positive signal about future profitability; asset prices increase; this adds fuel to the initial increase in investment. We next show that this mechanism can exacerbate non-fundamental movements in real investment and asset prices, and can distort allocative efficiency. This mechanism seems particularly relevant in periods of intense technological change, when information regarding the viability and profitability of new technologies is widely dispersed across the economy.

Preview. We conduct our exercise within a neoclassical economy in which allocations would be first-best efficient if all agents had the same information. A large number of "entrepreneurs" gets the option to invest in a new technology. They have dispersed information about the profitability of this technology and may sell their capital in a competitive financial market before uncertainty is realized. The "traders" who participate in the financial market are also imperfectly informed, but they observe aggregate investment, which provides a summary statistic of the information dispersed among the entrepreneurs. In this environment, movements in real investment and asset prices are driven by two types of shocks: "fundamental shocks," reflecting actual changes in the long-run profitability of investment, and "expectational shocks," reflecting correlated mistakes in individual assessments of this profitability.

The positive contribution of the paper is to study how the interaction between real and financial activity affects the transmission of these shocks in equilibrium. Because high aggregate investment is “good news” for profitability, asset prices increase with aggregate investment. As a result, an endogenous complementarity emerges in investment decisions. An entrepreneur anticipates that the price at which he might sell his capital will be higher the higher the aggregate level of investment. He is thus more willing to invest when he expects others to invest more. In equilibrium, this complementarity induces entrepreneurs to rely more on common sources of information regarding profitability, and less on idiosyncratic sources of information. This is because common sources of information are relatively better predictors of other entrepreneurs' investment choices, and hence of future financial prices. For the same reason, the entrepreneurs' choices become more anchored to the common prior, and hence less sensitive to changes in the underlying fundamentals. It follows that the feedback between the real and the financial sector of the economy amplifies the impact of common expectational shocks while also dampening the impact of fundamental shocks.

The normative contribution of the paper is to study whether the reaction of the economy to different shocks is optimal from a social perspective. The mere fact that entrepreneurs care about the financial market's valuation of their investment does not, on its own, imply any inefficiency. Indeed, as long as all agents share the same information, equilibrium asset prices coincide with the common expectation of profitability; whether entrepreneurs try to forecast fundamentals or asset prices is then completely irrelevant for efficiency.

This is not the case, however, when information is dispersed. The sensitivity of asset prices to aggregate investment induces a wedge between private and social returns to investment: while the fundamental valuation of the investment made by a given entrepreneur is independent of the investments made by other entrepreneurs (i.e., there are no production externalities or spillovers), the market valuation is not. By implication, the complementarity that emerges in equilibrium due to the dispersion of information is not warranted from a social perspective. It then follows that the positive effects documented above are also symptoms of inefficiency: equilibrium investment reacts too little to fundamental shocks and too much to expectational shocks.

We conclude by examining policies that improve efficiency without requiring the government to have any informational advantage vis-a-vis the market. We first consider interventions “during the fact,” while information remains dispersed. In particular, we consider a tax on financial trades or other policies aimed at stabilizing asset prices. By moderating the reaction of asset prices to aggregate investment, these policies dampen the equilibrium impact of non-fundamental shocks, which improves efficiency. In so doing, however, these policies also dampen the equilibrium impact of fundamental shocks, which was inefficiently low to start with. It follows that these policies can raise welfare, but never achieve full efficiency.

We next consider interventions “after the fact,” when uncertainty has been resolved. Building on results from Angeletos and Pavan (2007b), we show that full efficiency can be achieved by introducing a tax on capital holdings that is contingent on both realized aggregate investment and realized profitability. Although real and financial decisions are sunk by the time these taxes are collected, the *anticipation* of these contingencies affects the incentives entrepreneurs and traders face "during the fact." By appropriately designing these contingencies, the government can induce agents to respond efficiently to different sources of information, even if it can not directly monitor these sources of information.

Discussion. The US experience in the second half of the 90's has renewed interest in investment and asset-price booms driven by apparent euphoria regarding new technologies (e.g., the Internet), and on the optimal policy response to these episodes. A common view in policy discussions is that entrepreneurs and corporate managers are driven by noise traders and other irrational forces in financial markets, or are irrational themselves. Elements of this view are formalized in Shiller (2000), Cecchetti et al (2000), Bernanke and Gertler (2001), and Dupor (2005). Similar concerns are currently raised for the investment boom in China. The presumption that the government can detect "irrational exuberance" then leads to the result sought—that it should intervene.

While we share the view that expectational errors may play an important role in these episodes, we also recognize that these errors may originate from noise in information rather than irrationality. Furthermore, we doubt the government's ability to assess fundamentals better than the market as a whole. Our approach is thus different. On the normative side, we identify an informational externality that can justify intervention even by a policy maker with no superior information. At the same time, on the positive side, we show that the interaction between real and financial activity can amplify the impact of noise and that this amplification is stronger when information is more dispersed. This helps explain, without any departure from rationality, why periods of intense technological change, like the 90's, may feature significant non-fundamental volatility.

Because the source of both amplification and inefficiency in our model rests on the property that investment is largely driven by expectations about others' choices rather than about fundamentals, our results are reminiscent of Keynes' famous beauty-contest metaphor:

> "...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors..." Keynes (1936, p.156).

Implicit in Keynes' argument appears to be a normative judgement that something goes wrong when investment is driven by higher-order expectations. However, Keynes does not explain why this might be the case. More recently, Allen, Morris and Shin (2005), Bacchetta and Wincoop (2005) and Cespa and Vives (2007) have revisited rational-expectations models of asset pricing and have shown that, at least in certain cases, a mechanism similar to the one articulated by Keynes increases the impact of the common prior and of common noise on equilibrium prices. However, these papers abstract from the real sector of the economy. They also do not address whether the positive effects they document are symptoms of allocative inefficiency. To the best of our knowledge, our paper is the first to provide a complete micro-foundation for beauty-contest-like inefficiencies in the interaction between real and financial activity.

Other related literature. By focusing on the two-way feedback between real and financial decisions as a potential explanation of "bubbly" episodes, the paper also relates to two other lines of work. One line studies rational bubbles in economies with financial frictions or asset shortages (e.g., Ventura, 2003; Caballero, 2006; Caballero, Farhi and Hammour, 2006). The mechanisms studied in these papers also generate significant non-fundamental movements, but they are unrelated to information. The second and more closely related line studies speculative fluctuations in prices and investment due to heterogeneous priors regarding profitability (e.g., Scheinkman and Xiong, 2003; Gilchrist, Himmelberg, and Huberman, 2005; Panageas, 2005). In these papers, investment and prices are largely driven by expectational shocks regarding others' valuations. This is similar to the role of higher-order expectations in our paper. However, in these papers asset prices continue to reflect the social value of investment, ensuring that no inefficiency emerges.[1] In our paper, instead, the impact of higher-order expectations is also the source of inefficiency.

The paper also relates to the growing macroeconomic literature on heterogeneous information and strategic complementarities (e.g., Amato and Shin, 2006; Angeletos and Pavan, 2007a,b; Baeriswyl and Conrand, 2007; Hellwig, 2005; Hellwig and Veldkamp, 2007; Lorenzoni, 2006, 2007; Mackowiak and Wiederholt, 2006; Woodford, 2002). However, unlike the complementarities considered in these papers, which originate in monopolistic price competition, production or demand spillovers, or other payoff externalities, the complementarity documented here is due to an informational externality: it emerges only when information is dispersed and only because aggregate activity is then a signal of the underlying fundamentals. This specific source of complementarity is the key to both the positive and the normative results of our paper.[2]

Also related are Subrahmanyam and Titman (2001), Goldstein and Guembel (2003), and Ozde-

[1]In Gilchrist, Himmelberg, and Huberman (2005), inefficiency can arise due to the monopoly power of the owners of the firms issuing speculative stocks.

[2]Morris and Shin (2002) show that more precise public information can have a detrimental effect on equilibrium welfare in a game where the complementarity in individual actions reflects a discrepancy between private and social objectives; but they do not study the origin of this discrepancy and they assume it to be exogenous to the information structure. In our environment, instead, the discrepancy between private and social objectives, like the complementarity, originates in the two-way feedback between real and financial decisions and crucially depends on the information structure; this has also interesting implications for the social value of information (see Section 5.3).

noren and Yuan (2007). These papers study a variety of feedback effects between a financial market and a real sector that features of a form of network externality. Like the papers mentioned in the previous paragraph, the complementarity in these papers is exogenous to the information structure. On the other hand, a complementarity that originates in an informational externality is featured in the currency-crises model of Goldstein, Ozdenoren and Yuan (2007); but it is of a different kind than ours. In their model, the central bank looks at the size of attack to learn about the underlying fundamentals. The larger the attack, the worse the bank's perception of the fundamentals. But then also the higher the bank's willingness to abandon the peg and hence the higher the incentive for the individual speculator to attack.

Layout. Section 2 introduces the baseline model. Section 3 characterizes the equilibrium and derives the positive implications of the model. Section 4 characterizes the socially efficient use of information and contrasts it to the equilibrium. Section 5 discusses policy implications. Section 6 considers a number of extensions. Section 7 concludes. All proofs are in the Appendix.

2 The baseline model

We consider an environment in which heterogeneously informed agents choose how much to invest in a "new technology" with uncertain returns. After investment has taken place, but before uncertainty is resolved, agents trade financial claims on the returns of the installed capital. At this point, the observation of aggregate investment partially reveals the information that was dispersed in the population during the investment stage.

Timing, actions, and information. There are four periods, $t \in \{0, 1, 2, 3\}$, and two types of agents: "entrepreneurs,"who first get the option to invest in the new technology, and "traders," who can subsequently purchase claims on the installed capital of the entrepreneurs. Each type is of measure 1/2; we index entrepreneurs by $i \in [0, 1/2]$ and traders by $i \in (1/2, 1]$.

At $t = 0$, nature draws a random variable θ from a Normal distribution with mean $\mu > 0$ and variance $1/\pi_\theta$ (i.e., π_θ is the precision of the prior). This random variable represents the exogenous productivity of the new technology and is unknown to all agents.

At $t = 1$, the "real sector" of the economy operates: each entrepreneur decides how much to invest in the new technology. Let k_i denote the investment of entrepreneur i. The cost of this investment is $k_i^2/2$ and is incurred within the period. When choosing investment, entrepreneurs have access to various signals (sources of information) that are not directly available to the traders. Some of these signals may have mostly idiosyncratic noise, while others may have mostly common noise (correlated errors). To simplify, we assume that entrepreneurs observe two signals. The first one has only idiosyncratic noise and is given by $x_i = \theta + \xi_i$, where ξ_i is Gaussian noise, independently and identically distributed across agents, independent of θ, with variance $1/\pi_x$ (i.e., π_x is the precision of the idiosyncratic signal). The second has only common noise and is given by $y = \theta + \varepsilon$, where ε is Gaussian noise, common across agents, independent of θ and of $\{\xi_i\}_{i\in[0,1/2]}$, with variance $1/\pi_y$ (i.e., π_y is the precision of the common signal). The more general case where all signals have both idiosyncratic and common errors is examined in the Supplementary Material.

At $t = 2$, the "financial market" opens: some entrepreneurs sell their installed capital to the traders. In particular, we assume that each entrepreneur is hit by a "liquidity shock" with probability $\lambda \in (0, 1)$. Liquidity shocks are i.i.d. across agents, so λ is also the fraction of entrepreneurs hit by the shock. Entrepreneurs hit by the shock are forced to sell all their capital to the traders. For simplicity, entrepreneurs not hit by the shock are not allowed to trade any claims on installed capital.[3] The financial market is competitive and p denotes the price of one unit of installed capital. When the traders meet the entrepreneurs hit by liquidity shocks in the financial market, they observe the aggregate level of investment from period 1, $K = \int_0^1 k_i di$. They can then use this observation to update their beliefs about θ.[4]

Finally, at $t = 3$, θ is publicly revealed, each unit of capital gives a cash flow of θ to its owner, and this cash flow is consumed.

Payoffs. All agents are risk neutral and the discount rate is zero. Payoffs are thus given by $u_i = c_{i1} + c_{i2} + c_{i3}$, where c_{it} denotes agent i's consumption in period t. First, consider an entrepreneur. If he is not hit by the liquidity shock his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, 0, \theta k_i)$, so that his payoff is $u_i = -k_i^2/2 + \theta k_i$. If he is hit by the shock, he sells all his capital at the price p and his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, pk_i, 0)$, so that his payoff is $u_i = -k_i^2/2+pk_i$. Next, consider a trader and let q_i denote the units of installed capital he purchases in period 2. His consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (0, -pq_i, \theta q_i)$, so that his payoff is $u_i = (\theta - p)q_i$.

Remarks. The two essential ingredients of the model are the following: (i) the agents who make the initial investment decisions have dispersed private information, so that aggregate investment is a signal of the fundamental; (ii) there is some common source of "noise" that prevents aggregate investment from perfectly revealing the fundamental to all agents, so that the dispersion of information does not completely vanish by the time agents meet in the financial market.

The specific information structure we have assumed is a convenient way to capture these two properties. In particular, the role of the common signal y is to introduce correlated errors in the entrepreneurs' assessments of profitability in stage 1, thereby adding noise to the inference problem that the traders face in stage 2: in equilibrium, aggregate investment will move both with the

[3] We relax this assumption in Section 6.

[4] Letting the traders observe the entire cross-sectional distribution of investments does not affect the results. This is because, in equilibrium, this distribution is Normal with known variance; it then follows that the mean investment contains as much information as the entire cross-sectional distribution.

fundamental θ and with the common error ε, ensuring that aggregate investment reveals θ only imperfectly. As mentioned above, in the Supplementary Material we dispense with the common signal y and instead consider the case where entrepreneurs observe multiple private signals, all of which have both idiosyncratic and common errors. We also consider a variant that introduces unobserved common shocks to the entrepreneurs' cost of investment as an alternative source of noise in aggregate investment. In both cases, our main positive and normative results (Corollaries 1 and 2) remain intact, highlighting that the key for our results is the existence of a common source of noise, not the specific form of it.

A similar remark applies to other simplifying modeling choices. For example, we could have allowed the entrepreneurs that are not hit by a liquidity shock to participate in the financial market; we could further have allowed all entrepreneurs to observe a noisy signal of aggregate investment, or a noisy price signal, at the time they make their investment decisions.[5] What is essential for our results is only that the dispersion of information remains present both at the investment and at the trading stage.

Also note the "liquidity shock" need not be taken too literally. Its presence captures the more general idea that when an agent makes an investment decision, be him a start-up entrepreneur or the manager of a public company, he cares about the market valuation of his investment at some point in the life of the project. A start-up entrepreneur may worry about the price at which he will be able to do a future IPO; a corporate manager may be concerned about the price at which the company will be able to issue new shares. In what follows, we interpret λ broadly as a measure of the sensitivity of the firms' investment decisions to forecasts of future equity prices.[6]

Finally, note that there are no production spillovers and no direct payoff externalities of any kind: both the initial cost $(-k_i^2/2)$ and the eventual return on capital (θk_i) are independent of the investment decisions of other agents. The strategic complementarity that will be identified in Section 3.1 originates purely in an informational externality.

A benchmark with no informational frictions. Before we proceed, it is useful to examine what happens when the dispersion of information vanishes at the time of trading in the financial market. That is, suppose that all the information that is dispersed during period 1 (namely, the signals $\{x_i\}_{i\in[0,1/2]}$ and y) becomes commonly known in period 2. The fundamental θ then also becomes commonly known and the financial market clears if and only if $p = \theta$. It follows that the expected payoff of entrepreneur i in period 1 reduces to $\mathbb{E}[u_i|x_i, y] = \mathbb{E}[\theta|x_i, y]k_i - k_i^2/2$, which in

[5] See Section 6 for these extensions.

[6] See Baker, Stein and Wurgler (2003) for complementary evidence that the sensitivity of corporate investment to stock prices is higher in sectors with tighter financing constraints (which here can be interpreted as higher λ).

turn implies that equilibrium investment is given by

$$k_i = \mathbb{E}[\theta|x_i, y] = \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y}\mu + \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y}x_i + \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}y.$$

The key result here is that equilibrium investment is driven solely by first-order expectations regarding the fundamental and is independent of the intensity of the entrepreneurs' concern about financial prices (as measured by λ). This result does not require θ to be perfectly known in period 2. Rather, it applies more generally as long as the asymmetry of information about θ vanishes in period 2.[7] This case, which we henceforth refer to as the case with "no informational frictions," provides a convenient reference point for the rest of the analysis.

3 Equilibrium

Individual investment is described by a function $k : \mathbb{R}^2 \to \mathbb{R}$ so that $k(x, y)$ denotes the investment made by an entrepreneur with information (x, y). Aggregate investment is then a function of (θ, y):

$$K(\theta, y) = \int k(x, y)\, d\Phi(x|\theta), \tag{1}$$

where $\Phi(x|\theta)$ denotes the cumulative distribution function of x given θ. Since traders observe aggregate investment and are risk neutral, the unique market-clearing price is $p = \mathbb{E}[\theta|K]$, where the latter denotes the expectation of θ given the observed level of K.[8] Since K is determined by (θ, y), p is also a function of (θ, y). We thus define an equilibrium as follows.

Definition 1 *A (symmetric) equilibrium is an investment strategy $k(x, y)$ and a price function $p(\theta, y)$ that satisfy the following conditions:*

(i) for all (x, y),

$$k(x, y) \in \arg\max_k \mathbb{E}\left[\ (1-\lambda)\,\theta k + \lambda p(\theta, y)\, k - k^2/2 \mid x, y\ \right];$$

(ii) for all (θ, y),

$$p(\theta, y) = \mathbb{E}\left[\ \theta \mid K(\theta, y)\ \right],$$

where $K(\theta, y) = \int k(x, y)\, d\Phi(x|\theta)$.

[7] To clarify this point, consider an arbitrary information structure. Let $\mathcal{I}_{i,t}$ denote the information of agent i in period t. Impose that no agent has private information about θ in period 2 so that $\mathbb{E}[\theta|\mathcal{I}_{i,2}] = \mathbb{E}[\theta|\mathcal{I}_2]$ for all i. From market clearing we then have that $p = \mathbb{E}[\theta|\mathcal{I}_2]$. From the law of iterated expectations we then have that $\mathbb{E}[p|\mathcal{I}_{i,1}] = \mathbb{E}[\mathbb{E}[\theta|\mathcal{I}_2]|\mathcal{I}_{i,1}] = \mathbb{E}[\theta|\mathcal{I}_{i,1}]$ for all i. It follows that every entrepreneur chooses $k_i = \mathbb{E}[\theta|\mathcal{I}_{i,1}]$.

[8] Since the price is only a function of K and K is publicly observed, the price itself does not reveal any additional information. Therefore, we can omit conditioning on p. The case where p conveys additional information is examined in Section 6.

Condition (i) requires that the entrepreneurs' investment strategy be individually rational, taking as given the equilibrium price function. Condition (ii) requires that the equilibrium price be consistent with rational expectations and individual rationality on the traders' side, taking as given the strategy of the entrepreneurs.

As it is often the case in the literature, tractability requires that we restrict attention to equilibria in which the price function is linear.

Definition 2 *A linear equilibrium is an equilibrium in which* $p(\theta, y)$ *is linear in* (θ, y).

3.1 Endogenous complementarity

The optimality condition for the entrepreneurs' strategy can be written as

$$k(x, y) = \mathbb{E}[\ (1-\lambda)\,\theta + \lambda p(\theta, y)\ \mid x, y\ ]. \tag{2}$$

The linearity of $p(\theta, y)$ in (θ, y) and of $\mathbb{E}[\theta|x, y]$ in (x, y) then guarantees that the entrepreneurs' strategy is linear in (x, y); that is, there are coefficients $(\beta_0, \beta_1, \beta_2)$ such that

$$k(x, y) = \beta_0 + \beta_1 x + \beta_2 y. \tag{3}$$

By implication, aggregate investment is given by $K = \beta_0 + \beta_1\theta + \beta_2 y = \beta_0 + (\beta_1 + \beta_2)\theta + \beta_2\varepsilon$. Observing K is thus informationally equivalent to observing a Gaussian signal z with precision π_z, where

$$z \equiv \frac{K - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon \qquad \text{and} \qquad \pi_z \equiv \left(\frac{\beta_1 + \beta_2}{\beta_2}\right)^2 \pi_y. \tag{4}$$

Standard Gaussian updating then gives the expectation of θ given K as a weighted average of the prior and the signal z:

$$\mathbb{E}[\theta|K] = \frac{\pi_\theta}{\pi_\theta + \pi_z}\mu + \frac{\pi_z}{\pi_\theta + \pi_z}z.$$

Because market clearing in period 2 requires $p = \mathbb{E}[\theta|K]$, we conclude that the equilibrium price satisfies

$$p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y), \tag{5}$$

where

$$\gamma_0 \equiv \frac{\pi_\theta}{\pi_\theta + \pi_z}\mu - \frac{\pi_z}{\pi_\theta + \pi_z}\frac{\beta_0}{\beta_1 + \beta_2} \qquad \text{and} \qquad \gamma_1 \equiv \frac{\pi_z}{\pi_\theta + \pi_z}\frac{1}{\beta_1 + \beta_2}. \tag{6}$$

These results are summarized in the following lemma.

Lemma 1 *In any linear equilibrium, there are coefficients* $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)$ *such that*

$$k(x,y) = \beta_0 + \beta_1 x + \beta_2 y \quad \text{and} \quad p(\theta, y) = \gamma_0 + \gamma_1 K(\theta, y).$$

Moreover, $\gamma_1 > 0$ *if and only if* $\beta_1 + \beta_2 > 0$.

The key result here is the relation between K and p. Provided that high investment is "good news" for profitability (in the sense that a higher realization of K raises the traders' expectation of θ), financial prices increase with aggregate investment ($\gamma_1 > 0$). This in turn induces strategic complementarity in investment decisions. Indeed, when the entrepreneurs are choosing a higher level of investment, they are sending a positive signal to the financial market, thus increasing the price at $t = 2$. But then each entrepreneur's willingness to invest at $t = 1$ is higher when he expects a higher level of investment from other entrepreneurs, which means precisely that investment choices are strategic complements. We formalize these intuitions in the next result, which follows directly from replacing condition (5) into condition (2).

Lemma 2 *In any linear equilibrium, the investment strategy satisfies*

$$k(x,y) = \mathbb{E}[(1-\alpha)\kappa(\theta) + \alpha K(\theta, y) \mid x, y], \tag{7}$$

where $\alpha \equiv \lambda\gamma_1$ *and* $\kappa(\theta) \equiv \frac{(1-\lambda)\theta + \lambda\gamma_0}{1-\lambda\gamma_1}$.

Condition (7) can be interpreted as the best-response condition in the coordination game that emerges among the entrepreneurs for a *given* price function: it describes the optimal strategy for each individual entrepreneur as a function of his expectation of aggregate investment (the relevant summary of the strategy of other entrepreneurs), taking as given the impact of the latter on financial prices. The coefficient α then measures the degree of strategic complementarity in investment decisions: the higher α, the higher the slope of the best response of individual investment to aggregate investment, that is, the higher the incentive of entrepreneurs to align their investment choices. The function $\kappa(\theta)$, on the other hand, captures the impact of the fundamental on the individual return of investment for given K, normalized by $1 - \alpha$.[9]

A similar best-response condition characterizes the class of linear-quadratic games examined in Angeletos and Pavan (2007a), including the special case of Morris and Shin (2002). However, there are two important differences. First, while in those games the degree of strategic complementarity α is exogenously determined by the payoff structure, here it is endogenously determined as an integral

[9] This normalization serves two purposes. First, it identifies $\kappa(\theta)$ with the complete-information equilibrium level of investment in the game among the entrepreneurs, for *given* price function. Second, it ensures that the unconditional mean of investment is given by $\mathbb{E}k(x,y) = \mathbb{E}\kappa(\theta)$.

part of the equilibrium. Second, while in those games the degree of complementarity is independent of the information structure, here it actually originates in the dispersion of information. In fact, the complementarity in our setup is solely due to the informational content of aggregate investment. How much information aggregate investment conveys about θ determines the coefficient γ_1, which captures the sensitivity of prices to aggregate investment. In turn, the coefficient γ_1 pins down the value of α, which captures the degree of complementarity in the entrepreneurs' investment decisions. In the absence of informational frictions (the benchmark case examined in the previous section), aggregate investment provides no information to the traders, prices are thus independent of K, and the complementarity in investment decisions is absent. When instead information is dispersed, aggregate investment becomes a signal of θ, prices respond to aggregate investment and a complementarity in investment decisions emerges. The more informative aggregate investment is about θ, the stronger the complementarity.

Because the complementarity depends on the informational content of aggregate investment, which in turn depends on the entrepreneurs' strategies, to determine the equilibrium value of α we need to solve a fixed-point problem. Before doing so, we first show how this endogenous complementarity is instrumental in understanding the incentives entrepreneurs face in using their available sources of information.

Lemma 3 *In any linear equilibrium,*

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x} \frac{1}{1-\alpha}. \tag{8}$$

Therefore, provided that $\beta_1, \beta_2 > 0$, the sensitivity of the entrepreneurs' equilibrium strategy to the common signal relative to the idiosyncratic signal is higher the higher the equilibrium degree of complementarity.

Let us provide some intuition for this result. Consider an entrepreneur's best response to the strategy that other entrepreneurs follow, holding fixed the price function. Suppose that the other entrepreneurs' strategy is $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$, with $\beta_1, \beta_2 > 0$.[10] Aggregate investment is then given by $K(\theta, y) = \beta_0 + \beta_1 \theta + \beta_2 y$ and an agent's best predictor of aggregate investment is

$$\mathbb{E}[K|x, y] = \beta_0 + \beta_1 \mathbb{E}[\theta|x, y] + \beta_2 y.$$

[10] This condition means that investment responds positively to both signals. Since an entrepreneur's expectation of θ increases with either signal, it is quite natural to expect this condition to be satisfied in equilibrium. Below we will show that an equilibrium that satisfies this condition always exists and that this equilibrium is unique for λ small enough. For λ high enough, however, it is possible to construct equilibria in which entrepreneurs find it optimal to react negatively to a signal because they expect others to do the same.

The private signal x helps predict aggregate investment only through $\mathbb{E}[\theta|x,y]$, while the common signal y helps predict aggregate investment both through $\mathbb{E}[\theta|x,y]$ and directly through its effect on the term $\beta_2 y$. Therefore, relative to how much the two signals help predict the fundamental, the common signal y is a better predictor of aggregate investment than the private signal x. But now recall that a higher α means a stronger incentive for an individual entrepreneur to align his investment choice with that of other entrepreneurs. It follows that when α is higher entrepreneurs find it optimal to rely more heavily on the common signal y relative to the private signal x, for it is the former that best helps them align their choice with the choice of others.[11]

3.2 Equilibrium characterization

As noted earlier, completing the equilibrium characterization requires solving a fixed-point problem. On the one hand, how entrepreneurs use their available information depends on α, the endogenous complementarity induced by the response of prices to aggregate investment. On the other hand, how sensitive asset prices are to aggregate investment, and hence how strong α is, depends on how informative aggregate investment is about the fundamental, which in turn depends on how entrepreneurs use their available information in the first place. This fixed-point problem captures the essence of the two-way feedback between the real and the financial sector in our model. Its solution is provided in the following lemma.

Lemma 4 *There exist functions $F : \mathbb{R} \times (0,1) \times \mathbb{R}_+^3 \to \mathbb{R}$ and $G : \mathbb{R} \times (0,1) \times \mathbb{R}_+^3 \to \mathbb{R}^5$ such that the following are true:*

(i) *In any linear equilibrium, β_2/β_1 solves*

$$\frac{\beta_2}{\beta_1} = F\left(\frac{\beta_2}{\beta_1}; \lambda, \pi_\theta, \pi_x, \pi_y\right) \tag{9}$$

while $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1) = G(\beta_2/\beta_1; \lambda, \pi_\theta, \pi_x, \pi_y)$;

(ii) *Equation (9) has at least one solution at some $\beta_2/\beta_1 > \pi_y/\pi_x$;*

(iii) *For any $(\pi_\theta, \pi_x, \pi_x)$, there exists a cutoff $\bar{\lambda} = \bar{\lambda}(\pi_\theta, \pi_x, \pi_y) > 0$ such that (9) admits a unique solution if $\lambda < \bar{\lambda}$;*

(iv) *There exists an open set S such that (9) admits multiple solutions if $(\lambda, \pi_\theta, \pi_x, \pi_x) \in S$.*

The fixed-point problem that leads to the equilibrium characterization is set up in terms of the variable $b = \beta_2/\beta_1$, which represents the relative sensitivity of entrepreneurial investment to the two signals. Given b, we can determine the sensitivity of the price to aggregate investment γ_1.

[11] A similar property holds for the more general information structures considered in the Supplementary Material: a stronger complementarity shifts the use of information towards the signals whose errors are relatively more correlated across agents.

Given γ_1, we can then determine the complementarity α and then the sensitivity b of individual best responses to the two signals. These steps describe the mapping F used in Lemma 4 and provide the intuition for part (i) of the lemma: the fixed points of F identify all the linear equilibria of our economy. Parts (ii)-(iv) then characterize the fixed points of F, establishing than a linear equilibrium always exist and, although it is not always unique, it is unique for λ small enough.

The possibility of multiple equilibria for high values of λ is interesting for several reasons. First, it illustrates the potential strength of the two-way feedback between real and financial activity. Second, this multiplicity originates solely from an informational externality rather than from the more familiar payoff effects featured in coordination models of crises à la Diamond and Dybvig (1984) and Obstfeld (1996). Finally, this multiplicity can induce additional non-fundamental volatility in both real investment and financial prices.

However, the possibility of multiple equilibria is not central to our analysis. When there is a unique equilibrium, the key positive and normative predictions documented in Corollaries 1 and 2 below necessarily hold. When there are multiple equilibria, these predictions continue to hold for any equilibrium that satisfies the natural property that investment increases with both signals. For the rest of the paper we thus leave aside the possibility of multiplicity and focus on the case where the equilibrium is unique. The next proposition then summarizes some key equilibrium properties.

Proposition 1 *There always exists a linear equilibrium in which the following properties are true:*

(i) *Individual investment increases with both signals ($\beta_1, \beta_2 > 0$) and hence the equilibrium price increases with aggregate investment ($\gamma_1 > 0$);*

(ii) *The equilibrium degree of complementarity satisfies $0 < \alpha < 1$ and is increasing in λ;*

(iii) *The sensitivity of investment to the common signal relative to the private is higher than π_y/π_x and is increasing in λ.*

Moreover, λ small enough suffices for this equilibrium to be the unique linear equilibrium.

Part (i) guarantees that individual investment increases with both signals, which in turn ensures that high aggregate investment is necessarily "good news" for profitability and hence that α is positive. Part (ii) further establishes that α is higher the stronger the entrepreneurs' concern about financial prices. Combining this with Lemma 3 then gives part (iii), which spells out the implications for the equilibrium use of information.

3.3 Impact of fundamental and expectational shocks

To further appreciate the positive implications of informational frictions—and the complementarity thereof—it is useful to rewrite aggregate investment as

$$K = \beta_0 + (\beta_1 + \beta_2)\,\theta + \beta_2\varepsilon.$$

Aggregate investment thus depends on two types of shocks: *fundamental shocks*, captured by θ, and *expectational shocks*, captured by ε. How entrepreneurs use available information affects how investment respond to these shocks: the sensitivity to fundamentals is governed by the sum $\beta_1 + \beta_2$, while the sensitivity to expectational shocks is governed by β_2.

When information is dispersed, prices react positively to aggregate investment, the equilibrium degree of complementarity is positive, and hence the relative sensitivity to the common signal satisfies $\beta_2/\beta_1 > \pi_y/\pi_x$. In contrast, when there are no informational frictions, prices do not react to aggregate investment, the equilibrium degree of complementarity is zero, and hence $\beta_2/\beta_1 = \pi_y/\pi_x$. The following is then an immediate implication.

Corollary 1 (Main positive prediction) *The impact of expectational shocks relative to fundamental shocks is higher in the presence of informational frictions.*

This result is the key positive prediction of the paper: informational frictions amplify non-fundamental volatility relative to fundamental volatility; that is, they reduce the R-square of a regression of aggregate investment on expected profits. Importantly, because the equilibrium α increases with λ, this amplification effect is stronger the more entrepreneurs care about asset prices.[12]

Corollary 1 regards the *relative* impact of the two shocks. The next proposition reinforces this finding by examining the *absolute* impact of the two shocks.

Proposition 2 *There exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda}]$, there is a unique linear equilibrium and the following comparative statics hold:*

(i) *higher λ reduces $\beta_1 + \beta_2$, thus dampening the impact of fundamental shocks;*

(ii) *higher λ increases β_2, thus amplifying the impact of expectational shocks.*

The key intuition for these results is again the role of the complementarity for the equilibrium use of information. To see this, suppose for a moment that $\gamma_0 = 0$ and $\gamma_1 = 1$, meaning that $p(\theta, y) = K(\theta, y)$ in all states. The entrepreneurs' best response then reduces to

$$k(x, y) = \mathbb{E}\left[\ (1-\lambda)\,\theta + \lambda K(\theta, y)\ \mid x, y\ \right], \tag{10}$$

[12] This result extends to the more general information structures considered in the Supplementary Material. Because *all* signals have common errors, there are multiple "expectational shocks." We can then decompose the total variance of investment in two components: the one that is explained by θ (which defines "fundamental volatility"), and the one that is explained by the combination of all common errors (which defines "non-fundamental volatility"). We then show that the ratio of the latter to the former is higher with dispersed information (in which case $\alpha > 0$) than in the frictionless benchmark (in which case $\alpha = 0$).

so that the degree of complementarity now coincides with λ. One can then easily show that the unique solution to (10) is $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$, with

$$\beta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}\mu, \quad \beta_1 = \frac{\pi_x(1-\lambda)}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}, \quad \text{and} \quad \beta_2 = \frac{\pi_y}{\pi_\theta + \pi_x(1-\lambda) + \pi_y}.$$

It is then immediate that a higher λ increases the sensitivity to the prior (captured by β_0) and the sensitivity to the common signal (captured by β_2), while it decreases the sensitivity to the private signal (captured by β_1). We have already given the intuition for the result that a stronger complementarity amplifies the reliance on the common signal and dampens the reliance on the private signal. That it also increases the reliance on the prior is for exactly the same reason as for the common signal: the prior is a relative good predictor of others' investment choices.

However, note that the average return on investment coincides with the mean of θ, irrespective of λ. This is because the average price must equal the mean of θ, for otherwise the traders would make on average non-zero profits, which would be a contradiction. But then the average investment must also be equal to the average of θ, that is, $\beta_0 + (\beta_1 + \beta_2)\mu$ must equal μ. It is then immediate that, because a higher λ increases β_0, it also reduces the sum $\beta_1 + \beta_2$. In simple words, investment is less sensitive to changes in fundamentals simply because the complementarity strengthens the anchoring effect of the prior.

These intuitions would be exact if $\gamma_0 = 0$ and $\gamma_1 = 1$, or more generally if these coefficients were exogenous to λ. In our model, the price is an increasing function of aggregate investment, but the coefficients γ_0 and γ_1 depend on λ. This explains why these intuitions are incomplete and why the absolute effects documented in Proposition 2 hold only for a subset of the parameter space. However, the prediction regarding the amplification of the relative impact of non-fundamental shocks (Corollary 1) holds true more generally.

4 Constrained efficiency

The analysis so far has focused on the positive properties of the equilibrium. We now study its normative properties by examining whether there is an allocation that, given the underlying information structure, leads to higher welfare.

The question of interest here is whether society can do better, relative to equilibrium, by having the agents use their available information in a different way—not whether society can do better by giving the agents more information. We thus adopt the same constrained efficiency concept as in Angeletos and Pavan (2007a,b): we consider the allocation that maximizes ex-ante welfare subject to the sole constraint that the choice of each agent must depend only on the information available to that agent. In other words, we let the planner dictate how agents use their available information, but we do not let the planner transfer information from one agent to another. In so doing, we momentarily disregard incentive constraints; later on we will identify tax systems that implement the efficient allocation as an equilibrium.

Note that the payments in the financial market represent pure transfers between the entrepreneurs and the traders and therefore do not affect ex-ante utility.[13] We can thus focus on the investment strategy and define the efficient allocation as follows.

Definition 3 *The efficient allocation is a strategy $k(x, y)$ that maximizes ex-ante utility*

$$\mathbb{E}u = \int \left\{ \int \tfrac{1}{2} \left[(1-\lambda)\,\theta k\,(x,y) - \tfrac{1}{2} k\,(x,y)^2 \right] d\Phi(x|\theta) + \tfrac{1}{2}\left[\theta \lambda K\,(\theta, y)\right] \right\} d\Psi(\theta, y) \tag{11}$$

with $K\,(\theta, y) = \int k\,(x,y)\, d\Phi(x|\theta)$.

Condition (11) gives ex-ante utility for an arbitrary strategy. The first term in square brackets is the payoff of an entrepreneur with information (x, y); the second term in square brackets is the payoff of a trader when aggregate investment is $K(\theta, y)$; finally Ψ denotes the cumulative distribution function of the joint distribution of (θ, y). Note that the transfer of capital from the entrepreneurs that are hit by the liquidity shock to the traders does not affect the return to investment. It follows that (11) can be rewritten compactly as[14]

$$\mathbb{E}u = \tfrac{1}{2}\mathbb{E}[V\,(k\,(x,y)\,,\theta)\,] = \tfrac{1}{2}\mathbb{E}[\mathbb{E}[V\,(k\,(x,y)\,,\theta)\,|x,y]],$$

where $V\,(k, \theta) \equiv \theta k - \frac{1}{2}k^2$. From the society's viewpoint, λ is irrelevant and it is as if the entrepreneurs' payoffs are $V(k,\theta)$. It is then obvious that a strategy $k(x, y)$ is efficient if and only if, for almost all x and y, $k(x, y)$ maximizes $\mathbb{E}[V(k,\theta)|x, y]$. The following result is then immediate.

Proposition 3 *The efficient investment strategy is given by*

$$k\,(x,y) = \mathbb{E}\,[\theta|x,y] = \delta_0\mu + \delta_1 x + \delta_1 y,$$

where

$$\delta_0 \equiv \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_1 \equiv \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_2 \equiv \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}.$$

[13] By ex-ante utility, we mean before the realization of any random variable, including those that determine whether an agent will be an entrepreneur or a trader. However, note that, because utility is transferable, any strategy $k(x, y)$ that maximizes ex-ante utility also maximizes any weighted average of the expected utility of an entrepreneur and of a trader. By the same token, any strategy $k(x, y)$ that improves upon the equilibrium in terms of ex-ante utility can yield a Pareto improvement. It suffices, for example, to let the entrepreneurs and the traders continue to trade at a price $p(\theta, y) = \mathbb{E}(\theta|K(\theta, y))$, where $K(\theta, y) = \int k\,(x, y)\, d\Phi(x|\theta)$.

[14] It suffices to substitute the expression for $K\,(\theta, y)$ in (11).

Note that the efficient strategy would have coincided with the equilibrium strategy if it were not for informational frictions. It follows that our earlier positive results admit a normative interpretation.[15]

Corollary 2 (Main normative prediction) *In the presence of informational frictions, the impact of expectational shocks relative to fundamental shocks is inefficiently high.*

5 Policy implications

Having identified a potential source of inefficiency, we now analyze the effect of different policies. First, we consider interventions "during the fact," by which we mean interventions in the financial market at $t = 2$, when uncertainty about θ has not been resolved yet. Next, we consider policies "after the fact," by which we mean policies contingent on information that becomes public at $t = 3$, after uncertainty about θ has been resolved. In both cases, we impose that the government has no informational advantage vis-a-vis the private sector, which is our preferred benchmark for policy analysis. At the end of this section, however, we also consider situations where the government can directly affect the information available to the agents.

5.1 Interventions "during the fact": price stabilization

We start by considering policies aimed at reducing asset-price volatility. In particular, suppose the government imposes a proportional tax τ on financial trades (purchases of capital) at date 2. This tax can depend on the price, which is public information. For simplicity, it takes the following linear form:

$$\tau(p) = \tau_0 + \tau_1 p, \tag{12}$$

where (τ_0, τ_1) are scalars. Tax revenues are rebated as a lump sum.

The equilibrium price in the financial market is now given by $p = \mathbb{E}[\theta|K] - (\tau_0 + \tau_1 p)$; equivalently,

$$p = \frac{1}{1+\tau_1}\left(\mathbb{E}[\theta|K] - \tau_0\right) = \frac{1}{1+\tau_1}\left(\gamma_0 + \gamma_1 K - \tau_0\right), \tag{13}$$

where γ_0 and γ_1 are given, as before, by (6). When the tax is pro-cyclical (i.e., $\tau_1 > 0$), its effect is to dampen the response of asset prices to the traders' expectation of θ, and thereby their response to the news contained in aggregate investment. In equilibrium, this tends to reduce the degree of complementarity in investment decisions. To see this more clearly, note that the degree

[15] Corollary 2 presumes that the equilibrium is unique. When there are multiple equilibria, the result holds for any equilibrium in which $\beta_1, \beta_2 > 0$. Since the efficient allocation satisfies $\beta_1, \beta_2 > 0$, this also ensures that no equilibrium is efficient.

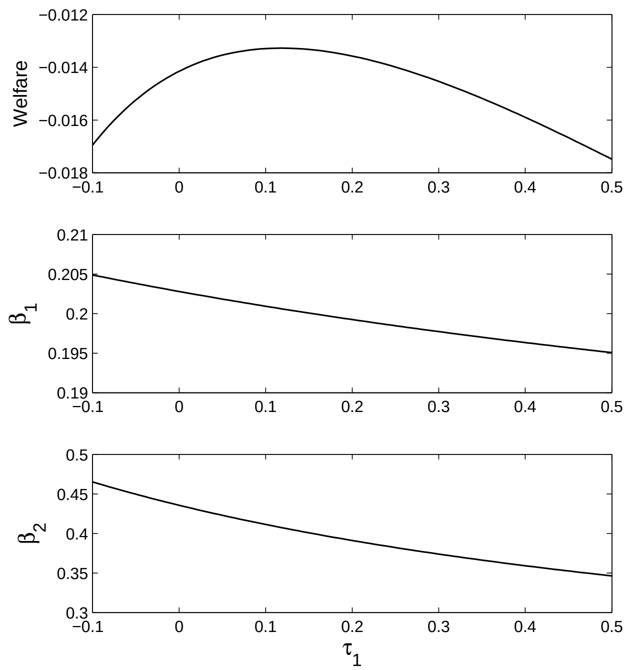

Figure 1: The impact of price-stabilization policies.

of complementarity is now given by

$$\alpha = \frac{\lambda \gamma_1}{1 + \tau_1}.$$

If γ_1 were exogenous, it would be immediate that α decreases with τ_1. But if α falls, we know from Lemma 3 that β_2/β_1, the relative weight on common sources of information, must also fall. This in turn means that aggregate investment becomes more informative about θ, so that γ_1 increases, counteracting the direct effect of τ_1 on α. However, one can show that, at least as long as the equilibrium is unique, the direct effect dominates, guaranteeing that the *equilibrium* degree of complementarity decreases with τ_1.[16]

We conclude that a higher τ_1, by reducing the degree of complementarity, necessarily reduces the relative impact of expectational shocks. However, by reducing the overall sensitivity of prices to all sources of variation in investment, a higher τ_1 also reduces the impact of fundamental shocks. As argued in the previous section, in the absence of policy intervention, investment is excessively sensitive to expectational shocks and insufficiently sensitive to fundamental shocks. It follows that the welfare consequences of the tax are ambiguous: while reducing the impact of expectational shocks improves efficiency, reducing the impact of fundamental shocks has the opposite effect.

These intuitions are illustrated in Figure 1 where for each value of τ_1, the value of τ_0 is chosen

[16]This follows from an argument similar to the one that establishes that α is monotonic in λ.

optimally to maximize welfare. The top panel depicts the difference in welfare under the stabilization policy considered here and under the constrained efficient allocation; the bottom panels depict the sensitivity to expectational shocks β_2 and to fundamental shocks $(\beta_1 + \beta_2)$.[17] The figure is drawn for a baseline set of parameters: $\pi_\theta = \pi_x = \pi_y = 1$ and $\lambda = 0.5$. However, its qualitative features are robust across a wide set of parametrizations. In particular, we have randomly drawn 10,000 parameter vectors $(\lambda, \pi_\theta, \pi_x, \pi_y)$ from $(0,1) \times \mathbb{R}^3_+$. For each such vector, we have found that the optimal τ_1 is positive and it induces a lower β_2 and a lower $\beta_1 + \beta_2$ as compared to the equilibrium without policy, reflecting the trade off discussed above.

While these numerical results, which span the entire parameter space, reveal that the optimal policy always involves a strictly positive degree of price stabilization (i.e., $\tau_1 > 0$), we have not been able to establish this result formally. However, it is easy to show that full price stabilization (i.e., $\tau_1 \to \infty$) is never optimal. In this limit, prices cease to react to aggregate investment, the strategic complementarity disappears, and equilibrium investment reduces to $k(x,y) = (1-\lambda)\,\mathbb{E}[\theta|x,y]$. By implication, the relative sensitivity of investment to expectational shocks $\beta_2/(\beta_1+\beta_2)$ is at its efficient level, but its overall sensitivity to the fundamental is λ times lower than at the efficient level. At this point, a marginal increase in the relative sensitivity implies only a second-order welfare loss, while a marginal increase in the overall sensitivity implies a first-order welfare gain. It follows that it is never optimal to fully stabilize the price.

Proposition 4 *A tax that stabilizes prices can increase welfare; however, a tax that completely eliminates price volatility is never optimal.*

5.2 Interventions "after the fact": corrective taxation

Suppose now that the government imposes a proportional tax τ on asset holdings in period 3. The tax is now paid by the entrepreneurs not hit by the liquidity shock and by the traders who acquired capital in period 2. The advantage of introducing a tax in period 3 is that the tax rate τ can now be made contingent on all information which is publicly available in that period, including K and θ. We focus on linear tax schemes of the form

$$\tau(\theta, K) = \tau_0 + \tau_1\theta + \tau_2 K, \tag{14}$$

where (τ_0, τ_1, τ_2) are scalars; tax revenues are again rebated in a lump-sum fashion. The following result shows that these simple tax schemes can implement the constrained efficient allocation.

[17] Note that τ_0 affects the unconditional average of $k(x,y)$, but has no effect on the sensitivity of investment to the signals x and y, i.e., on β_1 and β_2. We henceforth concentrate on τ_1.

Proposition 5 *There exists a unique linear tax scheme that implements the efficient allocation as an equilibrium. The optimal tax satisfies* $\tau_0 < 0$, $\tau_1 < 0$ *and* $\tau_2 > 0$.

The intuition behind this result is that the government can control the degree of strategic complementarity perceived by the agents by appropriately designing the contingency of the marginal tax rate τ on aggregate investment: the higher the elasticity τ_2 of the marginal tax rate with respect to K, the lower the degree of complementarity in investment choices α and the lower the sensitivity of equilibrium investment to common noise relative to idiosyncratic noise. This effect is analogous to that of the stabilization policies discussed above. However, the government now has an extra instrument available: the elasticity τ_1 of the tax to the realized fundamental. Through τ_2 the government can thus induce the optimal relative sensitivity to expectational shocks $\beta_2/(\beta_1+\beta_2)$ while, at the same time, adjust τ_1 to obtain the optimal absolute sensitivities to each shock.[18]

Although this result does not require any informational advantage on the government's side, it assumes that the government observes perfectly the fundamental θ and the agents' capital holdings at the time taxes are collected. However, the result easily extends to situations where these quantities are observed with measurement error. In particular, suppose that in stage 3 the government only observes $\tilde{\theta} = \theta + \epsilon$ and $\tilde{s}_i = s_i + \eta_i$ for each i, where s_i is the capital holding of agent i, while ϵ and η_i are measurement errors, possibly correlated with one another, but independent of θ and of the agents' information in period 2. Then let $\tilde{K}$ be the cross-sectional mean of $\tilde{s}_i$ (i.e., the government's measure of aggregate investment) and consider a proportional tax on $\tilde{s}_i$ of the following form: $\tau(\tilde{\theta}, \tilde{K}) = \tau_0 + \tau_1\tilde{\theta} + \tau_2\tilde{K}$. It is then easy to check that there continues to exist a unique set of coefficients (τ_0, τ_1, τ_2) that implement the efficient allocation as an equilibrium and that these coefficients continue to satisfy $\tau_1 < 0 < \tau_2$.

To recap, the key insight here is that the government can use the contingency of the tax rate on public signals of θ and K that will be revealed at stage 3 to achieve efficiency in the decentralized use of information during stage 1. Although this information becomes available only after all investment decisions are sunk, by promising a specific policy response to this information the government is able to manipulate how entrepreneurs use their available sources of information when making their investment decisions, even if it cannot directly monitor these sources of information.[19]

[18] The optimal τ_0 is then chosen to induce the optimal level of unconditional average investment. Similar tax schemes implement the efficient investment strategy in all the extensions considered in Section 6.

[19] These intuitions, and the implementation result in Proposition 5, build on Angeletos and Pavan (2007b). This paper considers optimal policy within a rich, but abstract, class of economies with dispersed information on correlated values. See also Lorenzoni (2007) for monetary policy in a business-cycle model with dispersed information on underlying productivity.

5.3 Optimal release of information

We now turn to policies that affect the information available to the agents. This seems relevant given the role of the government in collecting (and releasing) macroeconomic data.

To capture this role, suppose that in stage 2 traders can only observe average investment with noise, that is, they observe

$$\tilde{K} = K + \eta,$$

where η is aggregate measurement error, which is a random variable, independent of all other shocks, with mean zero and variance $1/\pi_\eta$.[20] Suppose now that the government can affect the precision π_η of the macroeconomic data available to financial traders. By changing π_η the government determines the weight that traders assign to $\tilde{K}$ when estimating future profitability. This is another channel by which the government is able to affect the degree of strategic complementarity in investment decisions.

Indeed, the choice of π_η is formally equivalent to the choice of τ_1 in the setup with a tax on financial transactions (Section 5.1). For each value π_η of the precision of the signal about aggregate activity, there is a value τ_1 of the tax elasticity that induces the same equilibrium strategy, and vice versa. To see this, note that in any linear equilibrium, the observation of $\tilde{K} = \beta_0+(\beta_1+\beta_2)\theta+\beta_2\varepsilon+\eta$ is informationally equivalent to the observation of a signal

$$z \equiv \frac{\tilde{K}-\beta_0}{\beta_1+\beta_2} = \theta + \frac{\beta_2}{\beta_1+\beta_2}\varepsilon + \frac{1}{\beta_1+\beta_2}\eta,$$

with precision

$$\pi_z = \left(\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^2 \pi_y^{-1} + \left(\frac{1}{\beta_1+\beta_2}\right)^2 \pi_\eta^{-1}\right)^{-1}.$$

The equilibrium price is then given by $p(\theta, y, \eta) = \gamma_0 + \gamma_1[K(\theta, y) + \eta]$, with γ_0 and γ_1 given by (6), and hence the degree of strategic complementarity remains equal to $\alpha \equiv \lambda\gamma_1$, as in the baseline model. By changing the value of π_η, the government can then directly manipulate γ_1 and thus the degree of strategic complementarity perceived by the entrepreneurs.[21]

We conclude that the choice of π_η is subject to the same trade-offs emphasized for the choice of τ_1: decreasing π_η reduces the relative response of investment to expectational shocks, but it also reduces its response to fundamental shocks. The results of Section 5.1 then imply that an intermediate degree of release of macroeconomic data may be optimal even when the cost of collecting such data is zero.[22]

[20] The equilibrium characterization for this case is a straightforward extension of the baseline case.

[21] See the Supplementary Material for the proof of this claim.

[22] Note, however, that this holds only as long as the equilibrium is inefficient. If, instead, the policies considered in Section 5.2 are in place, thus guaranteeing that the equilibrium is efficient, then a higher π_η is always welfare

Finally, one could consider policies which affect directly the agents' information regarding the fundamental θ. In particular, the government can collect some information about θ in period 1 and decide whether to disclose this information to the entrepreneurs, or to both the entrepreneurs and the traders. In the first case, the policy corresponds to an increase in the precision of the signal y in the baseline model. Although entrepreneurs have a more precise estimate of the fundamental, this information is not shared with the traders. Therefore, this policy could exacerbate the asymmetry of information and could magnify the feed-back effects between investment and asset prices, with possible negative consequences on social welfare.

In the second case, instead, the policy corresponds to an increase in the precision of the common prior in the baseline model. This policy is socially beneficial for two reasons: first, it improves the quality of the information available to the entrepreneurs and hence it permits them to better align their decisions to the fundamental. Second, it reduces the reliance of financial markets on the endogenous signal K in their estimate of the fundamental. This second effect tends to reduce the degree of strategic complementarity in investment decisions, and hence also the discrepancy between equilibrium and efficient allocations. Both effects then contribute to higher welfare.

6 Extensions

Our analysis has identified a mechanism through which the dispersion of information induces complementarity in real investment choices, amplification of non-fundamental disturbances, and inefficiency of market outcomes, all at once. In the baseline model, we have made a number of assumptions to illustrate this mechanism in the simplest possible way. In particular, we have assumed that the traders' demand for installed capital is perfectly elastic, that entrepreneurs who are not hit by the liquidity shock do not trade in the financial market, and that the traders' valuation of the asset coincides with that of the entrepreneurs. In this section, we relax each of these assumptions.

We first extend the model to allow for the traders' demand for capital to be downward sloping. This extension is interesting because it introduces a potential source of strategic substitutability in the entrepreneurs' investment decisions: when aggregate investment is higher, the supply of installed capital in the financial market is also higher, putting a downward pressure on asset prices and lowers the ex-ante incentive to invest.

In a second extension, we allow entrepreneurs not hit by the liquidity shock to participate in the financial market. This extension is interesting for two reasons: first, it allows for some of the entrepreneurs' information to be aggregated in the financial market; second, it introduces a non-trivial allocative role for prices.

improving.

Although some interesting differences arise, the key positive and normative predictions of the paper (Corollaries 1 and 2) remain valid in both extensions: as long as the dispersion of information does not completely vanish in the financial market, the signaling effect of aggregate investment continues to be the source of amplification and inefficiency in the response of the equilibrium to common sources of noise.

Finally, we consider a variant that introduces shocks to the financial-market valuation of the installed capital. This variant brings the paper closer to the recent literature on "mispricing" and "bubbly" asset prices. It also helps clarify that the details of the information structure we assumed in the baseline model are not essential: any source of common noise in the information that aggregate investment conveys about the underlying fundamentals opens the door to amplification and inefficiency.

6.1 The supply-side effect of capital: a source of strategic substitutability

We modify the benchmark model as follows. The net payoff of trader i, who buys q_i units of capital at the price p, is now given by

$$u_i = (\theta - p)\, q_i - \frac{1}{2\phi} q_i^2, \tag{15}$$

where ϕ is a positive scalar. The difference with the benchmark model is the presence of the last term in (15), which represents a transaction cost associated to the purchase of q_i units of capital. A convex transaction cost ensures a finite price elasticity for the traders' demand, which is now given by $q(p,K) = \phi\left(\mathbb{E}\left[\theta|K\right] - p\right)$. The parameter ϕ captures the price elasticity of this demand function and our benchmark model corresponds to the special case where the demand is infinitely elastic, i.e. $\phi \to \infty$.[23] As in the benchmark model, in any linear equilibrium the traders' expectation of θ is given by $\mathbb{E}[\theta|K] = \gamma_0 + \gamma_1 K$, for some coefficients γ_0 and γ_1. However, unlike in the benchmark model, the equilibrium price does not coincide with $\mathbb{E}[\theta|K]$. Market clearing now requires that $q(p,K) = \lambda K$, so the equilibrium price is

$$p = \mathbb{E}[\theta|K] - \tfrac{\lambda}{\phi} K = \gamma_0 + \left(\gamma_1 - \tfrac{\lambda}{\phi}\right) K. \tag{16}$$

It follows that aggregate investment has two opposing effects on the price of installed capital, p. On the one hand, it raises the traders' expectation of θ, thereby pushing the price up. On the other hand, it raises the supply of capital, thereby pulling the price down. The strength of these two effects determines whether investment choices are strategic complements or substitutes.

[23] A more familiar way of introducing a finitely elastic demand is to assume risk aversion. The alternative we use here captures the same key positive and normative properties—namely, demands are finitely elastic and individual payoffs are concave in own portfolio positions—but has the advantage of keeping the analysis tractable by making the elasticity of demands invariant to the level of uncertainty.

Proposition 6 ***(i)*** *In any linear equilibrium, the investment strategy satisfies*

$$k(x,y) = \mathbb{E}\left[\ (1-\alpha)\,\kappa(\theta) + \alpha K(\theta,y)\ \mid x,y\ \right], \tag{17}$$

with $\alpha \equiv \lambda\gamma_1 - \lambda^2/\phi$ *and* $\kappa(\theta) \equiv \frac{(1-\lambda)\theta+\lambda\gamma_0}{1-\lambda\gamma_1+\lambda^2/\phi}$.

(ii) λ *small enough suffices for the equilibrium to be unique, for investment to increase with* θ*, and for* γ_1 *to be positive.*

The degree of complementarity α is now the sum of two terms. The first term $\lambda\gamma_1$ captures the, by now familiar, informational effect of investment on asset prices documented in the benchmark model. The second term, $-\lambda^2/\phi$, captures the simple supply-side effect that emerges once the demand for the asset is finitely elastic. If either the information contained in aggregate investment is sufficiently poor (low γ_1) or the price elasticity of demand is sufficiently low (low ϕ), investment choices become strategic substitutes ($\alpha < 0$). However, the question of interest here is not whether investment choices are strategic complements or substitutes, but how the positive and normative properties of the equilibrium are affected by the dispersion of information. In this respect, the implications that emerge in this extension are essentially the same as in the benchmark model.

First, consider the positive properties of the equilibrium. Lemma 3 immediately extends to the modified model: equilibrium investment satisfies $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$ with

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x}\frac{1}{1-\alpha}. \tag{18}$$

Provided that investment increases with both signals, then aggregate investment is necessarily good news for θ (i.e., $\gamma_1 > 0$), in which case Proposition 6 implies that $\alpha > -\lambda^2/\phi$. In contrast, when there are no informational frictions, the equilibrium price is simply $p = \theta - (\lambda^2/\phi)K$, so that $\alpha = -\lambda^2/\phi$. It follows that the dispersion of information increases the value of α and hence amplifies the impact of common expectational shocks relative to that of fundamental shocks, even if α happens to be negative. We conclude that Corollary 1, which summarizes the key positive predictions of the model, continues to hold.

Next, consider the normative properties. Because of the convexity of the transaction costs, it is necessary for efficiency that all traders take the same position in the financial market: $q_i = \lambda K$ for all $i \in (1/2, 1]$. Ex-ante utility then takes the form

$$\begin{aligned}\mathbb{E}u &= \int \left\{ \tfrac{1}{2} \int \left[(1-\lambda)\,\theta k(x,y) - \tfrac{1}{2} k(x,y)^2 \right] d\Phi(x|\theta) + \tfrac{1}{2}\left[\theta\lambda K(\theta,y) - \tfrac{1}{2\phi}[\lambda K(\theta,y)]^2 \right] \right\} d\Psi(\theta,y) \qquad (19) \\ &= \frac{1}{2}\mathbb{E}\left\{ -\frac{k^2}{2} + \theta k - \frac{1}{2\phi}\left(\lambda K\right)^2 \right\}\end{aligned}$$

and the efficient investment strategy is the function $k(x, y)$ that maximizes (19).

Proposition 7 *The efficient investment strategy is the unique linear solution to*

$$k(x, y) = \mathbb{E}\left[(1 - \alpha^*)\, \kappa^*(\theta) + \alpha^* K(\theta, y) \mid x, y\ \right], \tag{20}$$

where $\alpha^* \equiv -\lambda^2/\phi < 0$, $\kappa^*(\theta) \equiv \theta / \left(1 + \lambda^2/\phi\right)$, *and* $K(\theta, y) = \int k(x, y) d\Phi(x|\theta)$.

To understand this result, note that the social return to investment is given by $(1 - \lambda)\theta + \lambda(\theta - \lambda K/\phi) = \theta - \lambda^2 K/\phi$. The new term, relative to the benchmark model, is $-\lambda^2 K/\phi$ and it reflects the cost associated with transferring λK units of the asset from the entrepreneurs to the traders. If information were complete, efficiency would require that each agent equates his marginal cost of investing to the social return to investment, which would give $k = \theta - \lambda^2 K/\phi$.[24] The analogue under incomplete information is that each agent equates the marginal cost to the expected social return:

$$k(x, y) = \mathbb{E}\left[\ \theta - \left(\lambda^2/\phi\right) K(\theta, y)\ \middle|\ x, y\ \right]. \tag{21}$$

Rearranging this condition gives (20).

The key finding here is that the introduction of downward sloping demands has a symmetric effect on the private and social returns to investment. This is simply because the negative pecuniary externality caused by the higher supply of capital perfectly reflects the social cost associated with having traders absorb this additional capital. As a result, it is only the informational effect that generates a discrepancy between the equilibrium and the efficient allocation.

As in the benchmark model, this discrepancy manifests itself in the response of equilibrium to expectational and fundamental shocks. Indeed, while equilibrium investment satisfies (18), efficient investment satisfies $k(x, y) = \beta_0^* + \beta_1^* x + \beta_2^* y$ with

$$\frac{\beta_2^*}{\beta_1^*} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha^*}.$$

Because in any equilibrium in which $\beta_1, \beta_2 > 0$ the complementarity satisfies $\alpha^* < \alpha < 1$, the relative sensitivity of the equilibrium strategy to common noise is inefficiently high. We conclude that Corollary 2, which summarizes the key normative predictions of the model, continues to hold.[25]

[24] Note that under full information the optimal level of investment would be equal to $\kappa^*(\theta)$.

[25] As common in competitive environments, there are other forms of pecuniary externalities that could induce strategic substitutability in the entrepreneurs' investment decisions, even with a perfectly elastic demand for capital. For example, suppose that, in order to complete their investment, entrepreneurs need to purchase certain inputs whose aggregate supply is imperfectly elastic (e.g., labor or land). Higher aggregate investment then implies higher aggregate demand for these inputs, and hence higher input prices and lower entrepreneurial returns, once again inducing strategic substitutability in the entrepreneurs' investment choices. However, such pecuniary externalities do not, on their own, cause discrepancies between private and social returns. Indeed, it is easy to construct variants of

6.2 Information aggregation through prices

The analysis so far has imposed that the entrepreneurs who are not hit by the liquidity shock can not access the financial market. Apart from being unrealistic, this assumption rules out the possibility that the price in the financial market aggregates, at least partly, the information that is dispersed among the entrepreneurs. To address this possibility, in this section we extend the analysis by allowing entrepreneurs not hit by the liquidity shock to participate in the financial market.

To guarantee downward sloping demands, we assume that traders and entrepreneurs alike incur a transaction cost for trading in the financial market of the same type as in the previous section.[26] Thus, the payoff of an entrepreneur i who is not hit by a liquidity shock, has invested k_i units in the first period, and trades q_i units in the second period, is given by

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \frac{1}{2\phi}q_i^2 + \theta\left(k_i + q_i\right),$$

while the payoff of a trader i is given by (15), as in the previous section.

Because the observation of K in the second period perfectly reveals θ to every entrepreneur,[27] their demand for the asset in the second period reduces to $q_E = \phi\left(\theta - p\right)$. The demand of the traders, on the other hand, is given by $q_T = \phi\left(\mathbb{E}\left[\theta|K,p\right] - p\right)$. Note that traders now form their expectation of θ based on K and on the information revealed by the equilibrium price p.[28] Because the aggregate demand for the asset is $\frac{1}{2}\left(1-\lambda\right)q_E + \frac{1}{2}q_T$ and the aggregate supply is $\frac{1}{2}\lambda K$, market clearing implies

$$p = \tfrac{1}{2-\lambda}\mathbb{E}\left[\theta|K,p\right] + \tfrac{1-\lambda}{2-\lambda}\theta - \tfrac{1}{\phi(2-\lambda)}\lambda K.$$

It follows that the joint observation of K and p perfectly reveals θ to the traders as well. The asymmetry of information thus vanishes and the equilibrium price satisfies $p = \theta - \frac{1}{\phi(2-\lambda)}\lambda K$. As in the previous section, this is just the social return to investment, adjusted for the fact that the total capital of the entrepreneurs hit by the liquidity shock $(\lambda K/2)$ is now equally distributed among the traders and the entrepreneurs not hit by the liquidity shock. Because the equilibrium price coincides with the social return to investment it follows that the equilibrium is efficient.

This result is no different from what we established for the frictionless benchmark at the end

the model that capture such sources of strategic substitutability while retaining the property that the informational effect of aggregate investment is the sole source of amplification and inefficiency, as in the example analyzed here.

[26] We assume that the entrepreneurs hit by the liquidity shock do not pay the transaction cost for the units of the asset that they *have* to sell in the second period; this simplification has no impact on the results.

[27] This presumes that entrepreneurs use their private information when deciding how much to invest (i.e. $\beta_1 \neq 0$), which is indeed true in equilibrium.

[28] In the benchmark model, as well as in the extension examined in the previous section, we did not condition on the information revealed by the equilibrium price simply because all agents trading voluntary in the financial market had symmetric information.

of Section 2: if the dispersion of information vanishes at the time of trade in the financial market, equilibrium investment is driven merely by first-order expectations of θ and is efficient. However, this result hinges on the equilibrium price perfectly revealing θ. To make this clear, in the subsequent analysis we introduce an additional source of noise, which prevents prices from being perfectly revealing.

Assume that the cost of trading for the entrepreneurs is subject to a shock ω, that is revealed to them at the time they trade but which is not observed by the traders. In particular, the payoff of an entrepreneur not hit by the liquidity shock is now given by

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \omega q_i - \frac{1}{2\phi}q_i^2 + \theta\left(k_i + q_i\right),$$

where ω is assumed to be independent of all other random variables, with $\mathbb{E}[\omega] = 0$ and $Var[\omega] = \sigma_\omega^2 \equiv \pi_\omega^{-1}$.

In what follows, we look at linear rational expectations equilibria; we continue to denote the investment strategy by $k\left(x,y\right)$ and we denote by $p\left(\theta,y,\omega\right)$ the equilibrium price. Because the observation of aggregate investment in the second period continues to reveal θ to the entrepreneurs (but not to the traders), asset demands can be written as $q_E = \phi\left(\theta - \omega - p\right)$ for the entrepreneurs and $q_T = \phi\left(\mathbb{E}\left[\theta|K,p\right] - p\right)$ for the traders. Market clearing then implies that the equilibrium price is

$$p = \tfrac{1}{2-\lambda}\mathbb{E}\left[\theta|K,p\right] + \tfrac{1-\lambda}{2-\lambda}\left(\theta - \omega\right) - \tfrac{1}{\phi(2-\lambda)}\lambda K. \tag{22}$$

Once again, the price is a weighted average of the traders' and of the entrepreneurs' valuation of the asset, net of trading costs. However, because the shock ω is not known to the traders, the price no longer perfectly reveals θ, ensuring that the informational effect of K on the traders' expectation of the fundamental reemerges. This effect is captured by the first term in the right-hand-side of (22). At the same time, the supply-side effect of K is also present and is captured by the last term in (22).

While the supply-side effect induces strategic substitutability, the informational effect induces complementarity. In any rational expectations equilibrium in which the price is linear in (θ, y, ω), $\mathbb{E}\left[\theta|K,p\right]$ is the projection of θ on (K,p) and, by (22), the price p can be expressed as a linear combination of (K,θ,ω). It follows that, for any linear equilibrium, there exist coefficients $(\gamma_0,\gamma_1,\gamma_2,\gamma_3)$ such that[29]

$$\mathbb{E}\left[\theta|K,p\right] = \gamma_0 + \gamma_1 K + \gamma_2\theta + \gamma_3\omega. \tag{23}$$

[29]Note that γ_1, which captures the effect of K on the traders' expectation of θ, now combines the information that is directly revealed to the traders by the observation of K with the information that is revealed to them through the observation of the equilibrium price.

Using (23), (22), and the fact that the private return to investment is the expectation of $(1-\lambda)\,\theta+\lambda p$, we reach the following characterization result.

Proposition 8 ***(i)*** *In any linear equilibrium, the investment strategy satisfies*

$$k\,(x,y)=\mathbb{E}[(1-\alpha)\,\kappa(\theta)+\alpha K\,(\theta,y)\ \mid x,y],$$

where $\alpha=\frac{\lambda}{2-\lambda}\gamma_1-\frac{\lambda^2}{\phi(2-\lambda)}$ *and* $\kappa(\theta)=\frac{\lambda\gamma_0+[2(1-\lambda)+\lambda\gamma_2]\theta}{2-\lambda-\lambda\gamma_1+\lambda^2/\phi}$.

(ii) *λ small enough suffices for the equilibrium to be unique, for investment to increase with θ, and for γ_1 to be positive.*

As in the previous section, α combines an informational effect (captured by $\frac{\lambda}{2-\lambda}\gamma_1$) with a supply-side effect (captured by $-\frac{\lambda^2}{\phi(2-\lambda)}$). The supply-side effect always contributes to strategic substitutability, while the informational effect contributes to strategic complementarity if and only if high investment is good news for θ (i.e. $\gamma_1>0$). Once again, the overall effect is ambiguous, but the role of informational frictions remains the same as before: Corollary 1 continues to hold.

We now turn to the characterization of the efficient allocation for this economy. The efficiency concept we use is the same as in the preceding sections; however, now we need to allow the planner to mimic the information aggregation that the market achieves through prices. We thus proceed as follows.

First, we define an *allocation* as a collection of strategies $k(x,y)$, $q_E\,(x,y,K,p,\omega)$ and $q_T\,(K,p)$, along with a shadow-price function $p\,(\theta,y,\omega)$ with the following interpretation: in the first period, an entrepreneur with signals (x,y) invests $k\,(x,y)$; in the second period, all agents observe the realizations of aggregate investment $K=K\,(\theta,y)$ and the shadow price $p=p\,(\theta,y,\omega)$; the amount of capital held by an entrepreneur not hit by a liquidity shock (in addition to the one chosen at $t=1$) is then given by $q_E\,(x,y,K,p,\omega)$, while the amount of capital held by a trader is given by $q_T\,(K,p)$.

Next, we say that the allocation is *feasible* if and only if, for all (θ,y,ω),

$$\lambda K\,(\theta,y)=(1-\lambda)\int q_E(x,y,K(\theta,y),\omega,p(\theta,y,\omega))d\Phi\,(x|\theta)+q_T(K(\theta,y),p(\theta,y,\omega)).\qquad(24)$$

As with equilibrium, this constraint plays two roles: first, it guarantees that the second-period resource constraint is not violated; second, it defines the technology that is used to generate the endogenous public signal (equivalently, the extent to which information can be aggregated through the shadow price).

Finally, for any given $k(x,y)$, $q_E\,(x,y,K,p,\omega)$ and $q_T\,(K,p)$, ex ante utility can be computed as $\mathbb{E}u = W(k, q_E, q_T)$, where

$$\begin{aligned} W(k, q_E, q_T) \equiv\ & \frac{1}{2}\int\int\{-\frac{1}{2}k(x,y)^2 + \lambda p(\theta,y,\omega)k(x,y) + (1-\lambda)\,\theta k(x,y) + \\ & + (1-\lambda)\, R\left(\theta-\omega, q^E(x,y,K(\theta,y),\omega,p(\theta,y,\omega))\right)\}d\Phi(x|\theta)\, d\Psi(\theta,y,\omega) \\ & + \frac{1}{2}\int R\left(\theta, q^T(K(\theta,y), p(\theta,y,\omega))\right)\}d\Psi(\theta,y,\omega), \end{aligned}$$

where $R(v,q) \equiv vq - q^2/(2\phi)$ and where $K(\theta,y) = \int k(x,y)d\Phi(x|\theta)$. We then define an efficient allocation as follows.

Definition 4 *An efficient allocation is a collection of strategies $k(x,y)$, $q_E(x,y,K,p,\omega)$ and $q_T(K,p)$, along with a shadow price function $p(\theta,y,\omega)$, that jointly maximize ex-ante utility, $\mathbb{E}u = W(k, q_E, q_T)$, subject to the feasibility constraint (24).*

Because utility is transferable, the shadow price does not affect payoffs directly; its sole function is to provide an endogenous public signal upon which the allocation of the asset in the period 2 can be conditioned. The next lemma then characterizes the efficient allocation of the asset.

Lemma 5 *The efficient allocation in the second period satisfies*

$$q_E^* = \frac{\lambda K}{2-\lambda} - \frac{\phi\omega}{2-\lambda} \quad \text{and} \quad q_T^* = \frac{\lambda K}{2-\lambda} + \frac{(1-\lambda)\,\phi\omega}{2-\lambda}. \tag{25}$$

To understand this result, suppose for a moment that information were complete in the second period. For any given K, efficiency in the second period would require that all entrepreneurs hold the same q_E and that (q_T, q_E) maximize

$$\left\{\theta q_T - \frac{1}{2\phi}q_T^2\right\} + (1-\lambda)\left\{\theta q_E - \omega q_E - \frac{1}{2\phi}q_E^2\right\}$$

subject to $(1-\lambda)\, q_E + q_T = \lambda K$. Clearly, the solution to this problem is (25). In our environment, information is incomplete but the same allocation can be induced through the following shadow-price and demand functions: $p(\theta,y,\omega) = -\frac{\phi(1-\lambda)\omega}{2-\lambda}$, $q_E(x,y,K,p,\omega) = \frac{\lambda K}{2-\lambda} - \frac{\phi\omega}{2-\lambda}$, and $q_T(K,p) = \frac{\lambda K}{2-\lambda} - p$.[30]

We now characterize the efficient investment decisions. Using Lemma 5, ex ante utility reduces to

$$\mathbb{E}u = \frac{1}{2}\mathbb{E}\left\{-\frac{1}{2}k^2 + \theta k - \frac{1}{2\phi(2-\lambda)}(\lambda K)^2\right\} + \frac{(1-\lambda)}{2\,(2-\lambda)}\phi\sigma_\omega^2. \tag{26}$$

[30] Note that the proposed shadow price is also the unique market-clearing price given the proposed demand functions. The efficient trades can thus be implemented by inducing these demand functions through an appropriately designed tax system and then letting the agents trade in the market.

Except for two minor differences—the smaller weight on $(\lambda K)^2$, which adjusts the cost associated with absorbing the fixed supply λK in the second period for the fact that now this quantity is split across a larger pool of agents, and the last term in (26), which captures how the volatility of ω affects the allocation of capital across entrepreneurs and traders in the second period—ex-ante utility has the same structure as in (19) in the previous section. The following result is then immediate.

Proposition 9 *The efficient investment strategy is the unique linear solution to*

$$k(x,y) = \mathbb{E}\left[(1-\alpha^*)\,\kappa^*(\theta) + \alpha^* K(\theta,y) \;\mid x,y\right], \tag{27}$$

where $\alpha^* \equiv -\frac{\lambda^2}{\phi(2-\lambda)}$, $\kappa^*(\theta) \equiv \frac{1}{1-\alpha^*}\theta$, *and* $K(\theta,y) = \int k(x,y)d\Phi(x|\theta)$.

Comparing the efficient strategy with the equilibrium one, we have that, once again, as long as investment increases with both signals, so that high investment is good news for profitability, then α remains higher than α^*, in which case the key normative prediction of the paper, as summarized by Corollary 2, continues to hold.

6.3 Financial-market shocks

In the specifications considered so far, entrepreneurs and traders share the same valuation for the installed capital. We now develop a variant of the model in which entrepreneurs and traders have different valuations. In this variant, additional non-fundamental volatility originates from correlated errors in the entrepreneurs' expectations about the traders' valuations; once again, our mechanism amplifies the impact of these errors. This variant thus helps connect our model to the recent work on speculative trading à la Harrison and Kreps (1978).[31]

We consider the following modification of the baseline model. The traders' utility in period $t=3$ is given by $(\theta+\omega)\,k_i$, where ω is a random variable, independent of θ and of any other exogenous random variable in the economy, Normally distributed with mean zero and variance σ_ω^2. This random variable is a private-value component in the traders' valuation. It can originate from the hedging motive of the traders, from a different discount factor, or from heterogeneous valuations à la Harrison and Kreps (1978). For our purposes, what matters is that the presence of ω in the traders' utility is taken as given by the social planner; that is, the planner respects the preference orderings revealed by the agents' trading decisions. We thus choose a neutral label for ω and simply call it a "financial market shock."

We also modify the entrepreneurs' information set, to allow for information regarding ω to affect investment decisions. In particular, the entrepreneurs observe a common signal $w=\omega+\zeta$, where ζ is

[31] See Scheinkman and Xiong (2003), Gilchrist, Himmelberg, and Huberman (2005), and Panageas (2005).

common noise, independent of any other exogenous random variable in the economy, with variance σ_ζ^2. The signal w is observed by the entrepreneurs but not by the traders; as in the baseline model, this is a shortcut for introducing correlated errors in the entrepreneurs' expectations regarding the financial-market shock. Finally, to focus on common expectational shocks about ω rather than about θ, we remove the common signal y : the entrepreneurs observe only private signals about θ, $x_i = \theta + \xi_i$, where ξ_i is idiosyncratic noise as in the baseline model.

In this environment, the asset price in period two is given by

$$p = \mathbb{E}\left[\theta | K, \omega\right] + \omega.$$

It follows that equilibrium investment choices depend not only on the entrepreneurs' expectations of θ, but also on their expectations of ω : there exist coefficients $(\beta_0, \beta_1, \beta_2)$ such that individual investment is given by $k\left(x, w\right) = \beta_0 + \beta_1 x + \beta_2 w$ and, by implication, aggregate investment is given by $K\left(\theta, w\right) = \beta_0 + \beta_1 \theta + \beta_2 w$. Following similar steps as in the baseline model, leads to the following result.

Proposition 10 ***(i)*** *In any equilibrium, there exist a scalar $\alpha > 0$ and a function $\kappa(\theta, \omega)$ such that*

$$k\left(x, w\right) = \mathbb{E}\left[\ (1-\alpha)\,\kappa\left(\theta, \omega\right) + \alpha K\left(\theta, w\right)\ \mid x, w\right].$$

(ii) *λ small enough suffices for the equilibrium to be unique and for investment to increase with both θ and w.*

(iii) *The efficient investment satisfies*

$$k\left(x, w\right) = \mathbb{E}\left[\ \theta + \lambda\omega \mid x, w\right].$$

(iv) *In any equilibrium in which investment increases with both θ and w, investment underreacts to θ and overreacts to w.*

In this economy, entrepreneurs pay too much attention to their signals regarding shocks in the financial market. The reason is essentially the same as in the benchmark model. When traders interpret high investment as good news for θ, financial prices increase with aggregate investment. Because the noise in the entrepreneurs' signals about the financial market shock ω is correlated, these signals are relatively better predictors of aggregate investment than the signals about θ. By implication, entrepreneurs' investment decisions are oversensitive to information about financial market shocks relative to information about their fundamental valuation θ. Through this channel, an increase in investment that was purely driven by expectations regarding financial market shock is amplified.

Absent informational frictions (i.e., if θ were known at the time of financial trade)), the response of investment to θ and ω would be efficient. Since ω can be interpreted as the difference between the traders' and the entrepreneurs' fundamental valuations of the asset, this case is reminiscent of the efficiency results obtained in richer models of "bubbles" based on heterogeneous priors; in particular, Panageas (2006) derives a similar efficiency result for a model that introduces heterogeneous valuations in a q-theory model of investment. The interesting novelty here is that inefficiency arises once we introduce dispersed information. Traders are then uncertain whether high investment is driven by good fundamentals or by the entrepreneurs' expectations of speculative valuations. This uncertainty opens the door to our feedback effect between financial prices and investment, creating inefficiency in the response of investment to different sources of information.

6.4 Other extensions

An important function of stock prices is to guide corporate investment choices by revealing valuable information that is dispersed in the marketplace and not directly available to corporate managers (e.g., Dow and Gorton, 1997; Subrahmanyam and Titman, 1999; Chen, Goldstein and Jiang, 2007). This effect is absent in the preceding analysis, because the entrepreneurs' investment choices are made before the opening of the financial market. However, we can easily incorporate such an effect by letting the entrepreneurs make an additional investment in stage two, after observing the price in the financial market.[32] Provided that the dispersion of information does not vanish, the source of complementarity and inefficiency we have documented remains. Interestingly, though, an additional information externality emerges: if all agents were to increase their reliance on idiosyncratic sources of information, then the information contained in prices would be more precise, which in turn would improve the efficiency of the investment decisions that follow the observation of these prices. Clearly, this informational externality only reinforces the conclusion that agents rely too much to common sources of information, and hence that non-fundamental volatility is inefficiently high.

Throughout the preceding extensions, we have maintained the assumption that traders cannot directly invest in the new technology during the first period. Clearly, our results do not hinge on this assumption. For example, consider the benchmark model and suppose that each trader j chooses first-period real investment k_j at cost $k_j^2/2$ and then trades an additional q_j units in the second-period financial market. Neither the equilibrium price in the financial market nor the entrepreneurs' choices in the first period are affected; all that happens is that aggregate investment now includes the investment of the traders, which is simply given by $k_T = \mathbb{E}\theta$, which does not affect the information structure in the second period. More generally, one could drop the distinction

[32]Alternatively, we could introduce a financial market in stage 1 or let entrepreneurs observe a noisy signal of K instead of a noisy price signal.

between entrepreneurs and traders altogether and simply talk about differentially informed agents who first make real investment decisions and then trade financial claims on the installed capital.

Next, consider the assumption that a fraction λ of the entrepreneurs is hit by a liquidity shock and is forced to sell their capital in the financial market; this was a modeling device that ensured that the private return to first-period investment depends on (anticipated) second-period financial prices while ensuring tractability. If one were to drop the assumption of risk neutrality, or assume that the second-period transaction costs depend on gross positions, or introduce short-sale constraints in the financial market, then the profits an agent could do in the financial market would depend on how much capital he enters the market with; this in turn would ensure that private returns to first-period investment depend on expectations of future financial prices, even in the absence of liquidity shocks.[33]

Finally, consider the assumption that profitability is perfectly correlated across entrepreneurs. Clearly, what is essential is only that there is a common component about which agents have dispersed information. For example, we could let the productivity of the new technology for entrepreneur i be $\tilde{\theta}_i = \theta + v_i$, where θ is the common component and v_i is an idiosyncratic component; we could then also let the entrepreneurs' signals be $\tilde{\theta}_i$ plus noise instead of θ plus noise. Alternatively, we could introduce common and idiosyncratic shocks to the entrepreneurs' cost of investment during period 1. In this case, unobservable common shocks to the cost of investment would also act as a source of noise in the information that aggregate investment conveys about θ, essentially playing the same role as the correlated errors in the entrepreneurs' signals about θ.[34]

7 Conclusion

This paper examined the interaction between real and financial decisions in an economy in which information about underlying profitability is dispersed. By conveying a positive signal about profitability, higher aggregate investment stimulates higher asset prices, which in turn raise the incentives to invest. This creates an endogenous complementarity, making investment decisions sensitive to higher-order expectations. In turn, this can dampen the impact of fundamental shocks and amplify the impact of common expectational shocks. Importantly, all these effects are symptoms of inefficiency.

These effects are likely to be stronger during periods of intense technological change, when the dispersion of information about the potential of the new technologies is particularly high. Our analysis therefore predicts that such periods come hand-in-hand with episodes of high non-fundamental

[33] Note, however, that these extensions may feature additional deviations from the first best (e.g., short-sale constraints), which may introduce novel effects in addition to the ones we have considered.

[34] Such an extension is studied in the Supplementary Material.

volatility and comovement in investment and asset prices. At some level, this seems consistent with the recent experiences surrounding the internet revolution or the explosion of investment opportunities in China. What looks like irrational exuberance may actually be the amplified, but rational, response to noise in information. While both explanations open the door to policy intervention, the one suggested by our theory is not based on any presumption of “intelligence superiority” on the government’s side.

Our mechanism also represents a likely source of non-fundamental volatility and inefficiency over the business cycle. Indeed, information regarding aggregate supply and demand conditions seems to be widely dispersed in the population, which explains the financial markets’ anxiety preceding the release of key macroeconomic statistics. Extending the analysis to richer business-cycle frameworks is an important direction for future research.

Appendix: Proofs omitted in the main text

Proof of Lemma 3. The derivations of β_1 and β_2 are in the proof of the Lemma 4. Rearranging (30), gives

$$\beta_2 = \frac{1}{1-\lambda\gamma_1}\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2.$$

Using (29), $\alpha = \lambda\gamma_1$ and $\delta_2/\delta_1 = \pi_y/\pi_x$, then gives the result. ■

Proof of Lemma 4. The proof proceeds in several steps. We start by proving part (i). We continue with some auxiliary results regarding the function F which are used in the last steps. We conclude by establishing parts (ii), (iii) and (iv). Throughout, to simplify notation, we suppress the dependence of F and G on $(\lambda, \pi_\theta, \pi_x, \pi_y)$ and let $\pi \equiv \pi_\theta + \pi_x + \pi_y$, $\delta_0 \equiv \pi_\theta/\pi$, $\delta_1 \equiv \pi_x/\pi$, and $\delta_2 \equiv \pi_y/\pi$.

Part (i). Substituting $K(\theta, y) = \beta_0 + \beta_1\theta + \beta_2 y$ into (7) and using $\mathbb{E}[\theta|x,y] = \delta_0\mu + \delta_1 x + \delta_2 y$ gives

$$\begin{aligned}
k(x,y) &= (1-\lambda)\,\mathbb{E}[\theta|x,y] + \lambda\gamma_0 + \lambda\gamma_1\left(\beta_0 + \beta_1\mathbb{E}[\theta|x,y] + \beta_2 y\right) \\
&= \left(1-\lambda+\lambda\gamma_1\beta_1\right)\mathbb{E}[\theta|x,y] + \lambda\gamma_0 + \lambda\gamma_1\beta_0 + \lambda\gamma_1\beta_2 y \\
&= \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_0\mu + \lambda\gamma_0 + \lambda\gamma_1\beta_0\right] + \\
&\quad + \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_1\right]x + \left[\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2 + \lambda\gamma_1\beta_2\right]y
\end{aligned}$$

Because in equilibrium the above must coincide with $\beta_0 + \beta_1 x + \beta_2 y$ for all x and y, the following conditions must hold

$$\beta_0 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_0\mu + \lambda\gamma_0 + \lambda\gamma_1\beta_0, \tag{28}$$

$$\beta_1 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_1, \tag{29}$$

$$\beta_2 = \left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_2 + \lambda\gamma_1\beta_2. \tag{30}$$

It is immediate that any equilibrium must satisfy $\beta_1 \neq 0$. Then let $b \equiv \beta_2/\beta_1$. From (4) and (6),

$$\gamma_1\beta_1 = h(b) \equiv \frac{\delta_2(1+b)}{\delta_0 b^2 + \delta_2(1+b)^2} \tag{31}$$

while from (29) and (30),

$$b = \frac{\delta_2}{\delta_1} + \frac{\lambda\gamma_1\beta_1 b}{\left(1-\lambda+\lambda\gamma_1\beta_1\right)\delta_1}. \tag{32}$$

Substituting (31) into (32) gives $b = F(b)$, where

$$F(b) \equiv \frac{\delta_2}{\delta_1}\left\{1 + \frac{\lambda(1+b)b}{(1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2}\right\}. \tag{33}$$

Note that the domain of F is the set of all $b \in \mathbb{R}$ such that $1 - \lambda + \lambda\gamma_1\beta_1 \neq 0$. Using (31), the latter is given by

$$\mathbb{B} \equiv \{b \in \mathbb{R} : (1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2 \neq 0\}.$$

It follows that, in any linear equilibrium, b is necessarily a fixed point of F, while the coefficients $(\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1)$ are given by the following conditions:

$$\beta_1 = [1 - \lambda + \lambda h(b)]\delta_1 \tag{34}$$

$$\beta_2 = b\beta_1 = b[1 - \lambda + \lambda h(b)]\delta_1 \tag{35}$$

$$\gamma_1 = \frac{\gamma_1\beta_1}{\beta_1} = \frac{h(b)}{[1 - \lambda + \lambda h(b)]\delta_1} \tag{36}$$

$$\beta_0 = (1 - \lambda + \lambda h(b))\,\delta_0\mu + \frac{\lambda\delta_0\mu}{\delta_0 + \delta_2\left(1+\frac{1}{b}\right)^2} \tag{37}$$

$$\gamma_0 = \frac{\delta_0}{\delta_0 + \delta_2(1+\frac{1}{b})^2}\mu - \gamma_1\beta_0 \tag{38}$$

Conditions (34)-(38) uniquely define the function G.

Auxiliary results. Let $g(b) \equiv (1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2$; the domain of F is $\mathbb{B} = \{b \in \mathbb{R} : g(b) \neq 0\}$ and its complement is $\mathbb{B}_c = \{b \in \mathbb{R} : g(b) = 0\}$. Note that the discriminant of $g(b)$ is $\Delta \equiv (\delta_2\lambda)^2 - 4\delta_0\delta_2(1-\lambda)$. If $\Delta < 0$, then $\mathbb{B}_c = \varnothing$; if $\Delta = 0$, then $\mathbb{B}_c = \left\{-\frac{(2-\lambda)\delta_2}{2(1-\lambda)(\delta_0+\delta_2)}\right\}$; finally, if $\Delta > 0$, then $\mathbb{B}_c = \left\{-\frac{(2-\lambda)\delta_2+\sqrt{\Delta}}{2(1-\lambda)(\delta_0+\delta_2)}, -\frac{(2-\lambda)\delta_2-\sqrt{\Delta}}{2(1-\lambda)(\delta_0+\delta_2)}\right\}$. Because there are values for $(\delta_0, \delta_2, \lambda)$ that make Δ negative, zero, or positive, all three cases are possible in general. However, because Δ is continuous in λ and $\Delta = -4\delta_0\delta_2 < 0$ when $\lambda = 0$, λ small enough suffices for $\mathbb{B}_c = \varnothing$. Moreover, because $g(b) \geq \delta_2 > 0$ for any $b \geq 0$, $\mathbb{R}_+ \subset \mathbb{B}$ always.

The function F is continuously differentiable over its entire domain, with

$$F'(b) = \lambda\frac{\delta_2}{\delta_1}\frac{\phi_1(b)}{g(b)^2}$$

where $\phi_1(b) \equiv [\delta_2 - (1-\lambda)\delta_0]b^2 + 2\delta_2 b + \delta_2$. Moreover,

$$\lim_{b\to-\infty} F(b) = \lim_{b\to+\infty} F(b) = F_\infty \equiv \frac{\delta_2}{\delta_1}\left\{1 + \frac{\lambda}{(1-\lambda)(\delta_0+\delta_2)}\right\} > \frac{\delta_2}{\delta_1},$$

and $F(-1) = F(0) = \delta_2/\delta_1 < F(\delta_2/\delta_1)$.

Consider the case $\delta_2 = (1-\lambda)\,\delta_0$. Then $\phi_1(b) = 0$ admits a unique solution at $b = -1/2$. Because $\Delta < 0$, the function F is defined over the entire real line, it is decreasing for $b < -1/2$ and increasing for $b > -1/2$. Next, consider the alternative case, $\delta_2 \neq (1-\lambda)\,\delta_0$. Then $\phi_1(b) = 0$ admits exactly two solutions, at $b = b_1$ and at $b = b_2$, where

$$b_1 \equiv \frac{-\delta_2 - \sqrt{(1-\lambda)\delta_0\delta_2}}{\delta_2 - (1-\lambda)\,\delta_0} \text{ and } b_2 \equiv \frac{-\delta_2 + \sqrt{(1-\lambda)\delta_0\delta_2}}{\delta_2 - (1-\lambda)\,\delta_0}.$$

The function F then reaches a local maximum at b_1 and a local minimum at b_2.

Part (ii). By the preceding results we have that F is continuous over $\mathbb{R}_+$, with $F(\delta_2/\delta_1) > \delta_2/\delta_1$ and $\lim_{b\to\infty} F(b) < \infty$. It follows that the equation $F(b) = b$ admits at least one solution at $b > \delta_2/\delta_1 = \pi_y/\pi_x$.

Part (iii). Fix any $(\delta_1, \delta_2) \in (0,1)^2$. If λ is such that $\delta_2 = (1-\lambda)\,\delta_0$, where $\delta_0 = 1-(\delta_1+\delta_2)$, then let $\underline{F} \equiv F(-1/2)$ and $\bar{F} \equiv F_\infty$. If, instead, λ is such that $\delta_2 \neq (1-\lambda)\,\delta_0$, then let $\underline{F} \equiv \min\{F_\infty, F(b_2)\}$ and $\bar{F} \equiv \max\{F_\infty, F(b_1)\}$. It is easy to check that both $\underline{F}$ and $\bar{F}$ converge to δ_2/δ_1 as $\lambda \to 0$. Since F is continuous over its entire domain, $\mathbb{B}$, and λ small enough suffices for $\mathbb{B} = \mathbb{R}$, we have that λ small enough also suffices for F to be bounded in $[\underline{F}, \bar{F}]$. But then F converges uniformly to δ_2/δ_1 as $\lambda \to 0$. It follows that for any $\varepsilon > 0$ there exists $\hat{\lambda} = \hat{\lambda}(\varepsilon) > 0$ such that, whenever $\lambda < \hat{\lambda}$, $\mathbb{B} = \mathbb{R}$ and F has no fixed point outside the interval $[\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$.

Now note that the function $F'(b;\lambda)$ is continuous at $(\delta_2/\delta_1, 0)$ with $F'(\delta_2/\delta_1; 0) = 0$. It follows that, for any $\eta \in (0,1)$, there exist $\tilde{\varepsilon} = \tilde{\varepsilon}(\eta) > 0$ and $\tilde{\lambda} = \tilde{\lambda}(\eta)$ such that $-1 < -\eta < F'(b;\lambda) < \eta < 1$ for all $b \in [\delta_2/\delta_1 - \tilde{\varepsilon}, \delta_2/\delta_1 + \tilde{\varepsilon}]$ and all $\lambda \in [0, \tilde{\lambda}]$.

Combining the aforementioned results with the continuity of F, we have that there exist $\bar{\varepsilon} > 0$ and $\bar{\lambda} > 0$ such that, for any $\lambda \in [0, \bar{\lambda}]$, the following are true: for any $b \notin [\delta_2/\delta_1 - \bar{\varepsilon}, \delta_2/\delta_1 + \bar{\varepsilon}]$, $F(b) \neq b$; for $b \in [\delta_2/\delta_1 - \bar{\varepsilon}, \delta_2/\delta_1 + \bar{\varepsilon}]$, F is continuous and differentiable in b, with $F'(b) < 1$. It follows that, for $\lambda < \bar{\lambda}$, F has at most one fixed point. Together with the fact that F necessarily has at least one fixed point (from part (ii)), this proves part (iii).

Part (iv). It is easy to check that $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$ implies that $\mathbb{B} = \mathbb{R}$ (so that F is continuous over the entire real line) and $F(b_2) < b_2 < 0$. These properties, together with the properties that $F(0) > 0$ and $\lim_{b\to-\infty} F(b) > 0 > -\infty$, ensure that, in addition to a fixed point in $(\delta_2/\delta_1, +\infty)$, F admits at least one fixed point in $(-\infty, b_2)$ and one in $(b_2, 0)$. Indeed, in this example F admits exactly three fixed point, which are "strict" in the sense that $F(b) - b$ changes sign around them. Because F is continuous in $(b, \delta_1, \delta_2, \lambda)$ in an open neighborhood of $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$, there necessarily exists an open set $S \subset (0,1)^3$ such that F admits three fixed points whenever $(\delta_1, \delta_2, \lambda) \in S$. ■

Proof of Proposition 1. From Lemma 4, there always equilibrium in which $b > \pi_y/\pi_x$. Pick the equilibrium that corresponds to the highest solution to $F(b;\lambda) = b$ and let $b(\lambda) \in (\pi_y/\pi_x, +\infty)$ denote this solution. Part (i) follows from conditions (34), (35) and (36) observing that $b > 0$ suffices for $h(b) > 0$ and hence for $\beta_1, \beta_2, \gamma_1 > 0$. For part (ii), note that $\alpha > 0$ follows from $\gamma_1 > 0$; that $\alpha < 1$ follows from (8) along with $\beta_2/\beta_1 > 0$; finally, that α increases with λ follows from Lemma 3 and part (iii), which we prove next. First, note that

$$\frac{\partial F(b;\lambda)}{\partial \lambda} = \frac{\delta_2}{\delta_1} \frac{b(1+b)\left(\delta_2(1+2b) + (\delta_0+\delta_2)b^2\right)}{g(b)^2},$$

so that $b > 0$ suffices for $\partial F(b;\lambda)/\partial\lambda > 0$. Next, note that the function $F(b;\lambda) - b$ is continuous in b over $(\pi_y/\pi_x, +\infty)$ and satisfies $\lim_{b\to+\infty}\{F(b;\lambda) - b\} = -\infty$. Since $b(\lambda) \in (\pi_y/\pi_x, +\infty)$ is the highest solution to $F(b;\lambda) - b = 0$, it is then necessarily the case that $F(b;\lambda) - b < 0$ for any $b > b(\lambda)$. Part (iii), then follows from this property together with the fact that F increases with λ. ■

Proof of Proposition 2. Take any $\lambda < \bar{\lambda}$. Let $b(\lambda)$ denote the unique fixed point to $F(b;\lambda) = b$ and denote by $\beta_0(\lambda), \beta_1(\lambda), \beta_2(\lambda), \gamma_0(\lambda)$ and $\gamma_1(\lambda)$ the corresponding equilibrium coefficients, as given by (34)-(38). Note that all these functions are continuous.

Part (i). Using conditions (34)–(38), the sensitivity of investment to the realization of θ is given by

$$\beta_1(\lambda) + \beta_2(\lambda) = W(\lambda)(\delta_1+\delta_2),$$

where $W(\lambda) \equiv w(b(\lambda),\lambda)$, with $w(b,\lambda) \equiv (1+b)(1-\lambda+\lambda h(b))\frac{\delta_1}{\delta_1+\delta_2}$ and $h(b)$ defined as in (31). We can compute $b'(\lambda)$ and $b''(\lambda)$ applying the Implicit Function Theorem to $F(b,\lambda) - b$. We can then use this to compute $W'(\lambda)$ and $W''(\lambda)$. After some tedious algebra (which is available upon request), we find that $W'(0) = 0$ and $W''(0) = -\frac{2\delta_0\delta_1\delta_2}{(\delta_1(\delta_1+\delta_2)+\delta_2)^2} < 0$. Together with the fact that $b(0) = \delta_2/\delta_1$ and hence $W(0) = 1$, this ensures that there exists $\hat{\lambda} \in (0, \bar{\lambda}]$ such that, for all $\lambda \in (0, \hat{\lambda})$, $W(\lambda) < W(0) = 1$ and $W'(\lambda) < 0$; that is, $\beta_1 + \beta_2$ is lower than $\delta_1 + \delta_2$, its value in the frictionless benchmark, and is decreasing in λ.

Part (ii). From condition (34), we have that $\beta_1(\lambda) = [1 - \lambda + \lambda h(b(\lambda))]\delta_1$ and hence $\beta_1'(\lambda) = \delta_1[-1 + h(b(\lambda)) + \lambda h'(b(\lambda) b'(\lambda))]$. Since $b(0) = \delta_2/\delta_1$ and $h(\delta_2/\delta_1) = \frac{\delta_1(\delta_1+\delta_2)}{\delta_1(\delta_1+\delta_2)+\delta_2} < 1$, we have that $\beta_1'(0) = \delta_1[-1 + h(\delta_2/\delta_1)] < 0$, which together with the result from part (i) that $\beta_1'(0) + \beta_2'(0) = 0$ gives $\beta_2'(0) > 0$. The result then follows from the local continuity of $\beta_2'(\lambda)$ in λ. ■

Proof of Proposition 4. The first claim is proved by the numerical example in the main text. Thus consider the second claim. Given any linear strategy $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$, ex-ante utility is given by

$$
\begin{aligned}
2\mathbb{E}u &= \mathbb{E}\left[-\frac{1}{2}k\left(x,y\right)^{2}+\theta k\left(x,y\right)\right] \qquad (39)\\
&= -\frac{1}{2}\beta_0^2+\beta_0\left(1-\beta_1-\beta_2\right)\mu-\frac{1}{2}\beta_1^2\pi_x^{-1}-\frac{1}{2}\left(\beta_1+\beta_2\right)^2\pi_\theta^{-1}\\
&\quad -\frac{1}{2}\beta_2^2\pi_y^{-1}+\left(\beta_1+\beta_2\right)\sigma_\theta^2+\left(\beta_1+\beta_2\right)\left[1-\frac{1}{2}(\beta_1+\beta_2)\right]\mu^2.
\end{aligned}
$$

Now suppose prices are fully stabilized at $p=\bar{p}$. Substituting $p(\theta,y)=\bar{p}$ into the entrepreneurs' best response (2) gives the following coefficients for the equilibrium investment strategy:

$$
\beta_0=\left(1-\lambda\right)\delta_0\mu+\lambda\bar{p},\quad \beta_1=\left(1-\lambda\right)\delta_1,\quad \text{and}\quad \beta_2=\left(1-\lambda\right)\delta_2. \qquad (40)
$$

Note that $\bar{p}$ affects only the first two terms in (40) through its effect on β_0. Hence, the maximal welfare that can be achieved with full price stabilization is obtained by choosing $\bar{p}$ so that $\beta_0=1-(1-\lambda)(\delta_1+\delta_2)$.

Next, note that for any $a\in(0,1)$ and any $b\in\mathbb{R}$, there exists a policy $\tau(p)=\tau_0+\tau_1 p$ that induces an equilibrium in which the investment strategy is given by[35]

$$
\beta_0=b\quad \beta_1=\frac{(1-\lambda)\delta_1}{1-a\delta_1}\quad \text{and}\quad \beta_2=\frac{(1-\lambda+a\beta_1)\delta_2}{1-a}. \qquad (41)
$$

To see this, suppose that, given (τ_0,τ_1), the entrepreneurs follow the linear strategy defined in (41). Then $\mathbb{E}[\theta|K]=\gamma_0+\gamma_1 K$, where (γ_0,γ_1) are obtained from (41) using the formulas given in (6). The market clearing price is then equal to

$$
p=\frac{1}{1+\tau_1}\left(\gamma_0+\gamma_1 K-\tau_0\right) \qquad (42)
$$

Replacing (42) and $K(\theta,y)=\beta_0+\beta_1\theta+\beta_2 y$ into (2), we then have that the best response for each entrepreneur consists in following the strategy $k\left(x,y\right)=\tilde{\beta}_0+\tilde{\beta}_1 x+\tilde{\beta}_2 y$ given by

$$
\begin{aligned}
\tilde{\beta}_0 &= \frac{\lambda(\gamma_0-\tau_0)}{1+\tau_1}+\left(1-\lambda\right)\delta_0\mu+\tilde{\alpha}\left[\beta_0+\beta_1\delta_0\mu\right]\\
\tilde{\beta}_1 &= \left(1-\lambda\right)\delta_1+\tilde{\alpha}\beta_1\delta_1\\
\tilde{\beta}_2 &= \left(1-\lambda\right)\delta_2+\tilde{\alpha}\left[\beta_1\delta_2+\beta_2\right]
\end{aligned}
$$

where $\tilde{\alpha}=\lambda\gamma_1/(1+\tau_1)$. It is then immediate that there exists a (τ_0,τ_1) such that $\tilde{\beta}_0=\beta_0$, $\tilde{\beta}_1=\beta_1$

[35] Equivalently, for any $a\in(0,1)$ and any $\kappa_0\in\mathbb{R}$, there exists a policy (τ_0,τ_1) that sustains an equilibrium in which the investment strategy satisfies $k(x,y)=\mathbb{E}[(1-a)\tilde{\kappa}(\theta)+aK(\theta,y)\mid x,y]$ with $\tilde{\kappa}(\theta)\equiv\left(\frac{1-\lambda}{1-a}\right)\theta+\kappa_0$.

and $\tilde{\beta}_2 = \beta_2$ (it suffices to choose τ_1 so that $\tilde{\alpha} = a$ and then adjust τ_0 so that $\tilde{\beta}_0 = b$).

Now let $b_0 \equiv 1 - (1-\lambda)(\delta_1 + \delta_2)$ and for any $a \in [0,1)$ let

$$\tilde{\beta}_1(a) \equiv \frac{(1-\lambda)\,\delta_1}{1 - a\delta_1}, \qquad \tilde{\beta}_2(a) \equiv \frac{(1-\lambda+a\beta_1)\delta_2}{1-a},$$

and

$$\begin{aligned} W(a) \;\; \equiv \;\; & -\frac{1}{2}b_0^2 + b_0\left[1 - \tilde{\beta}_1(a) - \tilde{\beta}_2(a)\right]\mu - \frac{1}{2}\left(\tilde{\beta}_1(a)\right)^2 \pi_x^{-1} - \frac{1}{2}\left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a)\right)^2 \pi_\theta^{-1} \\ & -\frac{1}{2}\left(\tilde{\beta}_2(a)\right)^2 \pi_y^{-1} + \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a)\right)\pi_\theta^{-1} \\ & + \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a)\right)\left[1 - \frac{1}{2}\left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a)\right)\right]\mu^2 \end{aligned}$$

Note that welfare under full price stabilization is given by $W(0)$, whereas welfare under any policy (τ_0, τ_1) that implements a linear strategy as in (41) with $a \in (0,1)$ and $b = b_0$ is given by $W(a)$. Next note that W is continuously differentiable over $[0,1)$. To prove the second claim in the proposition it thus suffices to show that

$$\frac{dW}{da} = \frac{\partial W}{\partial \tilde{\beta}_1}\frac{d\tilde{\beta}_1}{da} + \frac{\partial W}{\partial \tilde{\beta}_2}\frac{d\tilde{\beta}_1}{da} > 0$$

at $a = 0$. First note that

$$\begin{aligned} \frac{\partial W}{\partial \tilde{\beta}_1} &= -b_0\mu - \tilde{\beta}_1\pi_x^{-1} - \left(\tilde{\beta}_1 + \tilde{\beta}_2\right)\pi_\theta^{-1} + \pi_\theta^{-1} + \left[1 - \left(\tilde{\beta}_1 + \tilde{\beta}_2\right)\right]\mu^2 \\ \frac{\partial W}{\partial \beta_2} &= -b_0\mu - \tilde{\beta}_2\pi_y^{-1} - \left(\tilde{\beta}_1 + \tilde{\beta}_2\right)\pi_\theta^{-1} + \pi_\theta^{-1} + \left[1 - \left(\tilde{\beta}_1 + \tilde{\beta}_2\right)\right]\mu^2 \end{aligned}$$

Using $\tilde{\beta}_1(0) = (1-\lambda)\,\delta_1$, $\tilde{\beta}_2(0) = (1-\lambda)\,\delta_2$ and $b_0 \equiv 1 - (1-\lambda)(\delta_1 + \delta_2)$, we thus have that

$$\left.\frac{\partial W}{\partial \tilde{\beta}_1}\right|_{\tilde{\beta}_1 = \tilde{\beta}_1(0)} = \left.\frac{\partial W}{\partial \tilde{\beta}_2}\right|_{\tilde{\beta}_2 = \tilde{\beta}_2(0)} = -b_0\mu + \left[1 - (1-\lambda)\left(\delta_1 + \delta_2\right)\right]\mu^2 + \lambda\pi_\theta^{-1} = \lambda\pi_\theta^{-1} > 0.$$

Because $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are both increasing in a, it follows that $dW(0)/da$ is positive, which establishes the result. ■

Proof of Proposition 5. Rewrite the tax rule as

$$\tau(\theta, K) = -\phi_0 - (\phi_1 - 1)\theta - \phi_2 K,$$

where $\phi_0 \equiv -\tau_0$, $\phi_1 \equiv 1 - \tau_1$, and $\phi_2 \equiv -\tau_2$. Suppose that all other entrepreneurs follow the efficient strategy $k(x,y) = \mathbb{E}[\theta|x,y]$. The equilibrium price is then given by

$$p(\theta,y) = \mathbb{E}[\theta - \tau(\theta, K(\theta,y)) \mid K(\theta,y)] = \phi_0 + \phi_1 \mathbb{E}[\theta \mid K(\theta,y)] + \phi_2 K(\theta,y)$$
$$= \phi_0 + \phi_1\gamma_0 + (\phi_1\gamma_1 + \phi_2) K(\theta,y)$$

where we have used $\mathbb{E}[\theta \mid K(\theta,y)] = \gamma_0 + \gamma_1 K(\theta,y)$, with (γ_0,γ_1) determined by substituting $(\beta_0,\beta_1,\beta_2) = (\delta_0\mu, \delta_1, \delta_2)$ into (6). The best response for each individual entrepreneur is then to follow the strategy

$$\begin{aligned} k(x,y) &= \mathbb{E}[(1-\lambda)(\theta-\tau) + \lambda p \mid x,y] \\ &= \mathbb{E}[(1-\lambda)(\phi_0 + \phi_1\theta + \phi_2 K) + \lambda(\phi_0 + \phi_1\gamma_0 + (\phi_1\gamma_1 + \phi_2) K) \mid x,y] \\ &= (1-\lambda)\phi_0 + \lambda(\phi_0 + \phi_1\gamma_0) + (1-\lambda)\phi_1 \mathbb{E}[\theta|x,y] + [(1-\lambda)\phi_2 + \lambda(\phi_1\gamma_1 + \phi_2)] \mathbb{E}[K|x,y] \end{aligned} \tag{43}$$

For the tax $\tau(\theta,K)$ to implement the efficient allocation, it is thus necessary and sufficient that the strategy in (43) coincides with $k(x,y) = \mathbb{E}[\theta|x,y]$, which is possible if and only if

$$(1-\lambda)\phi_2 + \lambda(\phi_1\gamma_1 + \phi_2) = 0, \quad (1-\lambda)\phi_1 = 1, \quad \text{and} \quad (1-\lambda)\phi_0 + \lambda(\phi_0 + \phi_1\gamma_0) = 0.$$

Equivalently,

$$\tau_0 = -\phi_0 = \frac{\lambda}{1-\lambda}\gamma_0, \quad \tau_1 = 1 - \phi_1 = -\frac{\lambda}{1-\lambda}, \quad \text{and} \quad \tau_2 = -\phi_2 = \frac{\lambda}{1-\lambda}\gamma_1.$$

The result then follows from the fact that $\gamma_0 < 0 < \gamma_1$ when the entrepreneurs follow the efficient allocation. ■

Proof of Proposition 6. For part (i), it suffices to substitute the price as in (16) into the entrepreneurs' best response (2). Thus consider part (ii). Substituting (4) into (6) gives

$$\gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z}\frac{1}{\beta_1 + \beta_2} = \frac{(\beta_1+\beta_2)\pi_y}{\beta_2^2\pi_\theta + (\beta_1+\beta_2)^2\pi_y} = \frac{(\beta_1+\beta_2)\delta_2}{\beta_2^2\delta_0 + (\beta_1+\beta_2)^2\delta_2}.$$

In the limit, as $\lambda \to 0$, we have that $\beta_0 \to \delta_0$, $\beta_1 \to \delta_1$, $\beta_2 \to \delta_2$, and hence $\gamma_1 \to \frac{(\delta_1+\delta_1)\delta_2}{\delta_2^2\delta_0 + (\delta_1+\delta_1)^2\delta_2} > 0$. By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0,\hat{\lambda})$, $(\beta_1 + \beta_2) > 0$, i.e. investment increases with θ, $\gamma_1 > 0$, i.e. the traders' expectation of θ increases with K, and $\alpha = \lambda(\gamma_1 - \lambda/\phi) > 0$, i.e. entrepreneurs perceive a complementarity in their investment decisions. ■

Proof of Proposition 7. Let $V(k,K,\theta) \equiv -\frac{1}{2}k^2 + \theta k - \frac{\lambda^2}{2\phi}K^2$. From (19), $\mathbb{E}u = \frac{1}{2}\mathbb{E}V(k,K,\theta)$. The result then follows from Proposition 3 in Angeletos and Pavan (2007a), noting that $\kappa^*(\theta) \equiv$

$\arg\max_K V(K,K,\theta) = \frac{1}{1+\lambda^2/\phi}\theta$ and $\alpha^* \equiv 1 - \frac{V_{kk}+2V_{kK}+V_{KK}}{V_{kk}} = V_{KK} = -\lambda^2/\phi$. ■

Proof of Proposition 8. From (2), in any equilibrium in which p is linear in (θ, y, ω), there are coefficients $(\beta_0, \beta_1, \beta_2)$ such that $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$. From (22) and (23), the equilibrium price is then

$$p(\theta, y, \omega) = P(K(\theta, y), \theta, \omega) \equiv \eta_0 + \eta_1 K(\theta, y) + \eta_2 \theta + \eta_3 \omega. \tag{44}$$

for some $(\eta_0, \eta_1, \eta_2, \eta_3)$.

Now consider the optimality of the traders' strategies. As in the benchmark model, the information that $K(\theta, y)$ reveals about θ is the same as that of a signal

$$z \equiv \frac{K(\theta, y) - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon$$

whose precision is $\pi_z \equiv \left(\frac{\beta_1+\beta_2}{\beta_2}\right)^2 \pi_y$,while the information that $p(\theta, y, \omega)$ reveals about θ given $K(\theta, y)$ is the same as that of a signal

$$s = \frac{1}{\eta_2}[p(\theta, y, \omega) - \eta_0 - \eta_1 K(\theta, y)] = \theta + \frac{\eta_3}{\eta_2}\omega$$

whose precision is $\pi_s = \left(\frac{\eta_2}{\eta_3}\right)^2 \pi_\omega$. A trader who observes K and p thus believes that θ is normally distributed with mean

$$\begin{aligned}
\mathbb{E}\left[\theta \mid K(\theta, y), p(\theta, y, \omega)\right] &= \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s}\mu_\theta + \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} z + \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} s \\
&= \gamma_0 + \gamma_1 K(\theta, y) + \gamma_2 \theta + \gamma_3 \omega
\end{aligned}$$

where

$$\gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s}\mu_\theta - \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s}\frac{\beta_0}{\beta_1 + \beta_2} \tag{45}$$

$$\gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s}\frac{1}{\beta_1 + \beta_2} \tag{46}$$

$$\gamma_2 = \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} \tag{47}$$

$$\gamma_3 = \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s}\frac{\eta_3}{\eta_2}. \tag{48}$$

Combining (22) with (44) we then have that

$$\eta_0 = \frac{\gamma_0}{2-\lambda} \tag{49}$$

$$\eta_1 = \frac{1}{2-\lambda}\left(\gamma_1 - \frac{\lambda}{\phi}\right) \tag{50}$$

$$\eta_2 = \frac{1}{2-\lambda}\left(\gamma_2 + 1 - \lambda\right) \tag{51}$$

$$\eta_3 = \frac{1}{2-\lambda}\left(\gamma_3 - 1 + \lambda\right). \tag{52}$$

Lastly, consider the optimality of the entrepreneurs' investment strategies. From condition (2), the strategy $k(x,y) = \beta_0 + \beta_1 x + \beta_2$ is individually rational if and only if $(\beta_0, \beta_1, \beta_2)$ satisfy $\beta_0 + \beta_1 x + \beta_2 y = (1-\lambda)\,\mathbb{E}[\theta|x,y] + \lambda \mathbb{E}[p(\theta, y, \omega)|x,y]$. That is, $(\beta_0, \beta_1, \beta_2)$ must satisfy the following:

$$\beta_0 = [1 - \lambda + \lambda\eta_1\beta_1 + \lambda\eta_2]\delta_0\mu_\theta + \lambda\eta_0 + \lambda\eta_1\beta_0 \tag{53}$$

$$\beta_1 = (1 - \lambda + \lambda\eta_1\beta_1 + \lambda\eta_2)\,\delta_1 \tag{54}$$

$$\beta_2 = (1 - \lambda + \lambda\eta_1\beta_1 + \lambda\eta_2)\,\delta_2 + \lambda\eta_1\beta_2 \tag{55}$$

A linear equilibrium is a thus a solution to (45)-(55).

The existence of a linear equilibrium and its uniqueness for λ small enough can be established following steps similar to those in the benchmark model. Here we prove that λ small enough suffices for $\gamma_1 > 0$, and even for $\alpha > 0$.

Substituting $\pi_z \equiv \left(\frac{\beta_1+\beta_2}{\beta_2}\right)^2 \pi_y$ and $\pi_s = \left(\frac{\eta_2}{\eta_3}\right)^2 \pi_\omega = \pi_\omega$ into (46) gives

$$\begin{aligned}\gamma_1 &= \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s}\frac{1}{\beta_1 + \beta_2} \\ &= \frac{(\beta_1+\beta_2)\,\pi_y}{\beta_2^2\pi_\theta + (\beta_1+\beta_2)^2\,\pi_y + \beta_2^2\pi_\omega} \\ &= \frac{(\beta_1+\beta_2)\,\delta_2}{\beta_2^2\delta_0 + (\beta_1+\beta_2)^2\,\delta_2 + \frac{\beta_2^2\pi_\omega}{\pi_\theta+\pi_y+\pi_x}}.\end{aligned}$$

In the limit, as $\lambda \to 0$, we have that $\beta_0 \to \delta_0$, $\beta_1 \to \delta_1$, $\beta_2 \to \delta_2$, and hence

$$\gamma_1 \to \frac{(\delta_1+\delta_1)\,\delta_2}{\delta_2^2\delta_0 + (\delta_1+\delta_1)^2\,\delta_2 + \frac{\delta_2^2\pi_\omega}{\pi_\theta+\pi_y+\pi_x}} > 0.$$

By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda})$, $(\beta_1 + \beta_2) > 0$, $\gamma_1 > 0$ and $\alpha = \frac{\lambda}{2-\lambda}\left(\gamma_1 - \frac{\lambda}{\phi}\right) > 0$. ■

Proof of Proposition 9. Let

$$V(k,K,\theta) \equiv \theta k - \frac{1}{2}k^2 - \frac{\lambda^2}{2\phi(2-\lambda)}K^2.$$

The result then follows for the same argument as in the proof of Proposition 7. ■

Proof of Proposition 10. *Part (i).* In any equilibrium in which the price $p(\theta,\omega,w)$ is linear in (θ,ω,w), there are coefficients $(\beta_0,\beta_1,\beta_2)$ such that the investment strategy can be written as

$$k(x,w) = \beta_0 + \beta_1 x + \beta_2 w,$$

implying that aggregate investment satisfies $K(\theta,w) = \beta_0 + \beta_1\theta + \beta_2\omega + \beta_2\zeta$. For the traders, who know ω but do not know either ζ or θ, observing K is then equivalent to observing a Gaussian signal z with precision π_z, where

$$z \equiv \frac{K-\beta_0-\beta_2\omega}{\beta_1} = \theta + \frac{\beta_2}{\beta_1}\zeta \quad \text{and} \quad \pi_z \equiv \left(\frac{\beta_1}{\beta_2}\right)^2 \pi_\zeta,$$

with $\pi_\zeta \equiv \sigma_\zeta^{-2}$. It follows that the equilibrium price satisfies

$$p(\theta,\omega,w) = \mathbb{E}[\theta|K,\omega] + \omega = \gamma_0 + \gamma_1 K(\theta,w) + (1-\gamma_1\beta_2)\,\omega, \tag{56}$$

where

$$\gamma_0 = \frac{\pi_\theta}{\pi_\theta+\pi_z}\mu - \frac{\pi_z}{\pi_\theta+\pi_z}\frac{\beta_0}{\beta_1} \text{ and } \gamma_1 = \frac{\pi_z}{\beta_1(\pi_\theta+\pi_z)} = \frac{\pi_\zeta}{\beta_1\left(\left(\frac{\beta_2}{\beta_1}\right)^2\pi_\theta+\pi_\zeta\right)}. \tag{57}$$

Substituting (56) into the entrepreneurs' best response gives

$$\begin{aligned} k(x,w) &= (1-\lambda)\,\mathbb{E}[\theta|x,w] + \lambda\mathbb{E}[p(\theta,\omega,w)|x,w] \\ &= (1-\lambda)\,\mathbb{E}[\theta|x,w] + \lambda\gamma_0 + \lambda\gamma_1\mathbb{E}[K(\theta,w)|x,w] + \lambda(1-\gamma_1\beta_2)\,\mathbb{E}[\omega|x,w] \end{aligned} \tag{58}$$

which can be rewritten as in part (i) of the proposition by letting

$$\alpha \equiv \lambda\gamma_1 \quad \text{and} \quad \kappa(\theta,\omega) \equiv \frac{(1-\lambda)\,\theta + \lambda\gamma_0 + \lambda(1-\gamma_1\beta_2)\,\omega}{1-\lambda\gamma_1}.$$

Finally, that $\alpha > 0$ is shown in the next part.

Part (ii). Substituting $K(\theta,w) = \beta_0 + \beta_1\theta + \beta_2 w$ into (58) gives

$$k(x,w) = \lambda(\gamma_0+\gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1)\,\mathbb{E}[\theta|x,w] + \lambda\gamma_1\beta_2 w + \lambda(1-\gamma_1\beta_2)\,\mathbb{E}[\omega|x,w]$$

Using the facts that $\mathbb{E}[\theta|x,w] = \mathbb{E}[\theta|x] = \delta_0 + \delta_1 x$ and $\mathbb{E}[\omega|x,w] = \mathbb{E}[\omega|w] = \eta w$, where $\delta_0 \equiv \sigma_\theta^{-2}/(\sigma_\theta^{-2}+\sigma_x^{-2})\mu$, $\delta_1 \equiv \sigma_x^{-2}/(\sigma_\theta^{-2}+\sigma_x^{-2})$, and $\eta \equiv \sigma_\zeta^{-2}/(\sigma_\omega^{-2}+\sigma_\zeta^{-2})$, the above reduces to

$$\begin{aligned} k(x,w) &= \lambda(\gamma_0+\gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1)\delta_0 + (1-\lambda+\lambda\gamma_1\beta_1)\delta_1 x \\ &\quad + \lambda[\eta+(1-\eta)\gamma_1\beta_2]w \end{aligned}$$

For this strategy to coincide with $k(x,w) = \beta_0+\beta_1 x+\beta_2 w$, it is necessary and sufficient that the coefficients $(\beta_0,\beta_1,\beta_2)$ solve the following system:

$$\beta_0 = \lambda(\gamma_0+\gamma_1\beta_0) + (1-\lambda+\lambda\gamma_1\beta_1)\delta_0 \tag{59}$$

$$\beta_1 = (1-\lambda+\lambda\gamma_1\beta_1)\delta_1, \tag{60}$$

$$\beta_2 = \lambda[\eta+(1-\eta)\gamma_1\beta_2]. \tag{61}$$

By (57),

$$\gamma_1\beta_1 = \frac{\pi_\zeta}{\left(\frac{\beta_2}{\beta_1}\right)^2\pi_\theta+\pi_\zeta} \in (0,1), \tag{62}$$

which together with (60) guarantees that $\beta_1 \in (0,\delta_1)$. From (60) and (61) we then get

$$\frac{\beta_2}{\beta_1} = \lambda\frac{\eta\left(\left(\frac{\beta_2}{\beta_1}\right)^2\pi_\theta+\pi_\zeta\right)+(1-\eta)\pi_\zeta\left(\frac{\beta_2}{\beta_1}\right)}{\delta_1\left((1-\lambda)\left(\left(\frac{\beta_2}{\beta_1}\right)^2\pi_\theta+\pi_\zeta\right)+\lambda\pi_\zeta\right)}$$

or equivalently

$$\frac{\beta_2}{\lambda\beta_1} = F\left(\frac{\beta_2}{\lambda\beta_1};\lambda\right)$$

where

$$F(b;\lambda) \equiv \frac{\eta}{\delta_1}\left\{1+\lambda\frac{\lambda^2\pi_\theta b^2+\pi_\omega b}{(1-\lambda)\lambda^2\pi_\theta b^2+\pi_\zeta}\right\}.$$

It is then easy to show that, for λ small enough, F has a unique fixed point and this fixed point is in a neighborhood of

$$\frac{\beta_2}{\lambda\beta_1} = \frac{\eta}{\delta_1}.$$

Along with the fact that $\beta_1 > 0$ always, this guarantees that $\beta_2 > 0$ for λ small enough.

Part (iii). The social planner's problem can be set up as in the baseline model, giving the optimality condition stated in part (iii) of the proposition.

Part (iv). From part (iii) the efficient strategy is given by

$$k(x,w) = \beta_0^* + \beta_1^* x + \beta_2^* w$$

with

$$\beta_0^* = \delta_0, \quad \beta_1^* = \delta_1 \quad \text{and} \quad \beta_2^* = \lambda\eta.$$

We have already shown, in the proof of part (ii), that $\beta_1 < \delta_1 = \beta_1^*$, which means that investment underreacts to θ. Next, note that $\beta_1 > 0$ implies $\gamma_1 > 0$. From (61) it then follows that, in any equilibrium in which $\beta_2 > 0$, it is also the case that

$$\beta_2 = \lambda\eta + \lambda(1-\eta)\gamma_1\beta_2 > \lambda\eta = \beta_2^*,$$

which means that investment overreacts to w. ■

References

[1] Abel, Andrew B., and Olivier J. Blanchard (1986), "The Present Value of Profits and Cyclical Movements in Investment," *Econometrica* 54, 249-273.

[2] Allen, Franklin, Stephen Morris, and Hyun Song Shin (2006) "Beauty Contests and Iterated Expectations," *Review of Financial Studies* 19, 161-177.

[3] Amato, Jeffrey, and Hyun Song Shin (2006), "Imperfect Common Knowledge and the Information Value of Prices," *Economic Theory* 27, 213-241.

[4] Angeletos, George-Marios, and Alessandro Pavan (2007a), "Efficient Use of Information and Social Value of Information," *Econometrica* 75, 1103-1142.

[5] Angeletos, George-Marios, and Alessandro Pavan (2007b), "Policy with Dispersed Information," MIT/Northwestern mimeo.

[6] Bacchetta, Philippe, and Eric van Wincoop (2005), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?," *American Economic Review* 95.

[7] Baeriswyl, Romain, and Camille Cornand (2007), "Monetary Policy and its Informative Value," University of Munich/Université Louis Pasteur mimeo.

[8] Baker, Malcolm, Jeremy Stein, and Jeffrey Wurgler (2003), "When Does the Market Matter? Stock Prices and the Investment of Equity-Dependent Firms," *Quarterly Journal of Economics* 118, 969-1005.

[9] Barro, Robert J. (1990), "The Stock Market and Investment," *The Review of Financial Studies* 3, 115-131.

[10] Bernanke, Ben S., and Mark Gertler (2001), "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review* 91, 253-257.

[11] Bosworth, Barry (1975), "The Stock Market and the Economy," *Brookings Papers on Economic Activity* 2:1975, 257-290.

[12] Brainard, W., and J. Tobin (1968), "Pitfalls in financial model building," *American Economic Review* 58, 99-122.

[13] Caballero, Ricardo (2006), "On the Macroeconomics of Asset Shortages," MIT mimeo.

[14] Caballero, Ricardo, Emmanuel Farhi, and Mohamad Hammour (2006), "Speculative Growth: Hints from the U.S. Economy," *American Economic Review* 96, 1159-1192.

[15] Cecchetti, Stephen, Hans Genberg, John Lipsky, and Sushil Wadhwani (2000), *Asset Prices and Central Bank Policy*, London, International Center for Monetary and Banking Studies.

[16] Cespa, Giovanni and Xavier Vives (2007) "Dynamic Trading and Asset Prices: Keynes vs. Hayek," mimeo IESE Business School and University of Salerno.

[17] Chen, Nai-Fu, Richard Roll, and Stephen A. Ross (1986) "Economic Forces and the Stock Market," *The Journal of Business* 59, 383-403.

[18] Chen, Qi, Itay Goldstein, and Wei Jiang (2007), "Price Informativeness and Investment Sensitivity to Stock Price," *Review of Financial Studies* 20, 619-650.

[19] Cutler, David M., James M. Poterba and Lawrence H. Summers (1989), "What Moves Stock Prices?" *The Journal of Portfolio Management* 15, 4-12.

[20] Dow, J., and Gorton, G. (1997), "Stock Market Efficiency and Economic Efficiency: Is There a Connection?," *Journal of Finance* 52, 1087–1129.

[21] Dupor, William (2005), "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information," *Journal of Monetary Economics* 62, 727-747.

[22] Gilchrist, Simon, Charles Himmelberg, and Gur Huberman (2005), "Do stock price bubbles influence corporate investment?" *Journal of Monetary Economics*, 52, 805-827.

[23] Goldstein, Itay, and Alexander Guembel (2007), "Manipulation and the Allocational Role of Prices, " *Review of Economic Studies*, forthcoming.

[24] Goldstein, Itay, Emre Ozdenoren and Kathy Yuan (2007) "Learning and Strategic Complementarities: Implications for Speculative Attacks," mimeo University of Pennsylvania and University of Michigan.

[25] Harrison, J. Michael and David M. Kreps, "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," (1978) *The Quarterly Journal of Economics*, Vol. 92(2), 323-336.

[26] Hellwig, Christian (2005), "Heterogeneous Information and the Benefits of Transparency," UCLA mimeo.

[27] Heinemann, Frank, and Camille Cornand (2004), "Optimal Degree of Public Information Dissemination," CESifo Working Paper 1353.

[28] Keynes, John Maynard (1936), *The General Theory of Employment, Interest and Money,* Macmillan, London.

[29] Lorenzoni, Guido (2006), "A Theory of Demand Shocks," MIT mimeo.

[30] Lorenzoni, Guido (2007), "News Shocks and Optimal Monetary Policy," MIT mimeo.

[31] Mackowiak, Bartosz and Mirko Wiederholt (2007) "Optimal Sticky Prices under Rational Inattention," Humboldt University/Northewestern University mimeo.

[32] Morck, Randall, Robert Vishny, and Andrei Shleifer (1990), "The Stock Market and Investment: Is the Market a Sideshow?," *Brookings Papers on Economic Activity* 2:1990, 157-215.

[33] Morris, Stephen, and Hyun Song Shin (2005), "Central Bank Transparency and the Signal Value of Prices," *Brookings Papers on Economic Activity.*

[34] Morris, Stephen, and Hyun Song Shin (2002), "The Social Value of Public Information," *American Economic Review* 92, 1521-1534.

[35] Ozdenoren, Emre, and Kathy Yuan, "Feedback Effects and Asset Prices" (2007), forthcoming in the *Journal of Finance.*

[36] Panageas, Stavors (2005), "The Neoclassical q Theory of Investment in Speculative Markets," Wharton School mimeo.

[37] Scheinkman, Jose, and Wei Xiong (2003), "Overconfidence and Speculative Bubbles," *Journal of Political Economy* 111, 1183-1219.

[38] Shiller, Robert (2000), *Irrational Exuberance*, Princeton University Press, Princeton.

[39] Subrahmanyam, Avanidhar, and Sheridan Titman (1999). "The Going-Public Decision and the Development of Financial Markets," *Journal of Finance* 54, 1045-1082.

[40] Subrahmanyam, Avanidhar, and Sheridan Titman (2001). "Feedback From Stock Prices to Cash Flows," *Journal of Finance* 56, 2389-2413.

[41] Tobin, James (1969), "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking* 1, 15-29.

[42] Ventura, Jaume (2003), "Economic Growth with Bubbles," CREI/Universitat Pompeu Fabra mimeo.

[43] Woodford, Michael (2002), "Imperfect Common Knowledge and the Effects of Monetary Policy," in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press.