The Effects of Progressive Taxation on Labor Supply when Hours and Wages are Jointly Determined

Daniel Aaronson and Eric French

REVISED July, 2004
WP 2002-22
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Daniel Aaronson and Eric French
Federal Reserve Bank of Chicago
July 22, 2004

Abstract

This paper extends a standard intertemporal labor supply model to account for progressive taxation as well as the joint determination of hourly wages and hours worked. We show, qualitatively and quantitatively, that these two factors have implications for estimating the intertemporal elasticity of substitution. Furthermore, we show how to use the intertemporal elasticity of substitution to interpret the labor supply response to a tax change. Failure to account for wage-hours ties within a progressive tax system leads to an hours response to a change in marginal tax rates that may be understated by as much as 10 percent for men and 17 percent for women.
1 Introduction

When evaluating the costs and benefits of modifications to the tax system, as in Altig et al (2001), a critical elasticity of interest is the intertemporal labor supply elasticity. While some recent research explicitly studies reactions to specific tax reforms, a more common approach to approximating these effects is to employ estimates of the labor supply response to wage changes using the methods of MaCurdy (1981), Altonji (1986), and Browning et al (1985). Among men, this labor supply elasticity is commonly believed to be low, with most estimates ranging from 0 to 0.5. For women, the estimate is considerably more uncertain but believed to be around 1. Yet, some recent studies find larger income responses to specific tax changes than what would be expected given the estimated labor supply responses to wage changes.\(^1\) This is potentially verification that estimated wage elasticities lead tax analysts to underpredict the labor supply response to specific tax changes.

In this paper, we emphasize two erroneous simplifying assumptions in standard labor supply models that could also contribute to different inferences about behavioral responses to tax changes. First, most labor supply models ignore the joint determination of hours worked and hourly wages.\(^2\) Second, many intertemporal models ignore progressive labor taxation.

In this paper, we examine how progressive taxation and the joint determination of hours and wages affects estimates of structural preference parameters. We also consider how to use estimated preference parameters to predict the likely labor supply responses to tax changes. We show that failure to account for progressive taxation and the joint determination of hours and wages leads to a small bias when estimating the intertemporal elasticity of substitution. However, it is important to consider tied wage-hours offers and progressive taxation when using this estimated preference parameter to predict the likely labor supply responses to tax changes.

\(^1\)Feldstein (1995) and others attribute this difference to tax avoidance and retiming and reshifting of transactions, rather than labor supply adjustments. See Slemrod (1998) for a useful nontechnical summary and discussion of the literature.

\(^2\)Aaronson and French (2004) discuss identification and estimation of a causal link from hours worked to hourly wages - the so-called part-time wage penalty. They identify this relationship using exogenous variation in hours worked resulting from social security rules. Papers that use other identification strategies, primarily related to mothers returning to the workforce, include Rosen (1976), Moffitt (1984), Lundberg (1985), Biddle and Zarkin (1989), Blank (1990), and Ermisch and Wright (1993).
Solving a standard life-cycle labor supply model, augmented to include tied wage-hours offers and progressive labor income taxation, illuminates two fundamental model misspecification problems. First, in a model where the wage is a function of hours worked, an increase in the post-tax wage resulting from a tax cut potentially leads to an increase in hours worked. This increase in hours worked leads to an increase in the pre-tax wage through the tied wage-hours effect, further escalating hours worked. Therefore, there is a larger labor supply response to a tax change than to an equally sized wage change. Since most models do not account for tied wage-hours offers, the latter effect (i.e. the effect of increased hours worked on increasing wages, which should in turn further increase hours worked) is ignored. Therefore, this model misspecification problem causes tax analysts to understate the labor supply response to a tax change.

However, a tax cut may increase hours and consequently income, which in turn can shift the individual into a higher tax bracket. This type of “bracket creep” reduces the variation in the post-tax wage, implying that progressive taxation should dampen the labor supply response to the tax cut. Consequently, the impact of tied wage-hours offers and progressive taxation on labor supply tends to offset one another. Nevertheless, since the progressive taxation effect seems less important than the effect of tied wage-hours offers, tax analysts are likely to continue to underpredict the labor supply response to tax changes.

We are not the first to observe that the labor supply function must be augmented to account for the marginal effect of work hours on wages and progressive tax schedules. However, we believe that we are the first to show analytically why failure to account for tied wage-hours offers in both proportional and progressive tax systems will produce labor supply elasticities that are different than the elasticity of interest to tax analysts.

We consider strategies for consistently identifying the structural preference parameter, the intertemporal elasticity of substitution, showing that many estimation schemes do not recover this parameter in the presence of progressive taxation and hours-wage ties. Because of the criticisms raised against maximum likelihood estimation of labor supply models using kinked budget constraints (MaCurdy et al. (1990)), we follow the approach of MaCurdy et al. (1990) and Ziliak and Kniesner (1999) and use smooth approximations to the tax

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code.\(^4\) In particular, we analyze a common instrumental variable strategy in the presence of progressive taxes and hours-wage offers. We then show how to use the intertemporal elasticity of substitution to interpret the labor supply response to a change in marginal tax rates. Using the Panel Study of Income Dynamics, labor supply responses to tax changes that account for tied wage-hours and progressivity are compared with those that do not and the resulting difference can be up to 10 percent for men.

Finally, we analytically evaluate the labor supply response to a tax change using a range of relevant parameter values for the labor supply response to a wage change, the tied wage-hours relationship, and the progressivity of the labor income tax schedule. With enough progressivity, the tied wage-hours and progressivity effects can completely offset each other. But assuming a level of progressivity observed, on average, in the U.S. over the last 30 years results in a difference of around 8 percent for men, and potentially up to 17 percent for women.

2 Dynamic intertemporal labor supply elasticities with tied wage-hours offers and progressive taxation

2.1 Model

We begin with the canonical intertemporal labor supply model,\(^5\) as in MaCurdy (1985), augmented to account for tied wage-hours offers and a potentially progressive labor income tax schedule. Preferences take the form:

\[
U = E_0 \sum_{t=1}^{T} \beta^t \left( v(c_{it}) - \exp\left(-\varepsilon_{it}/\sigma\right) \times \frac{h_{it}^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right)
\]

(1)

where \(U\) is the expected discounted present value of lifetime utility, \(c_{it}\) is consumption, \(v(.)\) is some increasing concave function, \(h_{it}\) is hours worked, and \(\varepsilon_{it}\) is the person and year specific preference for work. The parameter \(\sigma\) is the intertemporal elasticity of substitution, the usual

\(^4\)Alternative approaches to handling these criticisms are in Blundell et al. (1998) and Heim and Meyer (2003).

\(^5\)The key results from this section do not depend on whether the model is static or dynamic. However, the intertemporal model simplifies the analysis because it allows us to focus more on the substitution effect of a tax change. In static models and models with liquidity constraints, tax changes cause an additional change in the marginal utility of wealth. Moreover, if individuals do make forward looking decisions, many measures of non-labor income that are used in static models are endogenous and inconsistent estimates will result.
object of interest in dynamic labor supply studies.

Labor supply models typically assume that a worker receives a fixed wage offer, then chooses the number of hours to work given that wage. However, firms may not be indifferent to the number of hours worked. For example, Lewis (1969) and Barzel (1973) argue that the fixed cost involved in hiring and retaining workers, including the cost of training and aspects of compensation unrelated to hours worked, can be spread over more hours of work, causing the wage to be increasing in hours worked.\(^6\)

Operationally, it is typical in the empirical literature to specify the wage as a linear function of hours worked:

\[
\ln w_{it} = \alpha_{it} + \theta \ln h_{it} \tag{2}
\]

where \(\alpha_{it}\) represents an individual’s underlying productivity or technology during a specific year and \(\theta\) maps hours worked into the wage.

Two aspects of equation (2) are worth highlighting. First, the linearized relationship in equation (2) provides a good approximation to a structural relationship between the wage and hours worked, at least in the range of hours to which the majority of workers in our empirical example are situated. This case is made in detail in appendix A. Second, the estimate of \(\theta\) that we use in the analysis is based on samples of workers that do not switch employers. This is important because virtually all of the estimates in the literature, as well as the static models of Lewis and Barzel, call into question whether the estimated wage-hours relationship represents a long-run equilibrium, where hours and wages changes only happen across jobs. But in Aaronson and French (2004), workers who cut their hours receive wage reductions even when working for the same employer, consistent with the hypothesis that employers face fixed costs of work.

Finally, the individual faces the dynamic budget constraint:

\[
A_{it+1} = (1 + r_t(1 - \tau_A))(A_{it} + w_{it}(\log h_{it})h_{it} + y_{it} - \tau_{it} - c_{it}) \tag{3}
\]

where \(A_{it}\) are time \(t\) assets, \(r_t\) the interest rate, \(\tau_A\) is the tax rate on capital income, \(y_{it}\) is

\(^6\)Barzel also contends that exhaustion eventually causes marginal productivity (and thus the wage) to declines once the workday reaches a certain threshold.
spousal income, and $\tau_{it}$ denotes labor income taxes:\(^7\)

$$\tau_{it} = \tau(w_{it}(\log h_{it})h_{it} + y_{it})$$

(4)

Maximization of (1) subject to equations (2) and the dynamic budget constraint (3) yields the labor supply function:

$$\log h_{it} = \sigma\left[\log(1 - \tau'(.) + \log w_{it} + \log(1 + \theta))\right] + \sigma \log \lambda_{it} + \epsilon_{it}.$$  

(5)

The term in square brackets is the logarithm of the opportunity cost of time. The first part of this term reflects the cost of taxation that arises from additional working hours and is sometimes referred to as the log of the “net of tax price”. Note that $\tau'_{it}$ is the marginal tax rate and thus $1 - \tau'_{it}$ is the share of labor income that the individual keeps at the margin. The second part, the wage, arises because income increases with hours worked, holding the wage fixed. The third part occurs because the worker is paid a higher hourly wage when she works more hours, if hours and wages are tied. If changes in hours of work impact neither the wage (i.e. $\theta = 0$) nor the amount of taxes paid (i.e. $\tau'(. ) = 0$), equation (5) becomes the standard estimating equation in intertemporal labor supply models. The term $\lambda_{it} \equiv v'(c_{it})$ represents the marginal utility of wealth.

To estimate $\sigma$, we first difference equation (5):

$$\Delta \log h_{it} = \sigma\left[\Delta \log(1 - \tau'(.) + \Delta \log w_{it})\right] + \sigma \Delta \log \lambda_{it} + \Delta \epsilon_{it}.$$  

(6)

From equation (6), it is clear that obtaining consistent estimates of $\sigma$ requires valid controls for changes in marginal tax rates, preferences, and the marginal utility of wealth. For the latter, we follow MaCurdy (1985) and derive an estimating equation that controls for changes in the marginal utility of wealth:\(^8\)

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\(^7\)This analysis looks at anticipated changes in tax rates. If a tax change is unanticipated, we must consider both movements along and “parametric shifts” (e.g. MaCurdy, 1985) in the lifecycle wage profile. Furthermore, we assume that capital income does not affect labor income tax rates, which simplifies the analysis (Blomquist, 1985) but is problematic in that interest and dividends are taxed like ordinary income. Capital gains were taxed like ordinary income prior to 1997 and are still taxed that way for investments held less than one year. For long-term investments, there are currently two marginal rates. However, if capital gains are primarily concentrated among higher income households (see Burman and Ricoy (1997) for evidence), these rates could be considered significantly more proportional in practice than labor income. For tractability and due to limitations in the data, we therefore ignore these aspects of the progressive tax schedule.

\(^8\)He shows that the marginal utility of wealth, and in approximation its log, follows a random walk with
\[ \Delta \log h_{it} = \sigma [\Delta \log(1 - \tau'(\cdot)) + \Delta \log w_{it}] - \sigma \log (1 + r_{t-1}(1 - \tau_A)) + \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}} + \Delta \epsilon_{it}. \]  

(7)

where \( \epsilon_{it} \) is the innovation to the marginal utility of wealth.

The remainder of this paper examines two general questions: how to obtain consistent estimates of \( \sigma \) and how to use \( \sigma \) to infer the labor supply response to a tax change. Sections 2.2 and 2.3 consider, in turn, the roles of tied wage-hours offers and progressive taxation for these issues.

2.2 The case of proportional taxes

When taxation is progressive, analyzing the effects of taxes on labor supply becomes a bit complicated. In this section, we consider proportional taxation in order to develop intuition about the effect of tax changes on labor supply in the presence of tied wage-hours offers.

Proportional taxes imply that a constant share of labor income is taxed and therefore the marginal tax rate is a constant:

\[ \tau'_{it}(\cdot) = \tau'. \]  

(8)

In this case, marginal tax rates disappear from equation (7).

First, consider the problem of identifying \( \sigma \). Note from equations (2) and (5) that changes in \( \epsilon_{it} \) will affect hours, which will in turn affect the wage. Therefore, \( \log w_{it} \) is correlated with \( \epsilon_{it} \). This is the simultaneous equations bias problem. In addition, wage changes are likely correlated with the marginal utility of wealth. Consequently, a good instrument needs to be correlated with \( \Delta \ln w_{it} \) but uncorrelated with \( r_t, \epsilon_{it}, \) and \( \Delta \epsilon_{it} \). If such an instrument, \( Z_{it} \), can be found, then the instrumental variables estimator converges in probability to

\[ \sigma_{IV}^* = \frac{E[Z_{it}\Delta \log h_{it}]}{E[Z_{it}\Delta \log w_{it}]} = \sigma \]  

(9)

and thus \( \sigma_{IV}^* \) is a consistent estimator of \( \sigma \).\(^9\)

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\(^9\)This result relies on the assumption that the log wage increases linearly in log hours. However, Barzel...
However, the parameter $\sigma$ is no longer sufficient for understanding the labor supply response to taxation if wages are tied to hours. In particular, tax analysts are interested in the effect of taxes on labor supply,

$$\frac{\Delta \log h_{it}}{\Delta \log (1 - \tau')} :$$

$$\frac{\Delta \log h_{it}}{\Delta \log (1 - \tau')} = \sigma \left( 1 + \theta \frac{\Delta \log h_{it}}{\Delta \log (1 - \tau')} + \frac{\Delta \log \lambda_{it}}{\Delta \log (1 - \tau')} \right). \quad (10)$$

There are three pieces on the right hand side of equation (10), reflecting different labor supply incentives arising from a tax change. The first term reflects changes in the post-tax wage, holding the pre-tax wage fixed. A reduction in taxes causes an increase in the post-tax wage, which in turn affects labor supply. This is the usual object of interest in intertemporal labor supply studies. The second term arises from the effect of hours worked upon the wage. If $\sigma > 0$, reductions in taxes cause increases in hours worked, which in turn increases the pre-tax wage (because of tied wage-hours offers). Because the pre-tax wage increases, hours worked increase further. The final term is the effect of the tax change on the marginal utility of wealth. Increases in $1 - \tau'$ (i.e., decreases in marginal tax rates) tend to increase lifetime wealth and thus decrease its marginal utility, $\frac{\Delta \log \lambda_{it}}{\Delta \log (1 - \tau')} \leq 0$. Nevertheless, the labor supply response to tax changes, holding the marginal utility of wealth constant, is an important object since it is used to calibrate many of the important models used for tax analysis (Altig et al. (2001)) and it is a measure of the deadweight loss associated with tax changes (Ziliak and Kniesner, 1999). Therefore, we assume $\frac{d \log \lambda_{it}}{d \log (1 - \tau')} = 0$ and rearrange equation (10) as

$$\left. \frac{\Delta \log h_{it}}{\Delta \log (1 - \tau')} \right|_{\lambda_{it}} = \frac{\sigma}{1 - \sigma \theta}. \quad (11)$$

Equations (9) and (11) demonstrate that the labor supply response to a one percent increase in $1 - \tau'$ is larger than the labor supply response to a one percent wage increase, holding the marginal utility of wealth constant. Therefore, the strategy used to identify the labor supply elasticity can be critical. The magnitude of this difference, and identification strategies used to uncover it, are discussed further below.

(1973) speculates that at very long work weeks, an increase in hours might lower wages as exhaustion reduces productivity, so $w''(\log h_{it}) < 0$. Nevertheless, the existence of tied wage-hours offers need not necessarily lead to inconsistent estimates of $\sigma$. It is non-linearity in the wage-hours relationship that causes inconsistent estimates of $\sigma$. See appendix A for more discussion of this issue.

\(^{10}\)If $\theta > 0$ then the budget set is not convex. However, equation (11) still represents an equilibrium condition so long as $\sigma \theta < 1$. This condition is satisfied for reasonable parameter values.
2.3 The case of progressive taxes

The above analysis provides an assessment of the importance of model mis-specification introduced by wage-hours ties. In this section, we discuss a further complication, allowing for the possibility that increased hours of work push households into a higher tax bracket. This type of bracket creep reduces the variation in the post-tax wage, implying that progressive taxation should dampen the labor supply response to a pre-tax wage and tax change.\footnote{Of course, the extent of this effect depends on the distribution of taxpayers on the tax schedule. If most are far from the kinks, the effect will be small.}

Ignoring progressive taxation leads to a downward biased estimate of $\sigma$ and an upward biased estimate of the labor supply response to a tax change for a given $\sigma$. It is the latter effect that is more important, however. An increase in the marginal tax rate causes a decrease in work hours, naturally decreasing labor income and potentially lowering the marginal labor tax rate that the worker faces. Therefore, progressive taxation attenuates the effect of the initial increase in the marginal tax rate. Consequently, the impact of tied wage-hours offers and progressive taxation on labor supply tends to offset one another.

In order to capture a potentially progressive (or regressive through, for example, the Earned Income Tax Credit) tax schedule, we let the marginal tax rate depend on a polynomial in $\log(w_{it}h_{it} + y_{it})$:\footnote{This approach follows MaCurdy et al. (1990) and Ziliak and Kniesner (1999). In practice, we use a third order polynomial in log income. We also tried higher order polynomials, although this adjustment did not affect our results. A differentiable tax function makes the evaluation of the labor supply response to tax changes more straightforward, as in equation (14).}

$$\log(1 - \tau'(w_{it}h_{it} + y_{it})) = \sum_{k=0}^{K} \gamma_k [\log(w_{it}h_{it} + y_{it})]^k$$  \hspace{1cm} (12)

which can be approximated using a first order Taylor’s series approximation:

$$\sum_{k=0}^{K} \gamma_k [\log(w_{it}h_{it} + y_{it})]^k = \sum_{k=0}^{K} \gamma_k [\log((w_{it}h_{it})(1 + \frac{y_{it}}{w_{it}h_{it}}))]^k \approx \sum_{k=0}^{K} \gamma_k [\log(w_{it}) + \log(h_{it}) + \frac{y_{it}}{w_{it}h_{it}}]^k$$  \hspace{1cm} (13)

Recall that our interest is in the relationship between $\sigma^{*}_{IV}$ (the probability limit of the IV estimator using the pre-tax wage), the structural parameter $\sigma$, and the labor supply response...
to a tax change. However, with progressive taxation, it is impossible to know the relationship between $\sigma^*_{IV}$ and $\sigma$ without knowing the distribution of preference and productivity shocks, $\alpha_{it}$ and $\varepsilon_{it}$, as the higher order moments include covariances between income and $\alpha_{it}$ and $\varepsilon_{it}$. Unfortunately, no evidence exists on these parameters because it is difficult to distinguish variation in $\alpha_{it}$ and $\varepsilon_{it}$ from variation in hours and wages induced by measurement error.

Nevertheless, it is still possible to obtain consistent estimates of $\sigma$ using instrumental variables procedures. Instead of using the relationship between the pre-tax wage and labor supply, it is necessary to use the relationship between the post-tax wage and labor supply.

Next, we describe the association between $\sigma$ and a tax change, $\gamma_0$. Note that a one percentage point change in $\gamma_0$ increases the after tax wage by one percentage point, holding pre-tax income constant. Assuming $\frac{d\gamma_0}{d\gamma_0} = 0$ and combining equations (12), (13), and (5), it can be shown that the elasticity of hours worked with respect to $\gamma_0$ is

$$\frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} = \frac{\sigma}{1 - \sigma[\theta + (1 + \theta)(\sum_{k=1}^{K} k \gamma_k [\log(w_{it}) + \log(h_{it}) + \frac{y_{it}}{w_{it}h_{it}}]^{k-1})].}$$

(14)

Relative to equation (11), this derivative has an extra term, $\sigma(1 + \theta)(\sum_{k=1}^{K} k \gamma_k [\log(w_{it}) + \log(h_{it}) + \frac{y_{it}}{w_{it}h_{it}}]^{k-1})$. The first part of this term, $(1 + \theta)$, represents the percent increase in own labor income due to a one percent increase in hours. The second term depicts the percent change in the quantity $1 - \tau'_{it}$ caused by shifting own and spouse’s labor income by one percent. Therefore, the entire term is roughly the percent change in $1 - \tau'_{it}$ caused by changing hours by one percent. Intuitively, this term captures the result that when $\gamma_0$ increases (in other words, as marginal tax rates fall), individuals supply more hours to the market. However, this initial effect is dampened by progressive taxation since increased income pushes the worker into a higher marginal tax rate, thus attenuating the effect of $\gamma_0$.

Equations (11) and (14) differ only in that individuals are aware that changes in labor supply cause changes in the marginal tax rate in the latter equation. Equation (11) em-

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13This assumption implies that changes in the marginal tax rate will equally impact husband’s and wife’s labor supply, leaving the ratio of the wife’s to husband’s income unchanged.

14Note that the elasticity of interest is most likely with respect to a vertical shift in the marginal tax rate schedule. The connection between this elasticity and the one in equation (14) is

$$\frac{MTR - 1}{MTR} \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} = \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}}.$$ 

15Recall that progressive taxation implies that $\sum_{k=1}^{K} k \gamma_k [\log(w_{it}) + \log(h_{it}) + \frac{y_{it}}{w_{it}h_{it}}]^{k-1} < 0$. 

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phasizes only tied wage-hours and how failure to account for this relationship leads to an understatement of the importance of tax changes. Failure to account for progressive taxation, on the other hand, causes the researcher to overstate the importance of tax changes. Therefore, the two effects tend to offset.

Although the relationship between \( \sigma, \sigma^*_{IV}, \) and \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \) is complicated, it is still straightforward to estimate \( \sigma \) and \( \sigma^*_{IV} \) given the approaches we have discussed. Equation (14) and estimates of \( \{\gamma_k\}_{k=1}^K \) also allow us to predict \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \). We present such estimates in section 5.

Moreover, if \( \log(1 - \tau'(\cdot)) \) is linear in log labor income (i.e., \( \gamma_k = 0 \) for \( k > 1 \)), it is possible to obtain simple analytic solutions to help give our results some intuition. First, it is possible to qualitatively show that \( \sigma^*_{IV} < \sigma \). In particular, appendix C illustrates that

\[
\sigma^*_{IV} = \frac{\sigma(1 + \gamma_1)}{1 - \sigma \gamma_1}. \tag{15}
\]

Intuitively, \( \sigma \) measures the labor supply response to a change in the post-tax wage, whereas \( \sigma^*_{IV} \) measures the labor supply response to a change in the pre-tax wage. Note that a 1 percent increase in the pre-tax wage causes less than a 1 percent change in the post-tax wage. Therefore, an anticipated 1 percent change in the post-tax wage causes a \( \sigma \) percent change in hours worked. However, a 1 percent change in the pre-tax wage will lead to less than a 1 percent change in the post-tax wage and thus less than a \( \sigma \) percent change in hours worked.

Finally, the relationship between \( \sigma^*_{IV} \) and \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \) can be derived analytically using equations (14) and (15). Again assuming that \( \log(1 - \tau'(\cdot)) \) is linear in log labor income and contemporaneous and lagged preference changes are uncorrelated, we can show that:

\[
\frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} = \frac{\sigma^*_{IV}}{(1 + \gamma_1) - \sigma^*_{IV} \theta(1 + \gamma_1)}. \tag{16}
\]

After describing the estimation strategy and data in the next two sections, section 5 provides estimates of \( \sigma \) and the tax function directly. Section 6 uses plausible ranges of \( \gamma_1 \) and \( \sigma^*_{IV} \) to calibrate \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \).
3 Estimation Strategy

In Section 2, we pointed out problems with inferring the labor supply response to a tax change using the intertemporal elasticity of substitution. However, failure to account for progressive taxation also leads to inconsistent estimates of the intertemporal elasticity of substitution. Moreover, failure to account for tied wage-hours offers sometimes leads to inconsistent estimates, depending on the instrument set. These points are somewhat technical, so we derive the asymptotic properties of different estimators in Appendix C.

Our strategy for analyzing the importance of jointly determined hours and wages in a progressive tax world is to directly estimate \( \sigma \), accounting explicitly for jointly determined hours and wages and progressive taxes. We compare estimates that account for wage-hours ties and progressive taxes with those that ignore both factors. This allows us to assess the bias described in the previous section when data and other methodological choices are fixed.

There are five terms on the right hand side of our estimating equation (7). The first term, changes in the marginal tax rate, are explicitly simulated for each individual using the NBER’s Taxsim program, augmented with payroll tax rates obtained from the Tax Policy Center at the Urban Institute.\(^ {16}\) The third term, \( \log \beta (1 + r_{t-1} (1 - \tau)) \) is accounted for by including year dummies and education controls. The year dummies account for changes in the interest rate over time. The education group controls account for variation in subjective discount rates across education groups.\(^ {17}\) Health status change regressors capture the observed component of preference shifters, the fifth term, with the remaining portion of that term assumed to be white noise.

However, an important problem emerges with regard to the first, second and fourth terms of equation (7). First, the marginal tax rate is endogenous because hours choices affect this rate. Consequently, \( E[(\Delta \log (1 - \tau'))(\Delta \varepsilon_{it})] \neq 0 \). Second, the wage change is potentially correlated with the innovation to the marginal utility of wealth if the wage change is unanticipated, and thus \( E[(\Delta \log w_{it})\varepsilon_{it}] \neq 0 \). Therefore, we need anticipated sources of post-tax wage variation that are uncorrelated with preferences to identify \( \sigma \).

One common strategy to solve this problem is to exploit the life cycle wage profile and assume that workers are able to anticipate future post-tax wage growth based on their age,

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\(^{16}\)See [www.nber.org/taxsim/](http://www.nber.org/taxsim/) for more details. Marginal rates are computed relative to the next $1,000 in wage income. The data section describes the computations in more detail.

\(^{17}\)See Mulligan (1999) for a discussion of the cross-sectional evidence.
as in MaCurdy (1981) and Browning et al. (1985), among many others. The age profile will give consistent estimates of \( \sigma \) so long as age-specific variation in preferences is fully accounted for using health status and an age trend.\(^\text{18}\) Appendix C contains a more thorough discussion of the identification difficulties of standard instrumental variables strategies in a setting with tied wage-hours. It shows that using age as an instrument will yield consistent estimates of \( \sigma \). One important point of this discussion is that just as the effects of tied wage-hours offers and progressive taxation tend to offset when estimating the labor supply response to a tax change for a given \( \sigma \), the effects of these two factors are likely to offset when computing the bias in the estimate value of \( \sigma \).

### 4 Data

Similar to many previous studies of taxes and labor supply, we use the PSID to estimate \( \sigma \). Our sample consists of male household heads aged 25 to 60 between 1977 and 1989. We drop the self-employed because their capital and labor income (as well as taxes) is difficult to distinguish. We also drop those workers with fewer than 300 or more than 4,500 hours, as well as those who earn less than $3 or more than $100 per hour. Our selection criterion leads to a sample of 2,393 working men encompassing 15,989 person-years observations.

Two variables require further elaboration. First, we use a common measure of the hourly wage, annual earnings divided by annual hours. However, such a measure introduces a non-standard measurement error problem called “division bias” by allowing measurement error in hours to enter both the left hand and right hand side of the estimating equation (7). This can drive estimates of the wage elasticity to negative values.\(^\text{19}\)

\(^{18}\)An alternative strategy is to assume workers can anticipate future wage growth based on their current wage and thus use lagged wages or wage changes as instruments, as in Altonji (1986), Holtz-Eakin et al. (1988), and Ziliak and Kniesner (1999), among others. However, in the presence of tied wage-hours offers, changes in hours worked caused by changes in preferences will impact the wage. This violates the orthogonality assumptions of the life cycle labor supply model. Because lagged wages depend on lagged hours, lagged wages will only be a valid instrument for the current wage if \( E[\Delta w_t \mid \epsilon_{t-k}] = 0 \) for wages lagged \( k \) periods. It is possible to show that a slightly modified version of the lagged wage instrument that adjusts lagged wages by \( \theta \log h_{it} \) can potentially eliminate this feedback effect. Results are available upon request. But it appears to us that the age profile is clearly a cleaner instrument in a setting with tied wage-hours offers.

\(^{19}\)One potential solution we have tried is to instrument for the current wage change using twice lagged wages. If measurement error is white noise, twice lagged wages (or wage changes) will be uncorrelated with the current wage change. However, French (2004a) and Ziliak and Kneiser (1999) provide evidence that the measurement error in earnings and hours is autocorrelated and thus cannot solve inconsistency problems associated with \( \sigma \). We have also tried using the reported wage of hourly workers. Its advantage is that it overcomes the division bias problem since measurement error in the reported hourly wage is likely to be uncorrelated with both
Second, effective marginal rates are computed for each household using the NBER’s Taxsim program. We augment these rates with payroll tax schedules obtained from the Tax Policy Center at the Urban Institute. For the state and federal calculations, we assume that all married households file jointly and use the standard deduction. We also assume that income is provided solely through the head and spouse’s wages and salaries. The number of dependents, including those who qualify for the age 65 exemption, are provided by the PSID and accounted for in the computations.

Figure 1 displays marginal tax rates for individuals in our sample. Circles represent single filers, squares represent heads of household, and triangles represent joint filers. There is variation within income level due to cross-sectional differences in state tax law, variation over time in federal and state tax law, differences in the number of dependents across households, and filing status across households. Nevertheless, the dominant source of variation in marginal tax rates is from labor income. A simple regression of log(1 - τ′ it ) on log income has an $R^2$ of 0.49. A third order income polynomial, as we use, yields an $R^2$ of 0.52.

`hours and earnings. However, there are two distinct disadvantages. First, only hourly employees are included, which limits the sample size substantially and introduces potentially important nonrandomness to the sample. Second, overtime pay and bonuses are excluded. The latter concern is critical since overtime and bonuses are an important source of wage variation.

`20To account for substantial changes in the tax code introduced by the 1986 law changes, we show the rates separately pre- and post-reform. It is also important to note that there are few households facing negative marginal tax rates because we include payroll taxes and limit the sample to those households headed by men with at least $5,000 in annual income. However, the EITC is accounted for in the calculations.
Figure 1: MARGINAL TAX RATES
## 5 Results

Table 1 reports our estimates of the various labor supply elasticities. The first two columns report findings when the contemporaneous wage change is defined as annual earnings divided by annual hours and the parameter $\theta$, the wage-hours tie, is set to 0 in column 1 and 0.4 in column 2. The 0.4 estimate is in the middle to upper end of the estimates in the tied wage-hours offer literature.\(^{21}\) It implies that cutting weekly work hours from 40 to 20 leads to a 24 percent reduction in the offered hourly wage. A $\theta = 0$ assumes that the hourly wage is not a function of hours worked. In both columns, the findings are based on specifications that use a third order age polynomial as a means of exploiting the life cycle profile of wages.

The top panel displays the $F$ - statistic and $R^2$ from the first-stage regressions to show the power of this instrument. The instruments seems to be strongly associated with contemporaneous wage changes, with the $F$ - statistic exceeding standard thresholds.

The bottom panel reports the size of the four key labor supply parameters: $\sigma^\prime_{IV}$, $\sigma$, and the objects of interest to tax analysts, $\frac{d \log h_{it}}{d \log (1 - \tau_{it}^\prime)} \bigg|_{\lambda_{it}, e_{it}}$ and $\frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}}$. These elasticities are described in equations (11) and (14).\(^{22}\)


\(^{22}\)Recall that $\frac{d \log h_{it}}{d \log (1 - \tau_{it}^\prime)} \bigg|_{\lambda_{it}}$ is somewhat difficult to interpret because the marginal tax rate is a function of hours worked. However, for many cases, tax analysts are interested in $\frac{d \log h_{it}}{d \log (1 - \tau_{it}^\prime)} \bigg|_{\lambda_{it}}$, which can still be interpreted as $\frac{d \log (1 - \tau_{it}^\prime)}{d \gamma_0} \bigg|_{\lambda_{it}}$, or the percent increase in labor supply given a change in $\gamma_0$ that is sufficiently large to increase $\log (1 - \tau_{it}^\prime)$ by 1 percent.
<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Hourly wage</th>
<th>Hourly wage</th>
<th>Annual earnings</th>
<th>Annual earnings</th>
</tr>
</thead>
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<td>0</td>
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<tr>
<td>$\theta = 0.4$</td>
<td>0.4</td>
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</tr>
</tbody>
</table>

**First Stage Estimates, Dependent Variable is $\Delta \log w_{it}$**

| $F$ – statistic    | 5.4         | 5.4         | 18.6            | 18.6            |
| $R^2$              | 0.016       | 0.016       | 0.025           | 0.025           |
| $N$                | 15,989      | 15,989      | 15,989          | 15,989          |

**Second Stage Estimates, Dependent Variable is $\Delta \log h_{it}$**

| $\sigma^*_IV$      | 0.62        | 0.62        | 0.81            | 0.81            |
|                    | (0.16)      | (0.16)      | (0.06)          | (0.06)          |
| $\sigma$           | 0.64        | 0.64        | 1.13            | 1.13            |
|                    | (0.22)      | (0.22)      | (0.35)          | (0.35)          |

$$ \frac{d \log h_{it}}{d \log (1 - \tau'_{it})} \bigg|_{\lambda_{it}} $$

|                  | 0.64        | 0.86        | 1.13            | 2.06            |
|                  | (0.22)      | (0.40)      | (0.35)          | (1.16)          |

$$ \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} $$

|                  | 0.57        | 0.69        | 0.92            | 1.31            |
|                  | (0.17)      | (0.26)      | (0.23)          | (0.47)          |

Life cycle instrument set is a third order age polynomial.
Other right hand side variables are year dummies, health status change, and education.

Table 1: Estimated Labor Supply Elasticities, PSID 1977-1989
We find that \( \sigma^*_{IV} \) and \( \sigma \) are 0.62 (standard error of .16)\(^{23}\) and 0.64 (0.22).\(^{24}\) Note that, as argued in appendix C, failure to account for progressive taxation does lead to a downward biased estimate of \( \sigma \) (i.e. 0.64 versus 0.62). However, this effect is small. Allowing wage-hours ties (i.e., setting \( \theta = 0.4 \)) increases the hours response to a change in \((1 - \tau'_{it})\) by 34 percent, to 0.86, relative to \( \sigma \). That is, a 1 percent increase in \((1 - \tau'_{it})\) has an initial effect of increasing the after tax wage by 1 percent, which in turn increases hours by 0.64 percent. However, the longer workweek further increases the hourly wage, due to the wage-hours tie. This leads to a further increase in hours worked. Thus, the initial 1 percent increase in \((1 - \tau'_{it})\) increases hours by 0.86 percent.

But this is not the end of the story. When we introduce progressive taxation, the tax elasticity of interest, \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \), falls to 0.69, only 8 percent higher than \( \sigma \) and 11 percent higher than \( \sigma^*_{IV} \).\(^{25}\) This result arises from higher income leading to a higher marginal tax rate, which dampens the labor supply response to the original tax change. As it turns out, in this case, the effect of progressivity offsets much, but not all, of the tied wage-hours effect.\(^{26}\)

In the data section, we noted that division bias, in combination with small samples, leads to estimates that are biased downward. To minimize this problem, we respecify the labor supply function in terms of log earnings rather than log wages.\(^{27}\) It can be easily shown that this modification results in \( \sigma \) being biased to zero rather than -1 from measurement error.\(^{28}\) However, Ghez and Becker (1975) point out that omitted variables potentially lead to an

\(^{23}\)Standard errors are computed using the multivariate delta method and correct for arbitrary forms of heteroskedasticity and serial correlation.

\(^{24}\)These estimates are at the high end of the literature for men, although consistent with the findings of Lee (2001) who uses a similar sample and instrument set. Lee finds that using unbalanced data and a parsimonious instrument set overcomes small sample bias, and thus leads to higher estimates of the intertemporal elasticity of substitution.

\(^{25}\)The results are similar when we restrict our sample to those 12,533 workers with lagged earnings and hours, as in the lagged wage instrument regressions reported in columns 3 and 4. Here, \( \sigma = 0.70, \frac{d \log h_{it}}{d \log(1 - \tau'_{it})} \bigg|_{\lambda_{it}} = 0.98 \)

\(^{26}\)When there is no wage-hours tie, ignoring progressivity leads to a 8 percent reduction (from 0.62 vs. 0.57) in the labor supply response to a one percent change in marginal rates. This is in contrast to Mulligan (1999), who finds that progressivity biases downward labor supply responses. Mulligan emphasizes the difference between \( \sigma^*_{IV} \) and \( \sigma \), but not the difference between \( \sigma \) and \( \frac{d \log h_{it}}{d \gamma_0} \bigg|_{\lambda_{it}} \). Our results show that the latter effect is more important.

\(^{27}\)The estimating equation becomes

\[
\Delta \log h_{it} = \tilde{\sigma} [\Delta \log (1 - \tau'(\cdot)) + \Delta \log E_{it}] - \tilde{\sigma} \log \beta (1 + r_{i-1}(1 - \tau_A)) + \tilde{\sigma} \frac{\beta(1 + r_{i-1}(1 - \tau_A)) \epsilon_{it}}{\lambda_{it-1}} + \frac{1}{1 + \sigma} \epsilon_{it}
\]  

(17)
upward bias using this specification. Results are in columns 3 and 4 of table 1. Using the age polynomial instruments, substituting log earnings for log wages drives $\frac{d\log h_{it}}{d\log(1-\tau_{it})}\bigg|_{\lambda_{it}}$ to 0.80, \(\sigma\) to 1.13, and $\frac{d\log h_{it}}{d\gamma_{0}}\bigg|_{\lambda_{it}}$ to 1.30 when \(\theta = 0.4\).

We also estimated equation (7) on men in the outgoing rotation files of the Current Population Survey (CPS). The key advantage of the CPS, particularly the outgoing rotation files, is large samples. Using similar sample selection criterion as those in our PSID sample, almost 700,000 men between 1979 to 1999 can be used in the estimation. Although the questions are more limited than the PSID, we can recreate the PSID specification, less information on health status. The drawback is that only two observations per person are available. Our estimates, based on the age polynomial instruments, are smaller than the PSID. We get estimates of \(\sigma^{\ast}_{IV}\) of just below 0.20, which is inelastic enough that the bias that arises from tied wage-hours and progressive taxation is hard to detect.

6 Calibration

The estimation results suggest that progressive taxation offsets much but not all of the impact of wage-hours ties. We generalize this result in table 2 by describing calibrations of the key tax derivative, $\frac{d\log h_{it}}{d\gamma_{0}}\bigg|_{\lambda_{it}}$, when plausible ranges of the underlying parameters, \(\theta\), \(\sigma^{\ast}_{IV}\), and \(\gamma_{1}\) are introduced. For \(\theta\), we allow the wage-hours relationship to vary from 0 to 0.60, which seems to cover the range of estimates in the literature. Most studies measure \(\sigma^{\ast}_{IV}\) to be between 0 and 0.5 for continuously employed men but are often greater than 1 for women (e.g. Heckman and MaCurdy (1980)). Therefore, we allow this parameter to vary between 0 and 1.5 to account for the vast majority of estimates in the literature.

Finally, we allow \(\gamma_{1}\) to take on four values: 0, -0.10, -0.18, and -0.28. Zero represents a proportional tax schedule. Larger negative values of \(\gamma_{1}\) characterize more progressive tax systems. In the U.S., we estimate \(\gamma_{1}\) to be, on average, -0.18 for the 1977-1989 period.\(^{28}\)

\[\sigma = \frac{\hat{\sigma}}{1 - \sigma}\] (18)

---

\(^{28}\)This is based on a regression of the PSID respondents’ effective marginal tax rate on log income. Adding a more complicated log income polynomial has only a marginal impact on the progressivity parameters as well as the general fit of the regression.
Panel A displays the proportional tax case. When $\sigma^*_IV = 0.5$ and $\theta = 0.4$, the bias introduced by tied wage-hours offers is 26 percent (0.63 versus 0.50). With $\sigma^*_IV = 1$, a relevant case for women, the bias introduced by $\theta = 0.4$ is 67 percent. However, inelastic labor supply or a small wage-hours tie results in a smaller bias.

Panel B introduces progressive taxes but at a level almost half that of the U.S. The offsetting effect of progressivity is readily apparent. Rather than a 26 percent bias when $\sigma^*_IV = 0.5$ and $\theta = 0.4$, we see a 14 percent difference (0.57 versus 0.50). For $\sigma^*_IV = 1$ the bias drops from 67 to 35 percent. With no tied wage-hours relationship, ignoring progressivity leads to a 4 to 9 percent overstatement $\sigma^*_IV$ when it is between 0.5 to 1.0.

When progressivity is assumed to be at the average level in the U.S. during the 1977 to 1989 period (panel C), the bias introduced by $\theta = 0.4$ falls to 8 to 17 percent, for values of $\sigma^*_IV$ between 0.5 and 1.0. This is consistent with the empirical exercise of the last section. Finally, only when tax progressivity is almost 50 percent higher than what we have seen in the U.S. (i.e. $\gamma_1 = -0.28$) or when $\theta = 0.2$, roughly half of what is found in Aaronson and French (2004), does progressive taxation completely offset the impact of hours-wage ties.

7 Conclusions

There are two important caveats to our analysis. First, we consider the decision of how many hours to work (the “intensive margin”), not the decision of whether to work (the “extensive margin”).29 Heckman (1993) contends that most of the variability in labor supply is at the extensive margin. Furthermore, French (2004b) argues that a large fixed cost of work is necessary to reconcile a high labor supply elasticity at the extensive margin, but a low labor supply elasticity at the intensive margin. It is not clear to what extent the results in this paper extend to a model with a labor force participation decision when there are fixed costs of work.

The second concern is that we focus only on the substitution effect associated with tax wage changes. Understanding the substitution effects is arguably sufficient for understanding the labor supply response to short-term tax adjustments. However, to understand the im-

29See Kimmel and Kniesner (1998) for a decomposition of labor supply elasticities into the intensive and extensive margins).
Table 2: \[ \frac{\partial \log h_{it}}{\partial \log w_{it}} \bigg|_{\lambda_{it}} \]

portance of fundamental tax reform, it is necessary to recognize the wealth effects associated with tax changes.

Nevertheless, we believe that we have shown, both qualitatively and quantitatively, that augmenting a standard intertemporal labor supply model to account for tied wage-hours offers and progressive taxation affects estimates of the intertemporal elasticity of substitution and the labor supply response to tax changes. Using common methods to estimate men’s labor supply functions, we find that the hours response to a change in marginal tax rates may be biased by as much as 10 percent, relative to many of the estimates in the literature, when not accounting for these features of the data. The bias could be up to 20 percent or so for
populations with more elastic labor supply, such as women. Therefore, tax analysts inferring the extent of behavioral responses to tax changes should consider the source of variation used for identification.

**Appendix A: The specification of tied wage-hours offers**

To formally capture the link between hours worked and the offered wage, we first note that, in equilibrium, perfectly competitive firms cover their fixed costs so that total output equals the wage bill plus the fixed cost of work:

\[ p_{it} h_{it} = w_{it} h_{it} + \phi \]  

(19)

where \( \phi \) is the fixed cost per employee, \( p_{it} \) is productivity of worker \( i \) at time \( t \), \( h_{it} \) is hours worked, and \( w_{it} \) is the offered hourly wage. By rewriting equation (19) as

\[ w_{it} = p_{it} - \frac{\phi}{h_{it}} \]  

(20)

it is obvious that the offered hourly wage is rising in hours worked. This relationship implies that at points in the life cycle or tax cycle that hours worked are high, the offered wage should also be high.

Empirical research typically estimates a linearized version of the hours-wage relationship as in equation (2) in the text. For example, Aaronson and French (2004) estimate \( \theta = 0.4 \), a result that appears to be well within the bounds found in the literature. The only papers that we are aware of that test for the existence of a nonlinearity in \( \ln h_{it} \) are Moffitt (1984) and Biddle and Zarkin (1989). While both papers find that equation (2) is misspecified, we have been unable to find any evidence of nonlinearities in either the Panel Study of Income Dynamics (PSID) or Current Population Survey (CPS).\(^{30}\)

Regardless, it is straightforward to compute the approximation bias assumed in equation (2) at different hours levels. The left panel in figure 2 plots the estimated relationship between hours worked and the offered hourly wage, using equation (2), and an estimate of \( \theta = 0.4 \) derived from Aaronson and French (2004). It also presents the structural relationship

\[^{30}\text{Furthermore, at least in the case of Biddle and Zarkin, even their smallest estimates of the elasticity of wages with respect to hours worked appear implausibly large. As we show in section 6, their implied estimates would suggest huge biases to the estimation of intertemporal labor supply elasticities, in cases where this elasticity is sufficiently large.}\]
between hours worked and the offered hourly wage using equation (20), again fitted to match Aaronson and French’s estimate of θ. The right hand panel plots the elasticity of the wage with respect to hours worked implied by equations (20) and (2). Between 1,700 hours and 2,500 hours, encompassing 68 percent of our sample, the implied elasticity from equation (20) is 0.48 to 0.28, versus the constant elasticity implied by equation (2). Therefore, we conclude the linearized relationship in equation (2) provides a good approximation to the structural equation (20).

Moreover, the estimated value of θ seems to provide a plausible estimate of the fixed cost of work. We find \( \phi = $13,450 \) and \( p_{it} = $23.30 \), implying that 28 percent of firm’s labor costs \( \frac{13,450}{13,450+17.26 \times 1,941} \) are fixed. This accords reasonably well with the studies on recruitment and training costs cited in Malcomson (1999).

Figure 2: Offered Hourly Wage as a Function of Hours

---

We use our estimate of \( \theta = 0.4 \), and pick \( \alpha_{it} \) to match the average work year length (1,941 hours) and wage ($17.26, in 1996 dollars) from the sample of older PSID (age 50 to 70) males for equation (2). We pick \( p_{it} \) and \( \phi \) to match the average wage and an elasticity of 0.4 at 1,941 hours of work for our fitted equation (20).
Appendix B: Controlling for changes in the marginal utility of wealth

This appendix describes our approach for dealing with changes in the marginal utility of wealth in order to derive equation (7) from the first differenced labor supply function illustrated in equation (6). The discussion follows MaCurdy (1985), in which the marginal utility of wealth and, in approximation, the log of the marginal utility of wealth are shown to follow a random walk with drift. This result falls out of the Euler equation of the model described in section 2.1. In particular, the Euler equation indicates that individuals equate expected marginal utility across time according to:

$$\lambda_{it-1} = \beta(1 + r_{t-1}(1 - \tau_A))E_{t-1}\lambda_{it}$$

(21)

where rational expectations$^{32}$ implies that innovations to the marginal utility of wealth, denoted $\epsilon_{it}$, should be uncorrelated with lagged values of the marginal utility of wealth:

$$\lambda_{it} = E_{t-1}\lambda_{it} + \epsilon_{it}$$

(22)

Equations (21) and (22) can be rewritten as

$$\frac{\beta(1 + r_{t-1}(1 - \tau_A))\lambda_{it}}{\lambda_{it-1}} = \left(1 + \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}}\right)$$

(23)

Taking logarithms of both sides of (23) and approximating $\log(1 + \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}})$ yields

$$\log \lambda_{it} - \log \lambda_{it-1} + \log \beta(1 + r_{t-1}(1 - \tau_A)) = \log \left(1 + \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}}\right) \approx \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}}$$

(24)

We assume that the approximation in (24) holds with equality, a valid assumption as innovations in the marginal utility of wealth become arbitrarily small.

$^{32}$If workers have rational expectations then at time $t$ they know their state variables $\alpha_{it}, \theta, r_t, \epsilon_{it}, \tau_{it}$ the Markov process that determines the evolution of the state variables, and optimize accordingly.
Combining (24) and (6) results in

\[
\Delta \log h_{it} = \sigma [\Delta \log (1 - \tau'(.) + \Delta \log w_{it}] - \sigma \log \beta (1 + r_{t-1}(1 - \tau_A)) + \frac{\beta(1 + r_{t-1}(1 - \tau_A))\epsilon_{it}}{\lambda_{it-1}} + \Delta \epsilon_{it}.
\]

Because the innovation to the marginal utility of wealth is potentially correlated with wage changes if the wage change is unanticipated, the wage must be instrumented. See section 3 for a discussion on instrument selection.

**Appendix C: Bias from failure to control for tied wage-hours offers and progressive taxation when estimating the intertemporal elasticity of substitution**

In this appendix we consider the likely biases caused by failure to control for tied wage-hours offers and progressive taxation when estimating the intertemporal elasticity of substitution parameter \( \sigma \). We show that disregarding progressive taxation leads to a downward biased estimate of \( \sigma \), as the econometrician overstates the amount of post-tax wage variability that the individual faces. The intuition for this result is straightforward. An anticipated 1 percent change in the post-tax wage causes a \( \sigma \) percent change in hours worked. However, a 1 percent change in the pre-tax wage will lead to less than a 1 percent change in the post-tax wage and thus less than a \( \sigma \) percent change in hours worked.

We also show that overlooking tied wage-hours offers potentially leads to inconsistent estimates of \( \sigma \). The fundamental problem that the econometrician must face when estimating the labor supply response to a wage change is the simultaneous equations bias. Because hours and wages are jointly determined, the econometrician must be careful that that he is estimating a labor supply function (where hours are a function of the wage) rather than a labor demand function (where wages are a function of hours worked). Failure to properly control for the simultaneous equations bias likely leads to an upward bias in \( \sigma \), as we show below.

Therefore, just as the effects of tied wage-hours offers and progressive taxation tend to offset when predicting the labor supply response to a tax change for a given \( \sigma \), the effects of tied wage-hours offers and progressive taxation tend to offset when computing the bias in the estimated value of \( \sigma \).
In order to simplify the analysis, consider the case where \( \log(1 - \tau'(w_{it}h_{it} + y_{it})) \) is linear in the log of labor income, and that the marginal tax rate is unaffected by spousal income:

\[
\log(1 - \tau'(w_{it}h_{it} + y_{it})) = \gamma_0 + \gamma_1 \left( \log(w_{it}) + \log(h_{it}) \right).
\] (26)

Further, ignore the importance of variable interest rates and observable preference shifters.\(^{33}\) Therefore, equation (7) can be rewritten as:

\[
\Delta \log h_{it} = \sigma \left[ \Delta \log(1 - \tau'(\cdot)) + \Delta \log w_{it} \right] + \Delta u_{it}
\] (27)

where \( \Delta u_{it} = \sigma \beta (1 + r_t - 1) \lambda_{it-1} + \Delta \varepsilon_{it} \). Combining equations (??), (26), and (7) yields the reduced form equations of the system:

\[
\Delta \log h_{it} = \sigma \left[ (1 + \gamma_1) \Delta \alpha_{it} \right] + \Delta u_{it}
\] (28)

\[
\Delta \log w_{it} = \frac{[(1 - \sigma \gamma_1) \Delta \alpha_{it} + \theta \Delta u_{it}] + \Delta u_{it}}{1 - \sigma (\gamma_1 (1 + \theta) + \theta)}.
\] (29)

Typically, instrumental variables procedures are used to estimate \( \sigma \) within the misspecified model

\[
\Delta \log h_{it} = \sigma^* \left[ \Delta \log w_{it} \right] + \Delta u_{it}
\]

where \( \sigma^* \) is the wage coefficient on the misspecified model.

Next, we show derivations of the estimated coefficient \( \sigma^*, \sigma^*_{IV} \) using our instrumental variables procedure. Consider the case where \( \text{Cov}(\Delta u_{it}, Z_{it}) = 0 \) (i.e. the instrument is uncorrelated with preferences and the marginal utility of wealth) and and \( \text{Cov}(\log w_{it}, Z_{it}) = \sigma_Z^2 \neq 0 \) (i.e., it is correlated with the productivity parameter \( \Delta \alpha_{it} \)). For example, arguably, the life-cycle wage profile of men measures changes in life cycle productivity but not changes

\(^{33}\)In other words, consider a model where both the log post-tax wage and post-tax hours worked are the residuals from regressions of the log post tax wage and log hours worked on year dummies and observable preference shifters. Using the Frisch-Waugh-Lovell Theorem (Davidson and MacKinnon, 1993), it is straightforward to show that using this approach will still yield a consistent estimate of \( \sigma \).
in life cycle preferences. In this case, we can consider the correlation caused by $Z_{it}$.\textsuperscript{34} then

$$
\sigma_{IV}^* = \frac{\sigma(1 + \gamma_1)(1 - \sigma\gamma_1)\sigma_Z^2}{(1 - \sigma\gamma_1)^2\sigma_Z^2} = \frac{\sigma(1 + \gamma_1)}{1 - \sigma\gamma_1}
$$  \hspace{1cm} (31)

where $\sigma_{IV}^*$ is the probability limit of the estimate. Recall that $\gamma_1 < 0$, so the estimated labor supply elasticity is biased downwards. However, if $\gamma_1 = 0$, then $\sigma_{IV}^* = \sigma$. Therefore, many common instrumental variables strategies overcome problems generated by tied wage-hours offers. However, these strategies will not overcome the model misspecification problem of using the pre-tax wage rather than the post-tax wage.

Note that in this simplified version of the labor supply model, we can analytically show the relationship between $\sigma_{IV}^*$ and $\frac{d\log h_{it}}{d\gamma_0} \bigg|_{\lambda_{it}}$. Combining equations (14) and (31), and assuming $\gamma_2 = \gamma_3 = \ldots = \gamma_K = 0$, the relationship is

$$
\frac{d\log h_{it}}{d\gamma_0} \bigg|_{\lambda_{it}} = \frac{\sigma_{IV}^*}{(1 + \gamma_1) - \sigma_{IV}^*\theta(1 + \gamma_1)}.
$$  \hspace{1cm} (32)

Lastly, we note that instrumental variables estimation of equation (27) does yield consistent estimates of $\sigma$. Using equations (26), (27), (28) and (29), the estimate of $\sigma$ using $E(\Delta \alpha_{it})$ as the instrument for $[\Delta \log(1 - \tau'(\cdot)) + \Delta \log w_{it}]$ will converge to $\sigma_{E(\Delta \alpha_{it})}$:

$$
\sigma_{IV} = \frac{\sigma(1 + \gamma_1)}{\sigma_Z^2}(1 + \gamma_1)\sigma_Z^2 = \sigma.
$$  \hspace{1cm} (33)

By the Frisch-Waugh-Lovell Theorem, by using dummy variables for the interest rate, the procedure will provide consistent estimates of $\sigma$ in equation (7) also.

\textsuperscript{34}More precisely, we can think of an individual’s age-specific productivity as being the sum of two orthogonal components, or $\alpha_{it} = \alpha_t + \psi_{it}$ where $\alpha_t$ is the age-specific component of wages and $\psi_{it}$ is the idiosyncratic component of wages, and $E[\alpha_t\psi_{it}] = 0$. In this case using $\alpha_t$ as the instrument (which is another way of saying that we use the average age-specific wage) yields $\sigma_{IV}^* = \frac{\sigma(1 + \gamma_1)(1 - \sigma\gamma_1)Cov(\Delta \alpha_{it}, \Delta \alpha_{it})}{(1 - \sigma\gamma_1)^2 Cov(\Delta \alpha_{it}, \Delta \alpha_{it})} = \frac{\sigma(1 + \gamma_1)}{1 - \sigma\gamma_1}$.
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*Ruilin Zhou*
WP-99-15

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*Sujit Chakravorti and Ted To*
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George G. Kaufman and Steven A. Seelig
<table>
<thead>
<tr>
<th>Working Paper Series (continued)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Double Play: Simultaneous Speculative Attacks on Currency and Equity Markets</td>
</tr>
<tr>
<td><em>Sujit Chakravorti and Subir Lall</em></td>
</tr>
<tr>
<td>Capital Requirements and Competition in the Banking Industry</td>
</tr>
<tr>
<td><em>Peter J.G. Vlaar</em></td>
</tr>
<tr>
<td>Financial-Intermediation Regime and Efficiency in a Boyd-Prescott Economy</td>
</tr>
<tr>
<td><em>Yeong-Yuh Chiang and Edward J. Green</em></td>
</tr>
<tr>
<td>How Do Retail Prices React to Minimum Wage Increases?</td>
</tr>
<tr>
<td><em>James M. MacDonald and Daniel Aaronson</em></td>
</tr>
<tr>
<td>Financial Signal Processing: A Self Calibrating Model</td>
</tr>
<tr>
<td><em>Robert J. Elliott, William C. Hunter and Barbara M. Jamieson</em></td>
</tr>
<tr>
<td>An Empirical Examination of the Price-Dividend Relation with Dividend Management</td>
</tr>
<tr>
<td><em>Lucy F. Ackert and William C. Hunter</em></td>
</tr>
<tr>
<td>Savings of Young Parents</td>
</tr>
<tr>
<td><em>Annamaria Lusardi, Ricardo Cossa, and Erin L. Krupka</em></td>
</tr>
<tr>
<td>The Pitfalls in Inferring Risk from Financial Market Data</td>
</tr>
<tr>
<td><em>Robert R. Bliss</em></td>
</tr>
<tr>
<td>What Can Account for Fluctuations in the Terms of Trade?</td>
</tr>
<tr>
<td><em>Marianne Baxter and Michael A. Kouparitsas</em></td>
</tr>
<tr>
<td>Data Revisions and the Identification of Monetary Policy Shocks</td>
</tr>
<tr>
<td><em>Dean Croushore and Charles L. Evans</em></td>
</tr>
<tr>
<td>Recent Evidence on the Relationship Between Unemployment and Wage Growth</td>
</tr>
<tr>
<td><em>Daniel Aaronson and Daniel Sullivan</em></td>
</tr>
<tr>
<td>Supplier Relationships and Small Business Use of Trade Credit</td>
</tr>
<tr>
<td><em>Daniel Aaronson, Raphael Bostic, Paul Huck and Robert Townsend</em></td>
</tr>
<tr>
<td>What are the Short-Run Effects of Increasing Labor Market Flexibility?</td>
</tr>
<tr>
<td><em>Marcelo Veracierto</em></td>
</tr>
<tr>
<td>Equilibrium Lending Mechanism and Aggregate Activity</td>
</tr>
<tr>
<td><em>Cheng Wang and Ruilin Zhou</em></td>
</tr>
<tr>
<td>Impact of Independent Directors and the Regulatory Environment on Bank Merger Prices: Evidence from Takeover Activity in the 1990s</td>
</tr>
<tr>
<td><em>Elijah Brewer III, William E. Jackson III, and Julapa A. Jagtiani</em></td>
</tr>
<tr>
<td>Does Bank Concentration Lead to Concentration in Industrial Sectors?</td>
</tr>
<tr>
<td><em>Nicola Cetorelli</em></td>
</tr>
<tr>
<td>On the Fiscal Implications of Twin Crises</td>
</tr>
<tr>
<td><em>Craig Burnside, Martin Eichenbaum and Sergio Rebelo</em></td>
</tr>
</tbody>
</table>
Working Paper Series (continued)

Sub-Debt Yield Spreads as Bank Risk Measures
Douglas D. Evanoff and Larry D. Wall

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WP-01-26

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Yukako Ono  
WP-02-01

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Victor Stango  
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Fiorella De Fiore and Pedro Teles  
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Stefania Albanesi, V. V. Chari and Lawrence J. Christiano  
WP-02-04

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Lawrence J. Christiano, Christopher Gust and Jorge Roldos  
WP-02-05

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Raphael Bostic, Hamid Mehran, Anna Paulson and Marc Saidenberg  
WP-02-06

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WP-02-08
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