Federal Reserve Bank of Chicago

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Abstract

The Chicago Fed dynamic stochastic general equilibrium (DSGE) model is used for policy analysis and forecasting at the Federal Reserve Bank of Chicago. This article describes its specification and estimation, its dynamic characteristics and how it is used to forecast the US economy. In many respects the model resembles other medium scale New Keynesian frameworks, but there are several features which distinguish it: the monetary policy rule includes forward guidance, productivity is driven by neutral and investment specific technical change, multiple price indices identify inflation and there is a financial accelerator mechanism.

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1 Introduction

In this paper, we describe the Chicago Fed's estimated dynamic stochastic general equilibrium model. This framework yields a history of identified structural shocks, which we apply to illuminate recent macroeconomic developments. To aid in the understanding of these results, we follow them with summaries of the model's structure, the data and methodology employed for estimation, and the estimated model's dynamic properties.

In several respects, the Chicago Fed DSGE model resembles many other New Keynesian frameworks. There is a single representative household that owns all firms and provides the economy's labor. Production uses capital, differentiated labor inputs, and differentiated intermediate goods. The prices of all differentiated inputs are "sticky", so standard forward-looking Phillips curves connect wage and price inflation with the marginal rate of substitution between consumption and leisure and marginal cost, respectively. Other frictions include investment adjustment costs and habit-based preferences.

There are, however, several features of the model which distinguish it from these frameworks. For instance, in addition to the usual current monetary policy shock in the monetary policy rule, we account for short-term guidance regarding the future path of the federal funds rate. A factor structure estimated from federal funds and Eurodollar futures prices is used to identify both a *current policy factor* and a *forward guidance factor*.

Also included in our monetary policy rule is a shock which dominates changes in long-run expected inflation. We refer to this shock, captured in a shifting intercept in the monetary policy rule, as the *inflation anchor shock*, and we discipline its fluctuations with data on long-term inflation expectations from the Survey of Professional Forecasters.

Another distinguishing feature of the Chicago model is the use of multiple price indices. Alternative available indices of inflation are decomposed into a single model-based measure of consumption inflation and idiosyncratic (series specific) disturbances that allow for persistent deviations from this common component. Estimation uses a factor model with the common factor derived from the DSGE framework.

The model also incorporates a financial accelerator mechanism. We introduce risk-neutral entrepreneurs into the New Keynesian framework who purchase capital goods from capital installers using a mix of internal and external resources. These entrepreneurs optimally choose their rate of capital utilization and rent the effective capital stock to goods producing firms. The dependence on internal resources explicity links fluctuations in the external finance premium, private net worth, and the state of the economy.

To identify parameters governing the financial accelerator, we use multiple credit spreads and data on borrowing by nonfinancial businesses and households. Consistent with our definition of investment, which includes consumer durables and residential investment as well as business fixed investment, we relate the external finance premium to a weighted average of High Yield corporate bond and Asset-backed security spreads, where the weight each receives is derived from the shares of nonfinancial business and household debt in private credit taken from the Flow of Funds. To capture the impact of entrepreneurial leverage on financial conditions, we rely on the ratio of private credit to nominal GDP.

The remainder of the paper proceeds as follows. The following section describes a recent forecast derived from the model. Section 3 describes the shocks in the model and how they have contributed to business cycle fluctuations. Section 4 outlines the specification and estimation of the model and presents parameter estimates. The last two sections reviews key information required to understand the model's dynamics. In Section 5 we describe five of the model's key equations and Section 6 reports the impulse response functions for key variables and the five structural shocks which drive most of the model's fluctuations.

2 A Recent Forecast

Constructing forecasts based on this model requires us to assign values to its many parameters. We do so using Bayesian methods to update an uninformative prior with data from 1989:Q2 through 2011:Q4. All of our forecasts condition on the parameters equaling their values at the resulting posterior's mode. These parameter values together with the data yield a posterior distribution of the economy's state in the final sample quarter. Our point forecasts run the model forward from the mode of the posterior distribution of the economy's

Table 1: Model Forecasts Q4 over Q4

	2011	2012	2013	2014	2015
Real GDP	1.6	2.1	2.6	2.5	2.5
Federal Funds Rate	0.1	0.2	0.2	0.4	1.3
Core PCE Inflation	1.8	1.3	1.0	1.3	1.5
Consumption	2.3	1.7	2.1	2.1	2.1
Investment	6.9	3.4	2.9	3.0	3.2

state in the final sample quarter assuming all shock innovations are equal to zero from then on.

To address the unique nature of fluctuations in the recent period, we specify a sample break in our model that begins in 2008:Q1. At this point, we calibrate three parameters and re-estimate the parameters governing the decomposition of the current policy and forward guidance factors on the remaining sample. All other parameters are fixed at their pre-sample break values. The three parameters we calibrate involve a structural break in the persistence of the *discount shock* which affects households' rate of time preference, the variance of the *inflation anchor shock*, and in the output gap coefficient in the monetary policy rule.

Increasing the persistence of the shock to the discount rate captures the idea that deleveraging by households following a financial crisis is unusually slow. Its value in the second half of our sample period raises its half life from a little over half a year in the pre-crisis sample to more than three years in the second half of our sample. Similarly, lowering the variance of the inflation anchor shock reflects the fact that inflation expectations exhibit a downward trend in the early part of our sample, but have fluctuated considerably less since.

In the second half of our sample period, we also work with a coefficient on the output gap in our policy rule that is three times larger than its pre-crisis estimate. Our motivation for doing so is that the FOMC's policy response to the recent downturn in activity was more aggressive than in previous recessions in our sample, each of which was moderate by historical standards. Furthermore, in combination with the above, this assumption increases the likelihood that the zero lower bound on the federal funds rate is binding at any given date.

Table 1 presents data from 2011 and forecasts for the following four years. The first three rows correspond to three key macroeconomic observables, Real GDP growth (Q4-over-Q4), the Federal Funds Rate (Q4 average), and growth of the Core PCE deflator (Q4-over-Q4). The following rows report forecasts of Q4-over-Q4 growth for two model-defined aggregates of importance: Consumption of nondurable goods and non housing services and Investment in durable goods, residential housing, and business equipment and structures.

Figure 1 complements this with quarter-by-quarter data and forecasts of these series along with the log level of per capita hours worked in the nonfarm business sector. The plots' dashed grey lines indicate the series' long-run values. The economy's long-run GDP growth rate – which we identify with potential growth – equals 2.7 percent.

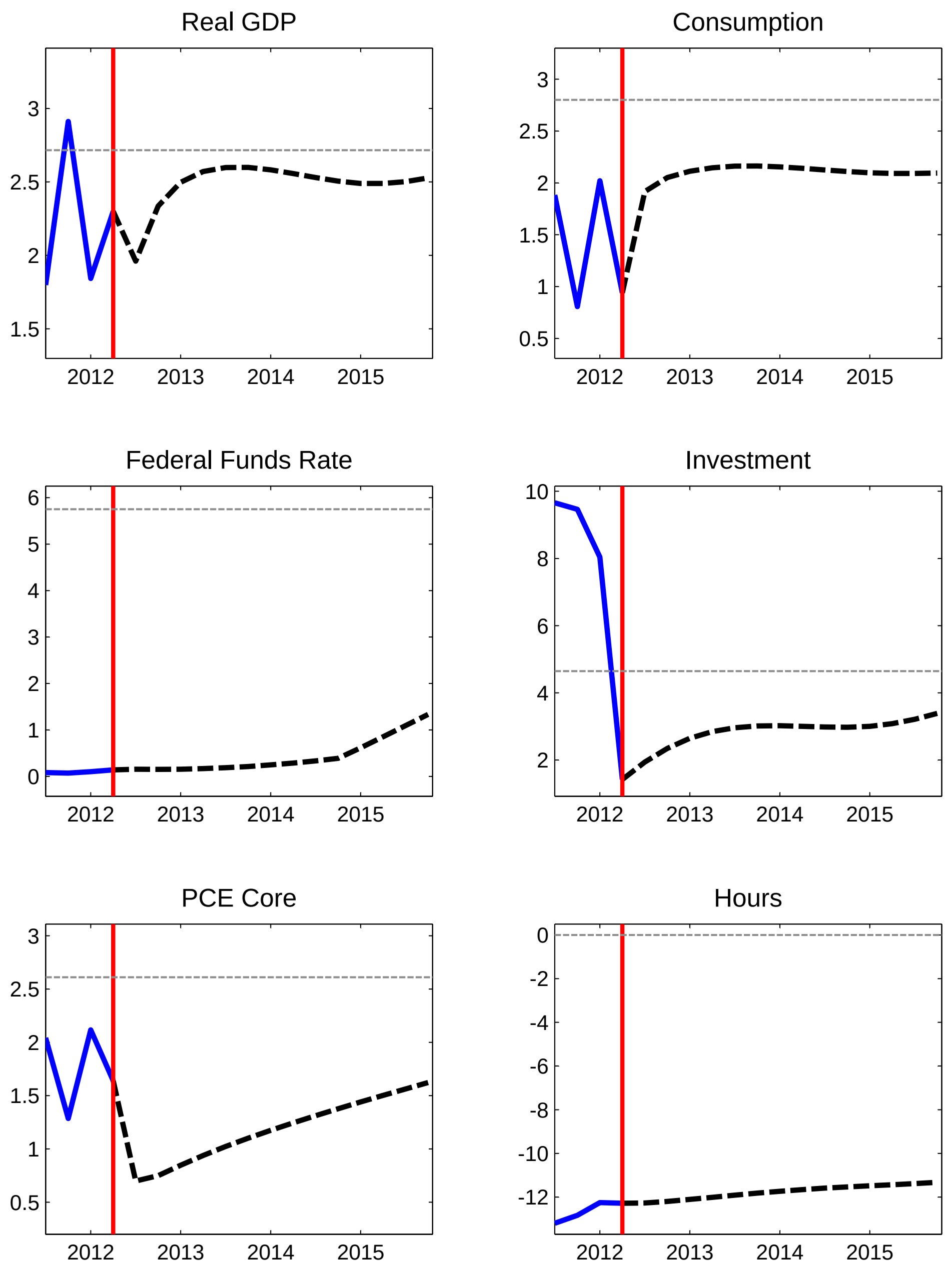
The economy grows just below potential throughout the forecast horizon. Consequently, per capita hours do not return to their steady-state by the end of 2015. The protracted weakness in the forecast arises from the model's *spread shock*. This shock, which embodies movements in the external finance premium beyond what is warranted by firms' balance sheets, has particularly persistent effects on economic activity.

The forecasted path for core PCE inflation remains in the range of 0.7 to 1.6 percent throughout the forecast horizon, well below the model's long-run expected inflation rate of 2.6 percent. Our forecast for mild inflation is explained by a recent negative realization of the model's *price mark-up shock* inferred from incoming Q2 data.

The contractionary forces shaping our forecast have been partially offset by monetary policy, which in our model captures policy makers' announcements regarding the path of the federal funds rate over the next ten quarters. Forward guidance has added about 0.4 percent to four quarter real GDP growth over the last year. The *forward guidance factor* has supported consumption and investment growth, as well as hours.

Our forecast for the federal funds rate is informed by futures prices which hold the funds rate in the range of zero to 0.25 percent through the end of 2014. Thereafter, the forecast rate begins to rise as the conventional monetary policy rule dynamics take over, increasing to 1.3 percent by the end of 2015. The expected output and inflation gaps are weak enough to merit only the gradual removal of policy accommodation. The increase in the funds rate

Figure 1: Quarterly Model Forecasts



in 2015 instead largely reflects mean reversion in our estimated interest rate rule.

3 Shock Decompositions

Our analysis identifies the structural shocks responsible for past fluctuations. In total, the model features eleven structural shocks and sixteen idiosyncratic disturbances without structural interpretations. For parsimony's sake, we group the shocks according to the following taxonomy.

Demand. These are the structural non-policy shocks that move output and consumption-based inflation in the same direction. The model features four of them. One changes the households' rate of time discount. We call this the Discount shock. The next two are financial disturbances. The Spread shock generates fluctuations in the external finance premium beyond the level warranted by current economic conditions, and the Net Worth shock generates exogenous fluctuations in private balance sheets. Finally, this category also includes a shock to the sum of government expenditures, net exports, and changes in the valuation of inventories.

Supply. Five shocks move real GDP and consumption-based inflation in opposite directions on impact. These *supply shocks* directly change

- Neutral Technology,
- Investment-Specific/Capital-Embodied Technology,
- Markups of Intermediate Goods Producers,
- Markups of Labor Unions, and
- Households' Disutility from Labor

The shock to households' disutility from labor is assumed to follow an ARMA(1,1) process, which is a parsimonious way of addressing low frequency movements in per capita hours worked and high frequency variation in wages.

Policy. The model's monetary policy follows an exogenous rule with interest-rate smoothing, a time varying intercept, and a factor structure which identifies a Current Policy factor and a Forward Guidance factor. The time varying intercept, or Inflation Anchor shock, is disciplined by equating model-based average expected consumer price inflation to a measure of long-term inflation expectations taken from the Survey of Professional Forecasters. The Current Policy shock and Forward Guidance factor are derived from contemporaneous federal funds futures prices zero to four quarters before they affect the federal funds rate. In the second half of the sample, we extend the number of futures contracts so as to capture developments which affect the federal funds rate up to ten quarters ahead.

Residual. We group the remaining shocks into a residual category. These include the idiosyncratic, that is series specific, shocks to the various price measures and monetary policy signals based on their factor structures, as well as the measurement errors in the interest rate spread and private credit-to-GDP ratio we use to capture the external finance premium and entrepreneurial net worth.

Table 2 reports the fraction of business-cycle variance attributable to shocks in each category for five key variables, the level of Real GDP, Real Consumption, and Real Investment, and the Federal Funds Rate and Core PCE Inflation. This decomposition is based on one-step-ahead forecast errors. As already mentioned, we introduce an unanticipated sample break in 2008:Q1 and hence report decompositions for both sub-samples. Demand shocks dominate business cycles. This is particulary true in the second half of our sample. Monetary policy shocks make only a minor contribution in the earlier sample period, but explain almost one-third of GDP's total business cycle variance in the later period, due largely to their effect on Investment.

Inflation fluctuations are dominated by supply shocks in the early part of the sample, with exogenous shocks to intermediate goods' markups almost entirely accounting for supply shocks' 63 percent contribution. In contrast, supply shocks account for between 7 and 12 percent of GDP's total business-cycle variance depending on the sample period. The accounting for the Federal Funds Rate's variance is also very sample-dependent. In the second half of the sample, demand shocks are the key driver, while policy shocks dominate in the earlier period. Perhaps this is unsurprising, considering that we classify the shock that directly moves households' rate of time preference as "demand," and increase the activity coefficient in our interest rate rule post-2007.

Table 2: The Model's Decomposition of Business-Cycle Variance

1989:Q2-2007:Q4

	Demand	Supply	Policy	Residual
Real GDP	0.73	0.12	0.12	0.02
Federal Funds Rate	0.20	0.04	0.77	0.00
PCE Core	0.15	0.63	0.13	0.09
Consumption	0.88	0.08	0.03	0.01
Investment	0.88	0.04	0.08	0.00

2008:Q1-2011:Q4

	Demand	Supply	Policy	Residual
Real GDP	0.62	0.07	0.31	0.01
Federal Funds Rate	0.78	0.01	0.21	0.00
PCE Core	0.95	0.03	0.01	0.01
Consumption	0.96	0.02	0.03	0.00
Investment	0.61	0.04	0.34	0.00

Note: For each variable, the table lists the fraction of variance at frequencies between 6 and 32 quarters attributable to shocks in the listed categories. The numbers may not add to one due to rounding.

4 The Model's Specification and Estimation

Our empirical work uses eighteen variables, measured from 1989:Q2 through the present:

- Growth of nominal per capita GDP,
- Growth of nominal per capita consumption, which sums Personal Consumption Expenditures on Nondurable Goods and Services;
- Growth of nominal per capita investment; which sums Business Fixed Investment, Residential Investment, and Personal Consumption Expenditures on Durable Goods
- Per capita hours worked in Nonfarm Business,
- Growth of nominal compensation per hour worked in Nonfarm Business,
- Growth of the implicit deflator for GDP,
- Growth of the implicit deflator for consumption, as defined above,

- Growth of the implicit deflator for investment, as defined above,
- Growth of the implicit deflator for core PCE,
- Growth of the implicit deflator for core CPI,
- The interest rate on Federal Funds,
- Ten-year ahead CPI forecasts from the Survey of Professional Forecasters,
- A weighted average of High-Yield corporate and Mortgage-backed bond spreads with the 10-year Treasury and an Asset-backed bond spread with the 5-year Treasury; where the weights equal the shares of nonfinancial business, household mortgage, and household consumer debt in private credit,
- Ratio of private credit-to-GDP; which sums household and nonfinancial business credit market debt outstanding and divides by nominal GDP,
- Quarterly averages of federal funds and Eurodollar futures contract rates one through four quarters ahead.

The ratio of private credit-to-GDP is detrended using the Hodrick-Prescott filter with smoothing parameter 1e5. We do not directly use data on government spending, net exports, or the change in the valuation of inventories. Their sum serves as a residual in the national income accounting identity. To construct series measured per capita, we used the civilian non-institutional population 16 years and older. To eliminate level shifts associated with the decennial census, we project that series onto a fourth-order polynomial in time.

Our model confronts these data within the arena of a standard linear state-space model. Given a vector of parameter values, θ, log-linearized equilibrium conditions yield a first-order autoregression for the vector of model state variables, ζ_t.

$$\begin{aligned} \zeta_t &= F(\theta)\zeta_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \Sigma(\theta)) \end{aligned}$$

Here, ε_t is a vector-valued innovation built from the model innovations described above. Many of its elements identically equal zero. Table 3 lists the elements of ζ_t. Habit puts lagged nondurable consumption into the list, and investment adjustment costs place lagged investment there. Rules for indexing prices and wages that cannot adjust freely require the state to include lags of inflation and technology growth. Financial frictions place lagged entrepreneurial borrowing and net worth in the state. The list includes the lagged policy rate because it appears in the monetary policy rule.

Gather the date t values of the fourteen observable variables into the vector y_t. The model analogues to its elements can be calculated as linear functions of ζ_t and ζ_{t-1}. We suppose that the data equal these model series plus a vector of "errors" v_t.

$$
\begin{aligned}
y_t &= G(\theta)\zeta_t + H(\theta)\zeta_{t-1} + v_t \\
v_t &= \Lambda(\varphi)v_{t-1} + e_t \\
e_t &\sim N(0, D(\varphi))
\end{aligned}
$$

Here, the vector φ parameterizes the stochastic process for v_t. In our application, the only non-zero elements of v_t correspond to the observation equations for the three consumption-based measures of inflation, the GDP deflator, and the spread and private credit-to-GDP measures. The idiosyncratic disturbances in inflation fit the high-frequency fluctuations in prices and thereby allow the price markup shocks to fluctuate more persistently. These errors evolve independently of each other. In this sense, we follow Boivin and Giannoni (2006) by making the model errors "idiosyncratic". The other notable feature of the observation equations concerns the GDP deflator. We model its growth as a share-weighted average of the model's consumption and investment deflators.

Table 4 displays the estimated modes for a number of model parameters. We denote the sample of all data observed with Y and the parameters governing data generation with $\Theta = (\theta, \varphi)$. The prior density for Θ is $\Pi(\Theta)$, which resembles that employed by Justiniano, Primiceri, and Tambalotti (2011). Given Θ and a prior distribution for ζ_0, we can use the model solution and the observation equations to calculate the conditional density of Y,

Table 3: Model State Variables

Symbol	Description	Disappears without
C_{t-1}	Lagged Consumption	Habit-based Preferences
I_{t-1}	Lagged Investment	Investment Adjustment Costs
π^p_{t-1}	Lagged Price Inflation	Indexing "stuck" prices to lagged inflation
K_t	Stock of Installed Capital	
A_t	Hicks-Neutral Technology	
a_t	Growth rate of A_t	Autoregressive growth of A_t
a_{t-1}	Lagged Growth Rate of A_t	Indexing "stuck" wages to lagged labor productivity growth
Z_t	Investment-Specific Technology	
z_t	Growth rate of Z_t	Autoregressive growth of Z_t
z_{t-1}	Lagged Growth Rate of Z_t	Indexing "stuck" wages to lagged labor productivity growth
ϕ_t	Labor-Supply Shock	
b_t	Discount Rate Shock	
$\lambda_{w,t}$	Employment Aggregator's Elasticity of Substitution	Time-varying Wage Markups
$\lambda_{p,t}$	Intermediate Good Aggregator's Elasticity of Substitution	Time-varying Price Markups
B_t	Entrepreneurial Borrowing	Need for external finance
B_{t-1}	Lagged Borrowing	
N_t	Entrepreneurial Net Worth	Risk-neutral entrepreneurs
N_{t-1}	Lagged Net Worth	
ν_t	Spread Shock	
ς_t	Net Worth Shock	
g_t	Government Spending Share Shock	
R_{t-1}	Lagged Nominal Interest Rate	Interest-rate Smoothing
$\varepsilon_{R,t}$	Monetary Policy Shock	
$\pi^\star_t$	Inflation Drift Shock	

Table 4: Selected Model Parameter Modes

Parameter	Description	Mode
ρ_π	Inflation anchor persistence	0.99
ρ_R	Inflation rate smoothing	0.85
ϕ_p	Inflation gap response	1.35
ϕ_y	Output gap response	0.10
α	Capital Share	0.17
δ	Depreciation rate	0.03
ι_p	Indexation Prices	0.08
ι_w	Indexation Wages	0.28
$\gamma_{\star 100}$	Steady state consumption growth	0.47
$\gamma_{\mu 100}$	Steady state investment-specific technology growth	0.60
$\mathcal{H}$	Habit	0.89
λ_p	Steady state price markup	0.10
π^{ss}	Steady state quarterly inflation	0.65
β	Steady state discount factor	0.997
$\mathcal{G}^{ss}$	Steady state residual expenditure share in GDP	0.22
ν	Inverse Frisch elasticity	2.17
κ_p	Price Phillip's curve slope	0.001
κ_w	Wage Phillip's curve slope	0.005
χ	Utilization elasticity	4.80
$\mathcal{S}$	Investment adjustment elasticity	7.84
$\frac{B}{N}$	Steady state borrowing to net worth ratio	1.11
$\mathcal{F}_{KN}$	Steady state spread	0.69
τ	Net worth elasticity	0.002
ζ	Entrepreneur survival probability	0.91
ρ_b	Discount factor persistence	0.76
ρ_υ	Spread persistence	0.99
ρ_ς	Net worth persistence	0.64
ρ_g	G + NX persistnce	0.99
ρ_z	Neutral technology growth persistence	0.10
ρ_μ	Investment technology growth persistence	0.73
ρ_{λ_p}	Price markup persistence	0.61
ρ_ψ	AR coefficient labor disutility	0.95
θ_ψ	MA coefficient labor disutility	0.98

$F(Y|\Theta)$. To form the prior density of ζ_0, we apply the Kalman filter. The actual estimation begins with 1989:Q2. Bayes rule then yields the posterior density up to a factor of proportionality.

$$P(\Theta|Y) \propto F(Y|\Theta)\Pi(\Theta)$$

Beginning in 2008:Q1, we set the persistence of the discount shock at 0.95 and scale the variance of the inflation anchor shock to be one quarter and the coefficient on the output gap in the monetary policy rule to be three times their earlier values. We re-estimate the volatility and factor loadings of the current policy and forward guidance factors and the standard deviations of the idiosyncratic shocks as well as the volatility of the discount shock. All remaining parameters are held fixed at their values in the first sub-sample. The Kalman filter is initialized with the necessary pre-sample data, and estimation on this second sample period proceeds as in the first except that as noted above we include signals up to ten quarters ahead in the estimation of the policy rule. We then calculate our forecasts with the model's parameter values set to this posterior distribution's mode.

Table 5 displays the estimate modes for both sample periods for the model parameters that are re-estimated on the second sub-sample.

5 Five Key Equations

This section summarizes the inferred parameters by reporting the estimates of five key equations: the two equations of the financial accelerator capturing the External Finance Premium and the evolution of private Net Worth, and the log-linearized forms of the Monetary Policy Rule, the Price Phillips Curve, and the Wage Phillips Curve.

5.1 Financial Accelerator

Financial frictions in the model arise from imperfections in private financial intermediation due to lenders' costly state verification of the returns realized by entrepreneurs' projects. We introduce risk neutral entrepreneurs into the model who at the end of period t purchase capital goods, $\overline{K_t}$, from the capital installers at the price Q_t, using a mix of internal and external

Table 5: Selected Modes for Re-estimated Parameters

Parameter	Description	First Mode	Second Mode
σ_b	Std. dev. Discount factor shock	0.14	0.06
σ_{f1}	Std. dev. Current Policy factor	0.04	0.05
σ_{f2}	Std. dev. Forward Guidance factor	0.06	0.07
σ_{u1}	Std. dev. 1st idiosyncratic shock	0.04	0.05
σ_{u2}	Std. dev. 2nd idiosyncratic shock	0.02	0.03
σ_{u3}	Std. dev. 3rd idiosyncratic shock	0.02	0.03
σ_{u4}	Std. dev. 4th idiosyncratic shock	0.05	0.03
σ_{u5}	Std. dev. 5th idiosyncratic shock		0.02
σ_{u6}	Std. dev. 6th idiosyncratic shock		0.02
σ_{u7}	Std. dev. 7th idiosyncratic shock		0.02
σ_{u8}	Std. dev. 8th idiosyncratic shock		0.09
σ_{u9}	Std. dev. 9th idiosyncratic shock		0.09
σ_{u10}	Std. dev. 10th idiosyncratic shock		0.09
$\mathcal{A}_1$	Current 1	1.25	1.25
$\mathcal{A}_2$	Current 2	0.69	0.43
$\mathcal{A}_3$	Current 3	0.42	0.18
$\mathcal{A}_4$	Current 4	-0.21	0.08
$\mathcal{A}_5$	Current 5		-0.01
$\mathcal{A}_6$	Current 6		0.02
$\mathcal{A}_7$	Current 7		0.01
$\mathcal{A}_8$	Current 8		-0.01
$\mathcal{A}_9$	Current 9		-0.00
$\mathcal{A}_{10}$	Current 10		-0.02
$\mathcal{B}_1$	Lead 1	0.80	0.16
$\mathcal{B}_2$	Lead 2	1.00	0.55
$\mathcal{B}_3$	Lead 3	0.92	0.78
$\mathcal{B}_4$	Lead 4	0.43	1.03
$\mathcal{B}_5$	Lead 5		1.00
$\mathcal{B}_6$	Lead 6		1.09
$\mathcal{B}_7$	Lead 7		1.03
$\mathcal{B}_8$	Lead 8		1.05
$\mathcal{B}_9$	Lead 9		0.91
$\mathcal{B}_{10}$	Lead 10		0.98

resources, given by end of period net worth, N_t, and borrowing B_t, such that $Q_t\overline{K_t} = N_t + B_t$.

In the next period, $t+1$, entrepreneurs optimally choose the rate of utilization, u_{t+1}, and rent the effective capital stock $K_{t+1} = u_{t+1}\overline{K_t}$ to the goods producing firms, receiving in return the gross rental rate of capital ω^k_{t+1}. At the end of period $t+1$ they resell the remaining capital stock, $(1-\delta)\overline{K_t}$ back to the capital producers at the price Q_{t+1}.

5.1.1 External Finance Premium

We assume that the external finance premium –the ratio of the equilibrium return to capital and the expected real interest rate– is an increasing function of the entrepreneurs' leverage ratio, $\frac{\overline{K_t}\ Q_t}{N_t}$, according to

$$\frac{E_t[1+r^k_{t+1}]}{E_t[\frac{1+R_t}{\pi_{t+1}}]} = F\left[\frac{\overline{K_t}\ Q_t}{N_t}\right]e^{\nu_t}$$

with R_t the nominal interest rate, π_{t+1} the gross inflation rate and $F(1) = 1$, $F' > 0, F'' > 0$.[1] The spread shock, e^{ν_t}, can be viewed as a disturbance to credit supply, moving the external finance premium beyond the level dictated by entrepreneurial net worth. We parameterize the steady state level of $\mathcal{F}_{KN}$ as well as its elasticity τ. We estimate the former to be 2.76 and the latter to be pretty small. The annualized steady state external finance premium is estimated to be 2.98 percent.

5.1.2 Net Worth

The law of motion for entrepreneurial net worth is given by

$$N_t = 0.91\left\{\overline{K}_{t-1}Q_{t-1}[1+r^k_t] - E_{t-1}[1+r^k_{t-1}]B_{t-1}\right\} + 0.09\Gamma_t + \varsigma_t$$

where Γ_t is the transfer from exiting to new entrepreneurs and ς_t is a shock to net worth that can arise for instance from time-varying survival probabilities for entrepreneurs. The AR(1) laws of motion for the spread and net worth shocks, ν_t and ς_t, are estimated to have independent autoregressive parameters $(0.99, 0.64)$ and volatilities i=$0.23, 0.37$.

[1] Notice that that if entrepreneurs are self-financed, which we rule out in steady state, $F(1) = 1$ and there is no external finance premium.

5.2 Monetary Policy Rule

$$R_t = 0.85R_{t-1} + 0.32\left(1.34\left(\frac{1}{4}\sum_{j=-1}^{2} E_t(\pi_{t+j}) - \pi_t^\star\right) + 0.11\left(\frac{1}{4}\sum_{j=-1}^{2} E_t(\hat{x}_{t+j})\right)\right) + \sum_{j=0}^{M} \xi_{t-j,j}$$

$$[1 + \lambda(1-L)^2(1-F)^2]\hat{x}_t = \lambda(1-L)^2(1-F)^2\hat{y}_t$$

$$\xi_{t,j} = \mathcal{A}_j f_t^c + \mathcal{B}_j f_t^F + u_{t,j}$$

Besides the lagged interest rate, the variables appearing on the right-hand side of our interest rate rule are an inflation gap, an output gap, and current and future deviations from the systematic component of the rule. For any variable v, $\hat{v}$ denotes deviations from steady state.

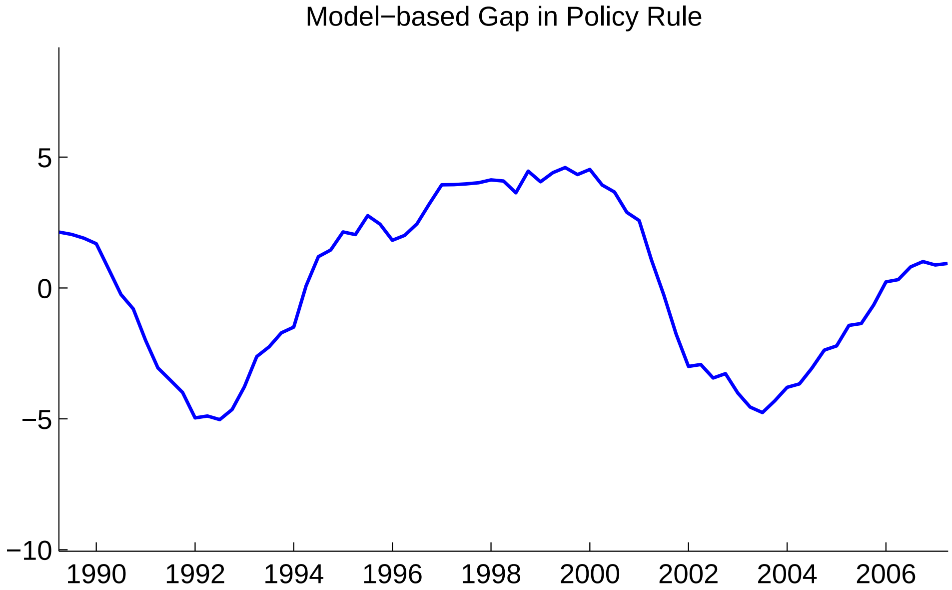
The inflation gap is the deviation of a four quarter average of model inflation from the time-varying inflation drift, or anchor, π_t^* which varies exogenously according to an AR(1) process. The four quarter moving average of inflation includes both lagged, current, and future values of inflation. The monetary authority uses the structure of the model to forecast the future terms.

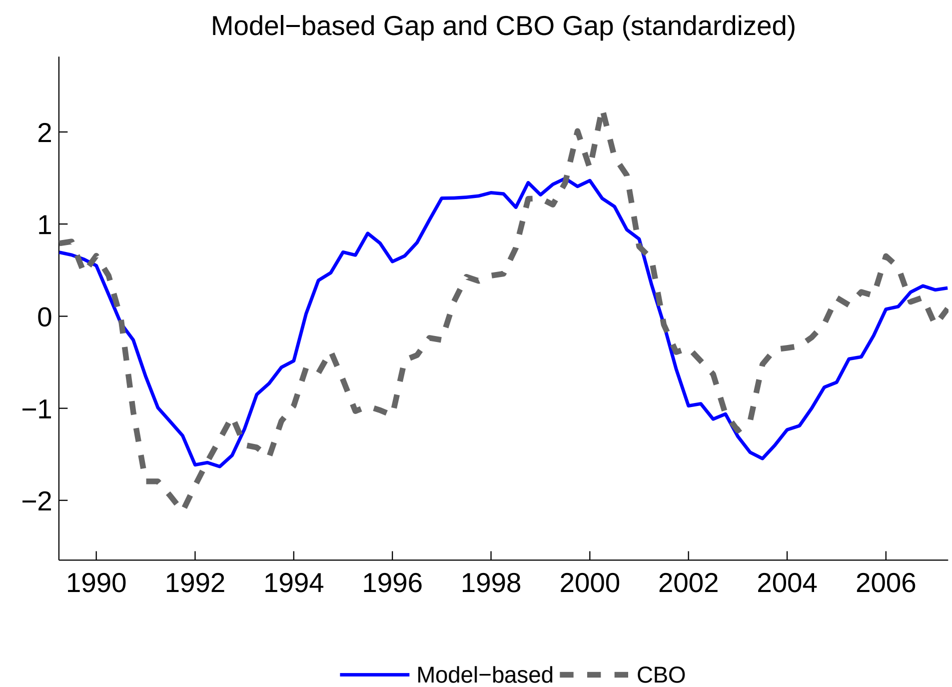
The inflation drift term can be interpreted in the context of the model as the monetary authority's medium-run desired rate of inflation. It is perfectly credible in the sense that we equate model-based average expected consumer price inflation over the next forty quarters to the ten-year ahead CPI forecast from the Survey of Professional Forecasters.

We define the output gap as the four-quarter moving average of detrended model output. Following Curdia, Ferrero, Ng, and Tambalotti (2011), the detrending is model-based where L and F represent the lag and lead operators and λ is a smoothing parameter that we estimate to be 9104. The filter above approximates the Hodrick-Prescott filter. While the methodologies differ, figure 2 demonstrates that our output gap also compares well with the CBO's output gap measure from 1989:Q2-2007:Q2.

Holding the economy's growth rate fixed, the long-run response of R_t to a permanent one-percent increase in inflation is 1.3 percent. Thus, the model satisfies the Taylor principle. Our estimated coefficient of the output response to our rule is 0.1. We scale this coefficient by a factor of 3 in the second half of our sample.

Figure 2: The Output Gap





Monetary policy shocks have a factor structure such that the factors f_t^c and f_t^F represent the *i.i.d. current policy shock* and the *forward guidance factor*. The disturbances $u_{t,j}$ are assumed uncorrelated across both j and t, and the factor structure identified by restricting the loading matrices, $\mathcal{A}$ and $\mathcal{B}$, such that the forward guidance factor only influences future values of the federal funds rate. Figure 3 depicts our estimates of both factors from 1989:Q2-2007:Q2.

By including forward looking terms for the inflation and output gaps in the interest rate rule, we account for news about both up to two quarters ahead from our forward guidance shocks. We estimate both the current policy and forward guidance factors using contemporaneous data on the federal funds rate and federal funds and Eurodollar futures contract prices. In the first sub-sample, this includes futures contracts one to four quarters ahead; while in the second sub-sample, we use futures contracts one to ten quarters ahead.

Historical decompositions highlighting the role played by forward guidance shocks for per capita GDP, core PCE inflation, and the federal funds rate from 1989:Q2-2007:Q2 are shown in figures, 4, 5, and 6. Forward guidance played a role in explaining each during the 1993-1995 and 2002-2004 periods as detailed in Campbell, Fisher, and Justiniano (2012). The first episode can be linked to statements by Chairman Greenspan extending expectations for increases in the funds rate, while the second is closely related to the extended period of low rates that followed 9/11.

5.3 Price Phillips Curve

$$\pi_t^p = 0.923 E_t \pi_{t+1}^p + 0.074 \pi_{t-1}^p + 0.002 s_t + \epsilon_t^p$$

Here, s_t represents intermediate goods producers' common marginal cost. The introduction of inflation drift does not alter the dynamic component of inflation indexation which is linked to the previous quarter's inflation rate.

- The slope of the estimated Phillips Curve is considerably flat compared to some other estimates in the literature. This reflects at least in part our sample period which starts in 1989.

Figure 3: Current Policy and Forward Guidance Factors

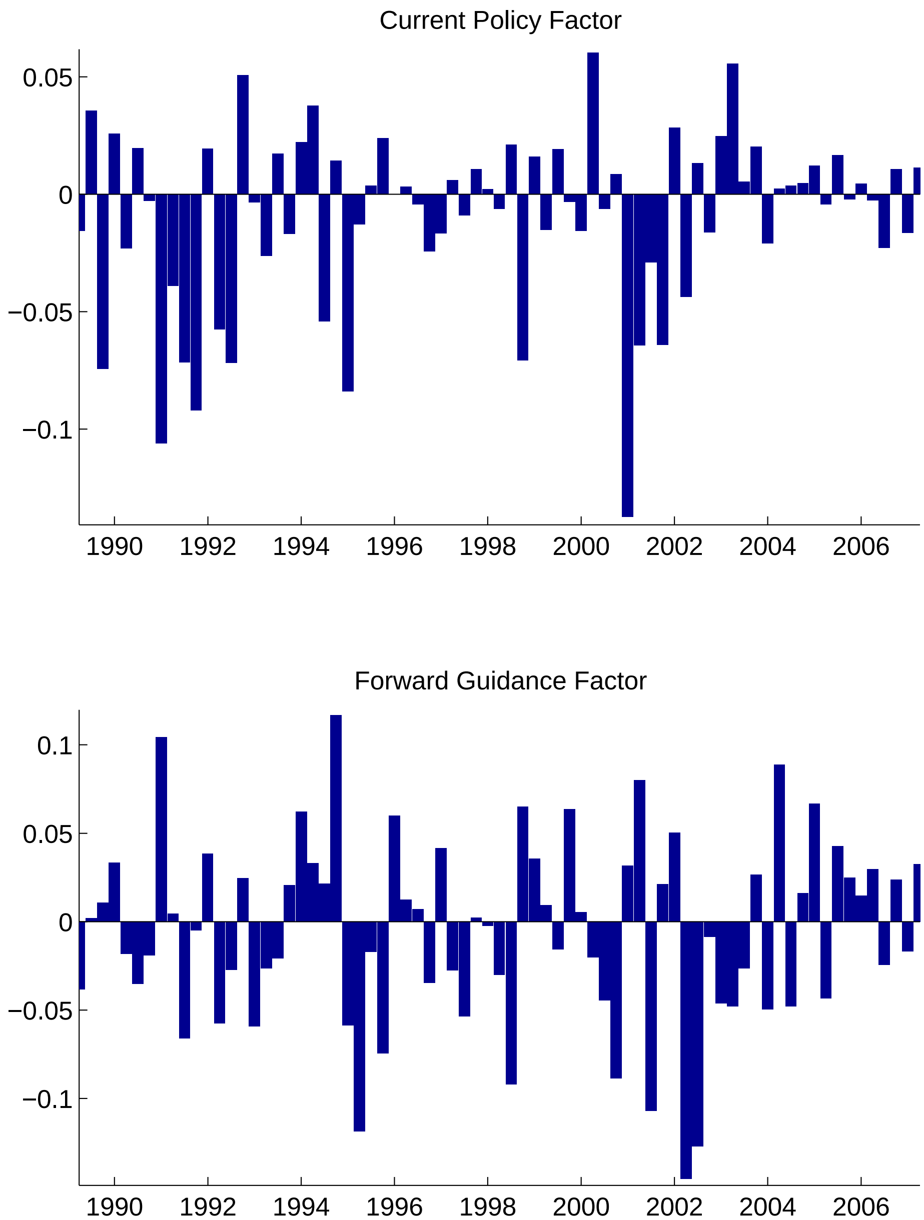


Figure 4: Historical Decomposition of per capita GDP

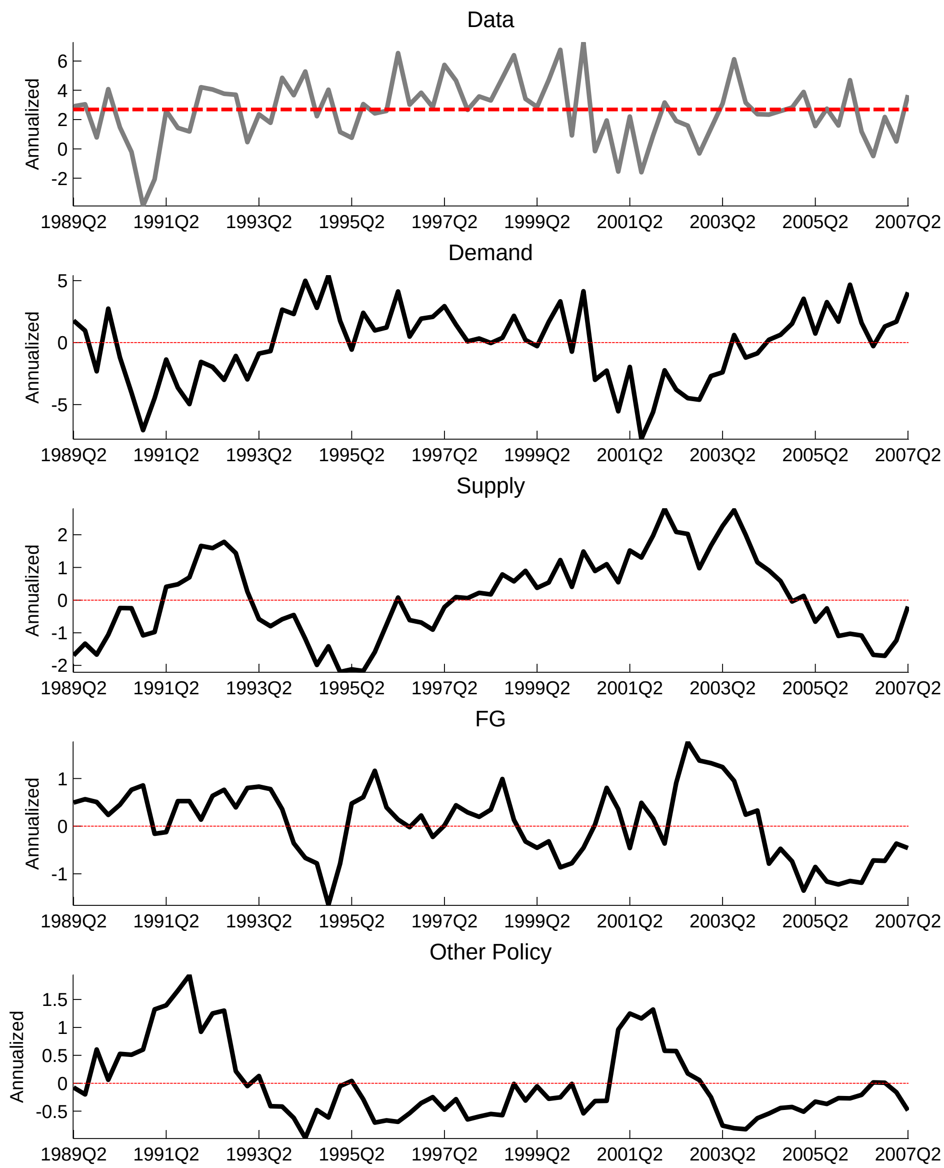


Figure 5: Historical Decomposition of Core PCE Inflation

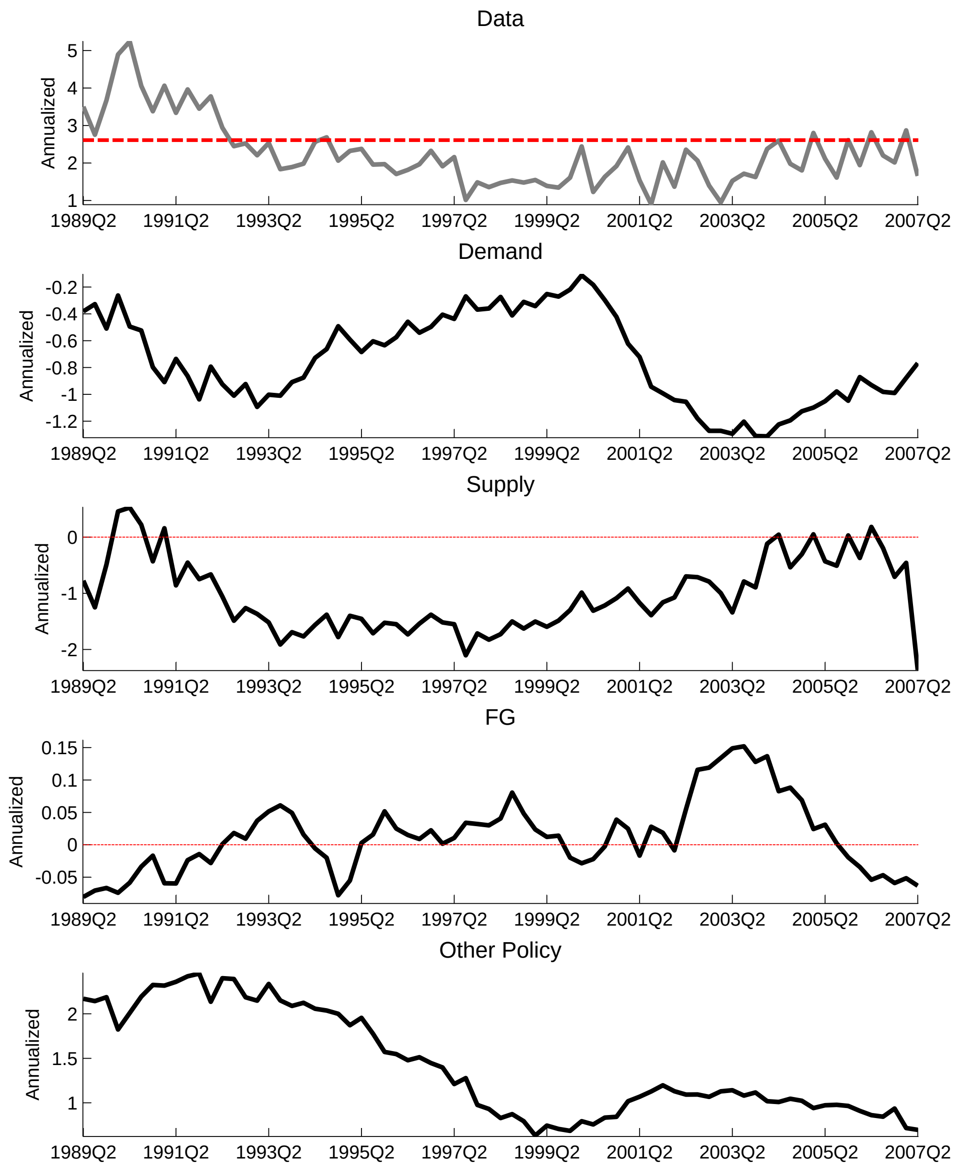
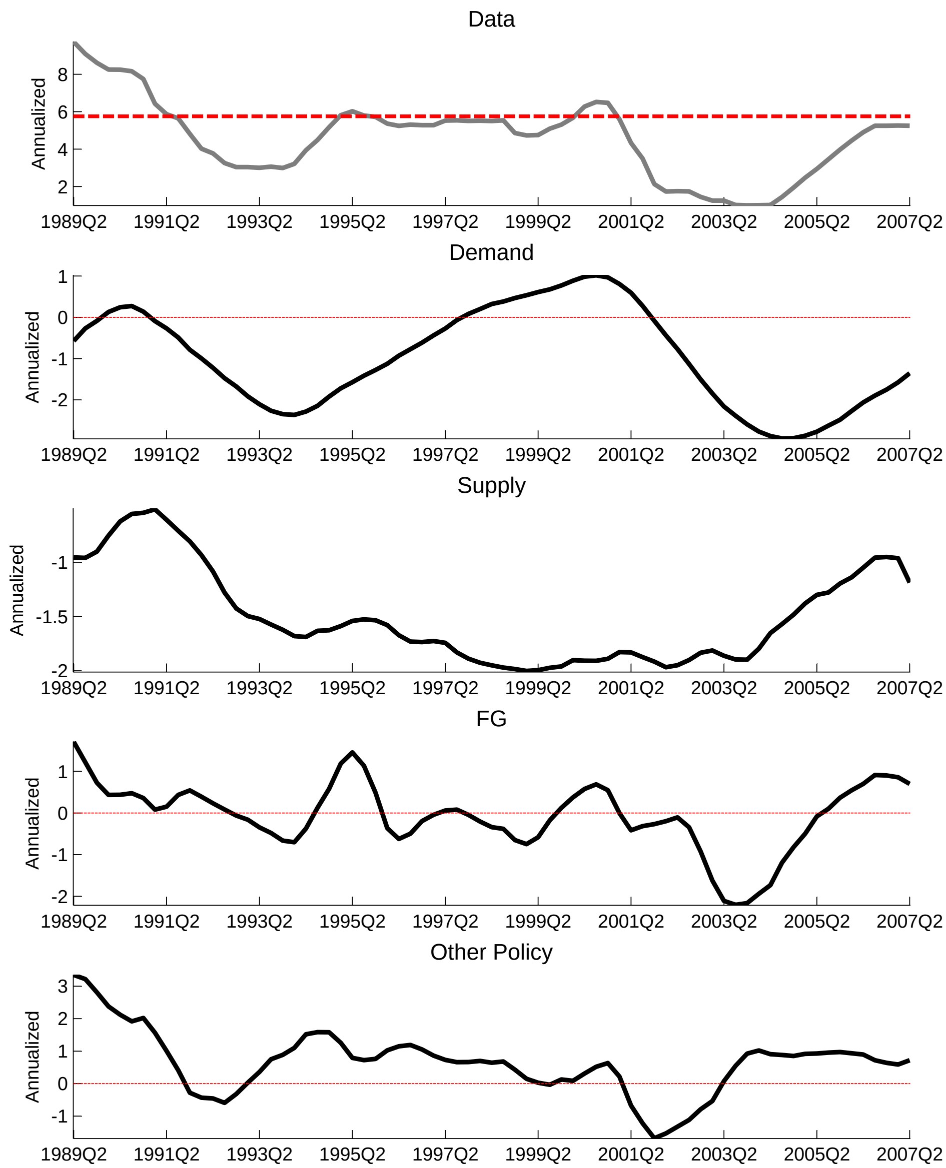


Figure 6: Historical Decomposition of the Federal Funds Rate



- Producers unable to update their price with all current information are allowed to index their prices to a convex combination of last quarter's inflation rate with the steady-state inflation rate. This places π_{t-1}^{p} in the Phillips curve. The estimated weight on steady-state inflation is 0.92.

5.4 Wage Phillips Curve

The Wage Phillips curve can be written as

$$\pi_t^w + \pi_t^p + j_t - \iota_w \left(\pi_{t-1}^p + j_{t-1}\right) = \beta E_t \left[\pi_{t+1}^w + \pi_{t+1}^p + j_{t+1} - \iota_w \left(\pi_t^p + j_t\right)\right] + \kappa_w x_t + \epsilon_t^w,$$

where π_t^w and π_t^p correspond to inflation in real wages and consumption prices respectively, $j_t = z_t + \frac{\alpha}{1-\alpha}\mu_t$ is the economy's technologically determined stochastic trend growth rate, with α equal to capital's share in the production function, z_t the growth rate of neutral technology, and μ_t the growth rate of investment-specific technical change. The term $\pi_{t-1}^p + z_{t-1} + j_t$ arises from indexation of wages to a weighted average of last quarter's productivity-adjusted price inflation and its steady state value. The estimated weight on the steady state equals 0.72. The log-linearized expression for the ratio of the marginal disutility of labor, expressed in consumption units, to the real wage is

$$x_t = b_t + \psi_t + \nu l_t - \lambda_t - w_t,$$

where b_t and ψ_t are disturbances to the discount factor and the disutility of working, respectively, l_t hours, λ_t the marginal utility of consumption and w_t the real wage. Finally, ϵ_t^w is a white noise wage markup shock.

Note that without indexation of wages to trend productivity, this equation says that nominal wage inflation (adjusted by trend growth) depends positively on future nominal wage inflation (also appropriately trend-adjusted), and increases in the disutility of the labor-real wage gap.

The estimated equation is given by

$$\pi_t^w + \pi_t^p + j_t - 0.28\left(\pi_{t-1}^p + j_{t-1}\right) = 0.997 \times E_t[\pi_{t+1}^w + \pi_{t+1}^p + j_{t+1} - 0.28\left(\pi_t^p + j_t\right)] + 0.01 x_t + \epsilon_t^w,$$

6 The Model's Responses to Key Shocks

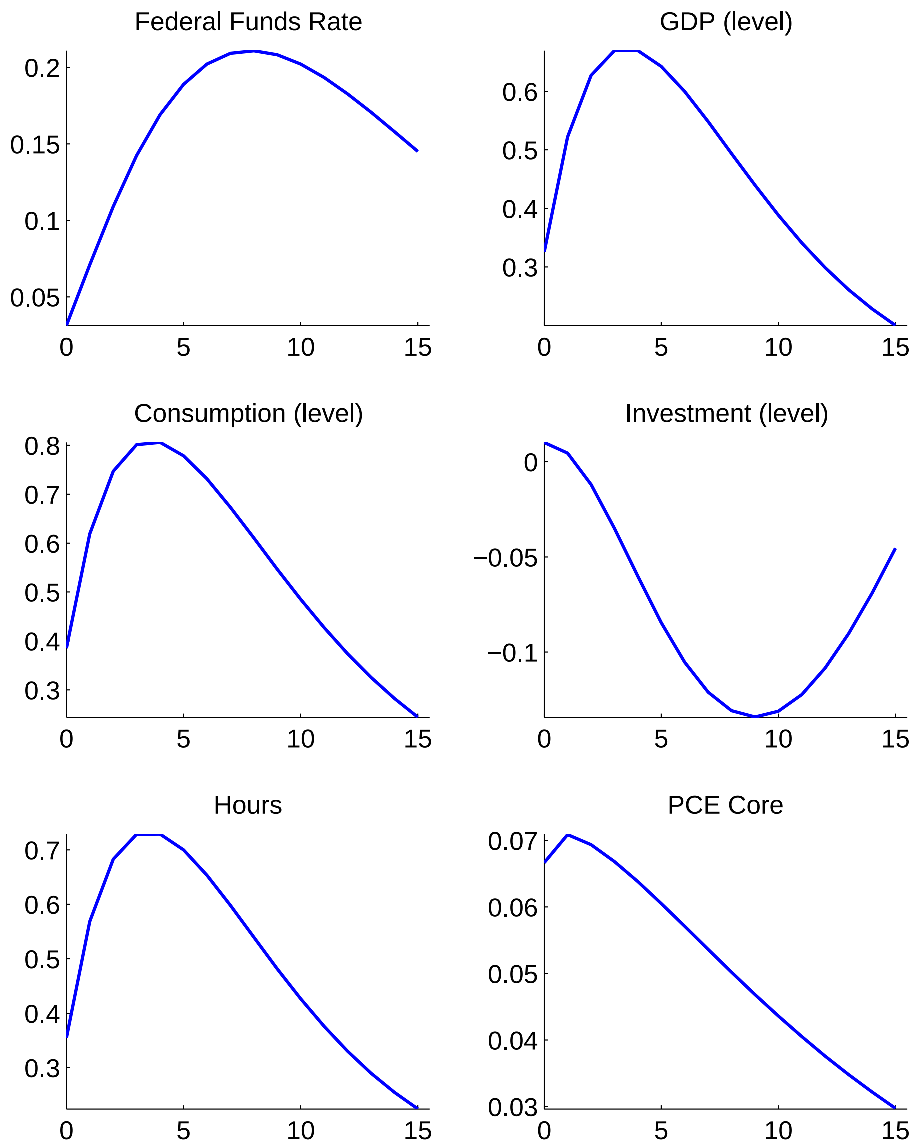
The following shocks figure prominently into explaining the structure of the model: The discount rate shock, the spread shock to the external finance premium, the neutral technology shock, the price mark-up shock, the monetary policy (current and forward guidance factor) and inflation anchor shocks. In this section, we provide greater detail on the model's responses to these seven shocks by presenting impulse response functions to a one standard deviation realization of each of these disturbances.

Figure 7 plots responses to a discount rate shock that increases impatience and tilts desired consumption profiles towards the present. The variables examined are real GDP, the federal funds rate, consumption, investment, inflation, and hours worked.

In a neoclassical economy, this shock would be contractionary on impact. Upon becoming more impatient, the representative household would increase consumption and decrease hours worked. To the extent that the production technology is concave, interest rates and real wages would rise; and regardless of the production technology both real GDP and investment would drop.

Increasing impatience instead *expands* activity in this New Keynesian economy. As in the neoclassical case, consumption rises on impact. However, investment remains unchanged as adjustment costs penalize the sharp contraction of investment from the neoclassical model. Instead, investment displays a hump-shaped response, exhibiting negative co-movement with consumption with a slight lag. Habit causes the consumption growth to persist for two more quarters before it begins to decline. Market clearing requires either a rise of the interest rate (to choke off the desired consumption expansion) or an expansion of GDP. By construction, the monetary policy rule prevents the interest rate from rising unless the shock is inflationary or expansionary. Therefore, GDP must rise. This in turn requires hours worked to increase.

Figure 7: Responses to a Discount Rate Shock



Two model features overcome the neoclassical desire for more leisure. First, some of the labor variants' wages are sticky. For those, the household is obligated to supply whatever hours firms demand. Second, the additional labor demand raises the wages of labor variants with wage-setting opportunities. This rise in wages pushes marginal cost up and lies behind the short-run increase in inflation. After inflation has persisted for a few quarters, monetary policy tightens and real rates rise.

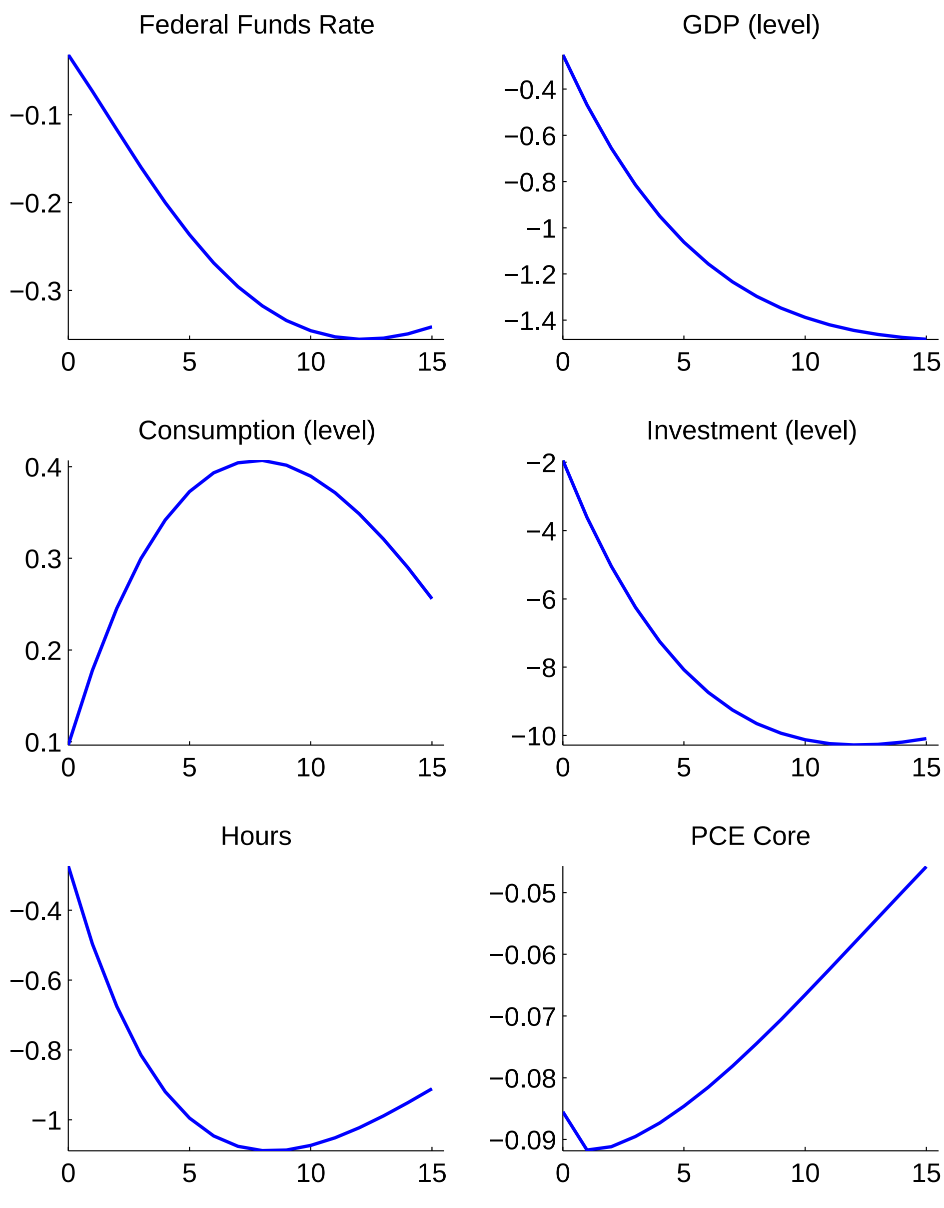
Since the discount rate shock moves output and prices in the same direction, a Keynesian analysis would label it a shift in "demand." In the neoclassical sense, it is also a demand shock, albeit a reduction in the demand for future goods. The matching neoclassical supply shock in our model is to the spread shock. A positive shock to it decreases the supply of future goods. Figure 8 plots the responses to such a shock.[2]

A positive spread shock reduces the supply of credit available to entrepreneurs, who are then forced to shrink their demand for capital. The price of installed capital drops sharply so that the return to capital collapses on impact and is followed by a prolonged contraction in borrowing by entrepreneurs. The decline in borrowing is initially smaller than in net worth, which results in a rising leverage ratio and a further tightening of the external finance premium. Investment and other measures of real activity, with the exception of consumption, all decline. In response to lower activity and inflation, monetary policy eases and real rates move lower.

Increasing the external finance premium thus lowers investment, hours worked, GDP, and the real interest rate. Two aspects of our model limit the response of consumption on the same shock's impact. First, habit-based preferences penalize an immediate increase in consumption. Second, monetary policy responds to the shock only slowly, so real interest rates are slow to adjust. Although this shock changes the economy's technology for intertemporal substitution – and therefore deserves the neoclassical label "supply" – it makes prices and output move in the same direction. For this reason, it falls into our Keynesian taxonomy's

[2]The interpretation of this shock is not unique. The negative spread shock resembles in nature a positive marginal efficiency of investment (MEI) shock. It could also be interpreted as a shock to the efficiency of channeling funds to entrepreneurs or, more broadly, variations in the supply of credit. Barro and King (1984) and Greenwood, Hercowitz, and Huffman (1988) consider the analogous responses to an MEI shock from a neoclassical model.

Figure 8: Responses to a Spread Shock



"demand" category.

Figure 9 displays the responses to a neutral technology shock. Measures of real activity, with the exception of hours, all rise after a positive technology shock. The effects are delayed, however, due to habit persistence in consumption and investment adjustment costs. As inflation declines on impact, monetary policy progressively eases over a period of 6 quarters before bringing real rates back to their steady-state as real activity picks up. This results in a hump-shaped response in GDP, consumption, and investment. Since the neutral technology shock moves output and prices in opposite directions, we label it a shift in "supply."

Figure 10 depicts the responses to a positive price mark-up shock. Inflation increases on impact and measures of real activity all decline, thereby resembling a transitory negative technology shock. Monetary policy tightens over a period of four quarters before real rates gradually return to their steady-state as real activity picks up.

Figures 11 and 12 present the impulse response functions for our two monetary policy shocks, the current policy and forward guidance factors. We begin with the forward guidance factor. A positive realization of this shock signals a hump-shaped increase in the interest rate given our estimated factor loadings with limited movement in the rate today. The gradual decline in the interest rate after four quarters is governed mostly by the autoregressive coefficient in the rule.

In response to the anticipated tightening, activity contracts immediately, afterward following a hump-shaped response. Inflation declines primarily on impact, as forward looking price setters incorporate the weaker outlook for activity into their decisions today. The current policy factor displays a similar pattern, except that compared with the forward guidance factor it accelerates the policy tightening. That is, it displays an immediate jump followed by a steeper rise and subsequent fall.

The responses to the current policy factor are standard, but those following a forward guidance shock require more explanation. At the announcement date, the expected value of the policy rate four quarters hence rises. Because both Phillips curves are forward looking, this expected contraction causes both prices and quantities to fall. This anticipated weakness then feeds through the monetary policy rule to create a gradual *easing* of policy.

Figure 9: Responses to a Neutral Technology Shock

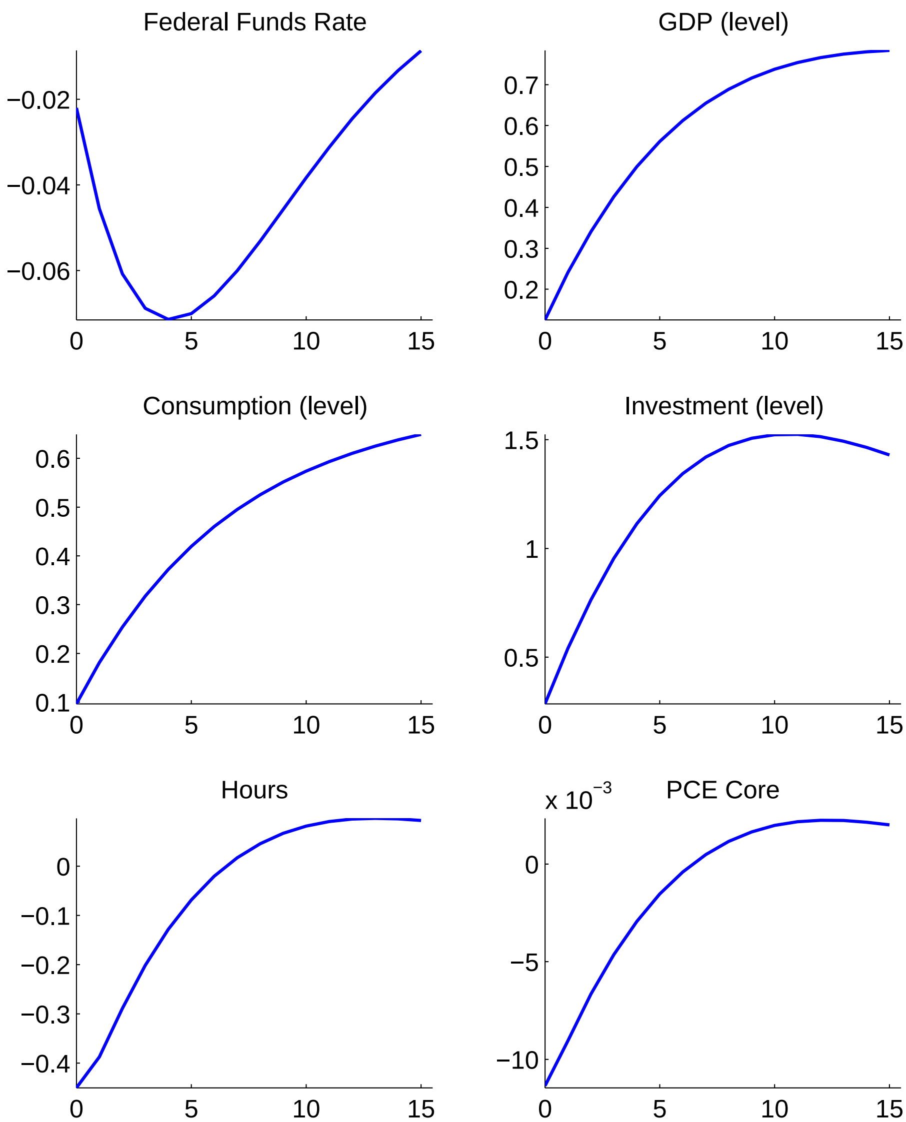


Figure 10: Responses to a Price Mark-up Shock

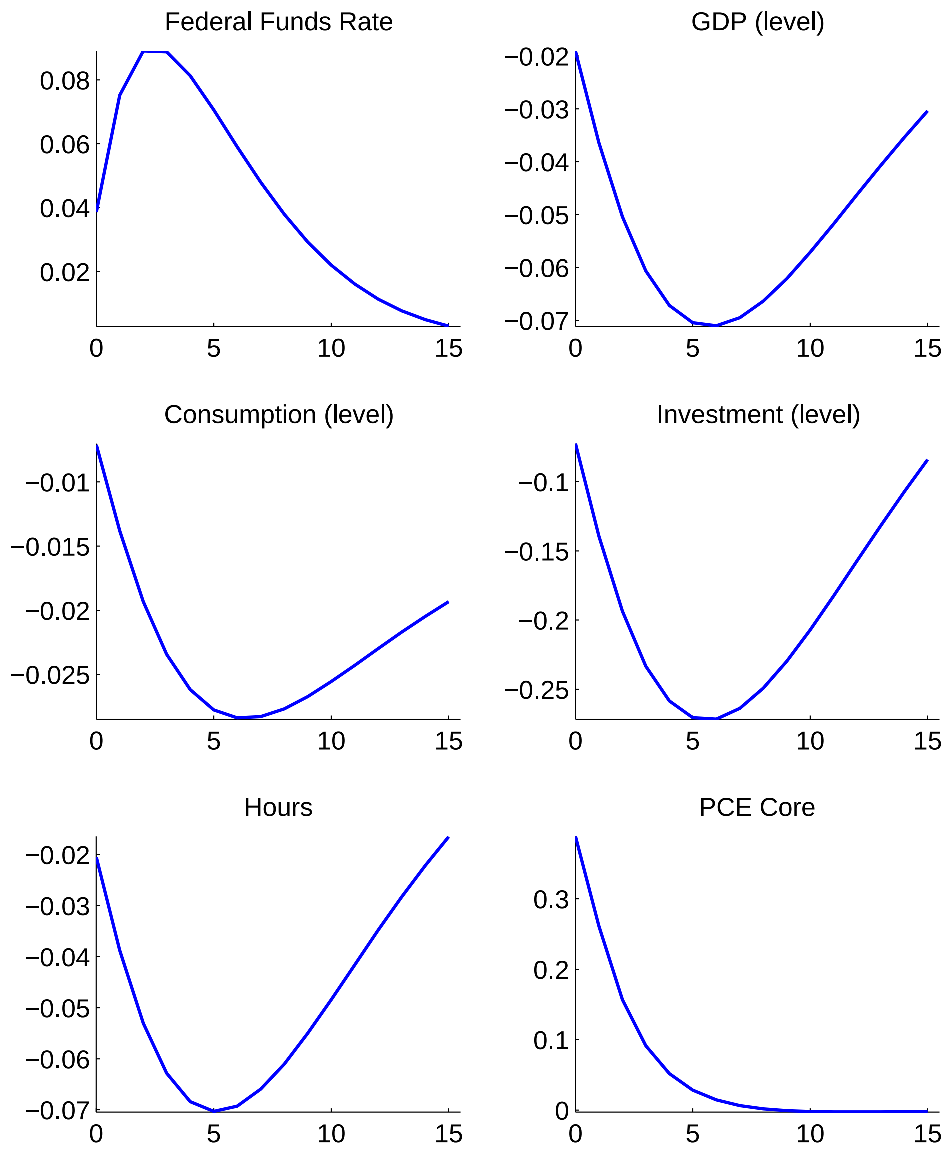


Figure 11: Responses to the Current Policy Factor

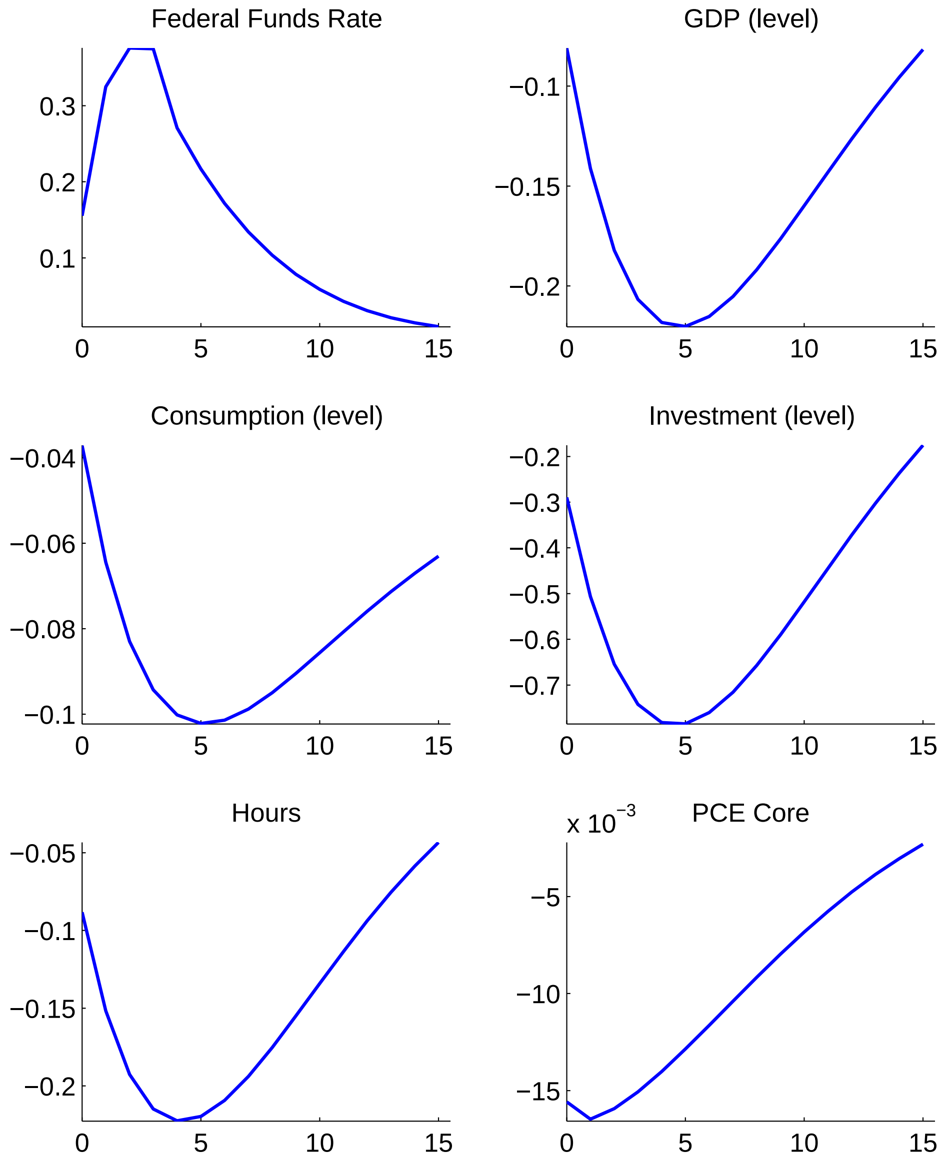


Figure 12: Responses to the Forward Guidance Factor

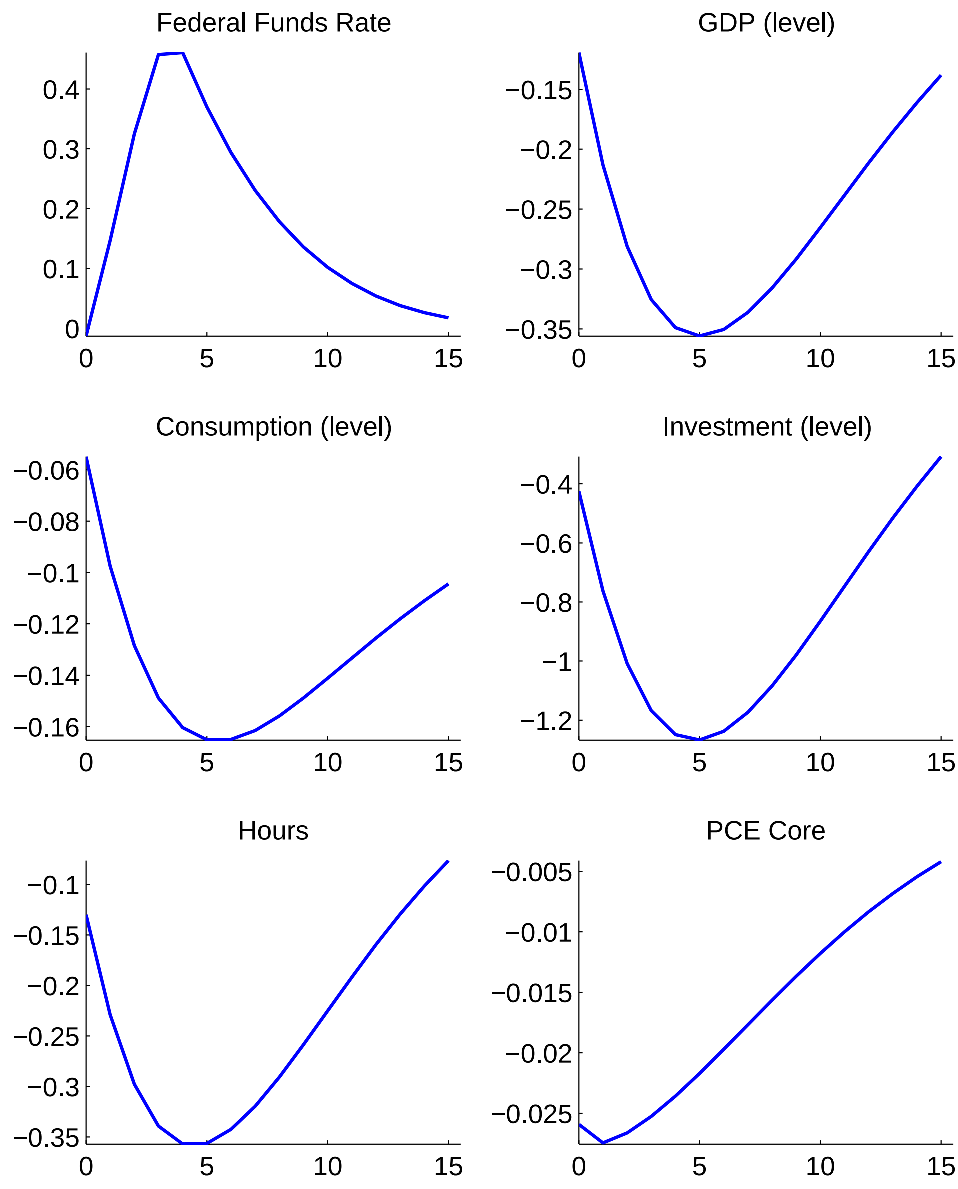
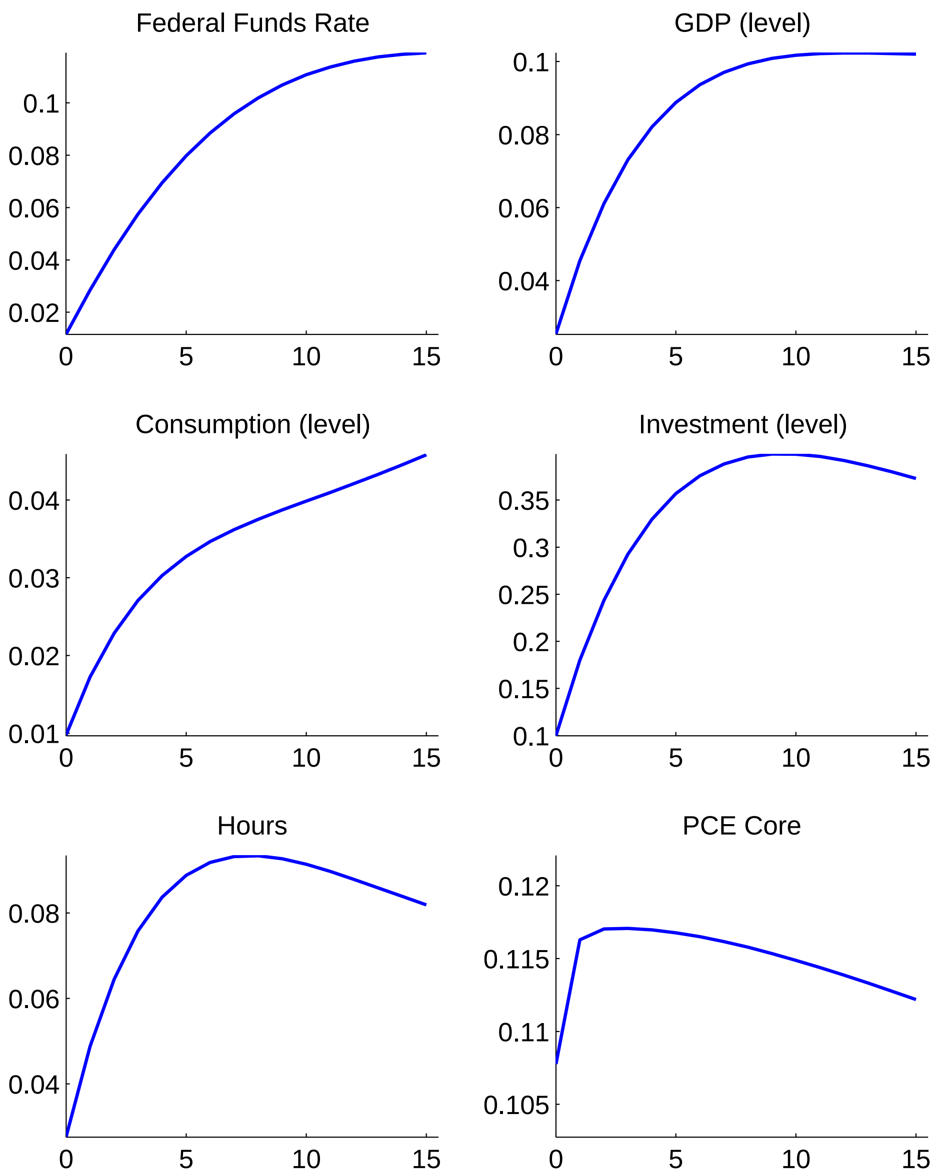


Figure 13 displays the impulse response functions for a positive inflation anchor shock. In response, inflation jumps on impact, as does expected long-run expected inflation (not shown). Under the assumption of perfect credibility, higher inflation is achieved without any contemporaneous movement in the federal funds rate. Although monetary policy does eventually tighten to return the real interest rate to its steady-state, lower real rates during the initial transition fuel an increase in consumption, investment, and hours. Therefore, GDP moves up as well. Given the high degree of persistence of this shock, its effects on real activity and inflation dissipate at a glacial pace.

Figure 13: Responses to an Inflation Drift Shock



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