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Gambling for Dollars: Strategic Hedge Fund Manager Investment

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Abstract

Hedge fund managers differ in ability and investors want to distinguish good ones from bad. Via the design of their investment strategies, better fund managers want to ease this inference problem while worse fund managers want to complicate it. We impose only the minimal restrictions on the nature the investment strategies that, on average, returns reflect the hedge fund manager's ability and that returns be bounded from below, and solve for the set of equilibria that emerge. We then show that under a variety of equilibrium refinements, a unique equilibrium obtains. In this equilibrium, investors set a cutoff standard for providing capital to a hedge fund: and invest if and only if returns exceed this cutoff. This induces less able hedge fund managers to adopt risky investment strategies that maximize the probability of meeting this cutoff by risking large losses if they fail. Over time, as investors learn about a hedge fund manager's ability and less able hedge fund managers are stochastically weeded out, investors set less demanding re-investment standards. Our economy reconciles many facts regarding hedge fund performance. For example, in a regression with fixed hedge fund manager effects, returns of more experienced hedge fund managers decline, even though the expected profits of investors rise with the hedge fund manager's experience; more experienced hedge funds deliver less volatile returns; persistence of returns is greater for better hedge funds; hedge fund failure rates are initially very high, but fall sharply with hedge fund manager experience; returns of exiting hedge funds are *substantially* worse than historical returns; and the longer is an investor's horizon, the lower is the expected return of the hedge funds in which he invests.

1 Introduction

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When hedge fund managers differ in their abilities to identify profitable investment strategies and profitable investment opportunities, investors have strong incentives to distinguish good hedge fund managers from bad, and to allocate their resources accordingly. Unfortunately, the very way in which hedge funds generate returns make this inference problem difficult for investors. In particular, hedge fund investment strategies are zealously concealed, because those strategies are the source of their profits. If revealed, successful strategies can be mimicked by others, thereby competing away those profits. To conceal investment strategies from other institutions, hedge funds must also conceal their strategies from investors. As a result, investors must look at realized hedge fund performance, which they observe on a periodic basis, to determine how to allocate their money. If a hedge fund does well, investors will infer that the hedge fund manager is more likely to be able, and shift investments toward the fund; if a fund does badly, investors will conclude that the hedge fund manager is less likely to be able, and move investments out. The (convex) flow of cash into better-performing funds and away from poorly-performing funds has been well documented (e.g., Agarwal et al. (2009), Chevalier and Ellison (1997)), and indicates that investors use hedge fund performance to update about a manager's investment skills.

In this paper, we determine how this endogenous fund flow relationship affects both the incentives of different fund manager types to take on risk and the nature of that risk, and how it varies over time according to historical fund performance. In turn, we derive the consequences for the evolution of hedge fund returns, as investors learn more from continued observation of a hedge fund's returns about its manager's skills.

The central idea is that a hedge fund manager knows his level of competence and recognizes that investors will shift investments away from poorly-performing funds toward better-performing funds. Less able fund managers can employ *appropriately* tailored investment strategies to try to mimic the performance of better fund managers; and good hedge fund managers can tailor investments to try to distinguish themselves from bad ones. Hedge fund managers have enormous discretion in the design of their investment strategies. To capture this discretion, we impose almost no restrictions on the possible investment strategies that a fund manager can employ, requiring only that an investment strategy have a payoff that is bounded from below, and that, on average, returns reflect a hedge fund manager's ability.

In this paper we focus on a simple investment technology, where if a fixed amount is invested, the hedge fund's "project" will pay off an amount that depends on the hedge fund manager's ability. Hence, each period, the investor's decision becomes whether to re-invest in the fund, rather than how much to invest. This allows us to characterize more easily the equilibrium dynamics on investor re-investment choices, the evolution of the distribution of fund performances (mean, volatility and persistence), the impact of survivorship bias on measured performance, investor horizon and unobserved, post-investment, idiosyncratic shocks to fund performance. We show that under a set of different equilibrium refinements, the equilibrium is uniquely pinned down, and investors set simple cutoff standards for re-investment that decline over time as investors have longer track records on which to assess performance. In this equilibrium, the investor is indifferent between investing in the hedge fund conditional on the hedge fund return achieving this re-investment standard, and investing all funds in alternative assets. Better hedge fund managers do not need to distort investments to receive continued funding. Less able fund managers, in contrast, will tailor their investment strategies to maximize the probability of meeting that re-investment standard, placing residual probability on "disaster", i.e., on the lower bound on payoffs. This reflects that the hedge fund will have to exit regardless of the degree of sufficiently poor performance. This equilibrium, which is the unique selection of the Grossman Perry perfect sequential equilibrium refinement (Grossman and Perry 1986), is also the one that is best for investors given the moral hazard problem they face from hedge fund managers.

It follows that there will be clusters of hedge fund returns slightly "above" expected performance. Consistent with this, Dimmock and Gerken (2013) and Bollen and Pool (2009) find that funds are *far* more likely to report small positive returns than small negative returns. While these papers argue that this reflects mis-reporting (by about 10% of hedge funds), our analysis shows that strategic design of investment strategies could account for much of this. It also there will be far fewer hedge funds with modestly poor returns, and 'unusually many' extremely poor performers. Indeed, Malkiel and Saha (2005) document that failing to account for these (liquidated) hedge funds that disappear from hedge fund databases as a result, biases up estimates of hedge fund returns by four percent.

We also predict that because good hedge fund managers do not need to strategically tailor investment strategies to receive continued funding, their returns will be less volatile and more persistent, consistent with the findings of Jagannathan et al. (2010). Moreover, because bad fund managers are stochastically weeded out over time, we predict that the reinvestment standards that investors set over time will decline. As a result, survival rates rise with the age of hedge funds—both because bad managers are differentially more likely to be weeded out, and because some intermediate hedge fund manager types cease to have to adopt riskier investment strategies. Further, consistent with Boyson (2005), those senior hedge funds that do fail are those that pursue riskier strategies—these are run by less able hedge fund managers who initially got lucky, but still have to have their riskier investment strategies succeed to win continued funding.

In turn, if one tracks the performance of a surviving hedge fund, we predict that more experienced hedge funds will have both less volatile returns, and lower average returns, as, for example Boyson (2005) documents in her fixed (hedge fund manager) effects regressions. This latter result reflects that hedge fund managers that had to employ risky investment strategies survived because they initially got lucky, and achieved a return that exceeded their expected return. Over time the upside of their risky investment strategies declines, so their returns will fall, regardless of whether they continue to get lucky or not. It follows that tracking a hedge fund's performance over time, the performance of older hedge funds will, on average, decline. This decline in returns for older hedge funds is reinforced when hedge funds are subject either to *pre-investment* idiosyncratic shocks that affect the quality of their investment opportunities at a moment in time, or to *post-investment* idiosyncratic shocks to returns after their investment strategies have been chosen. In both cases, ceteris paribus, surviving hedge funds will tend to be those that got positive idiosyncratic shocks, implying that future expected returns will be lower. This reduced performance of older funds would be further reinforced were our hedge fund investment technology to be partially scalable, so that taking larger positions in different assets have price impacts that are more limited in nature. In this case, an investor puts more money into better performing hedge funds, causing the expected marginal return to decline (see Berk and Green 2004, and Fung et al. 2008).

Our findings collectively resolve some seeming paradoxes: Why is investor learning over time about which hedge fund managers are better associated with *lower* expected future hedge fund manager returns for a given fund manager? If this is so, why shouldn't investors put their money into newer hedge funds? The answer is that, over time, the stochastic filtering of worse hedge fund managers means that the average quality of hedge fund managers rises with time—in the cross-section, more experienced hedge fund managers are better. However, the expected performance of any given hedge fund manager should decline, and this decline should be sharper for (low and intermediate quality) hedge fund managers who pursued (and possibly continue to pursue) riskier investment strategies. In turn, these predictions point to the very different theoretical predictions for fixed effect regressions (returns of more experienced fund managers should fall) versus cross-sectional regressions (returns of more experienced fund managers should rise). In particular, we resolve these "paradoxes", without appeal to the various biases in hedge fund databases (e.g., hedge funds only enter the databases if they succeed, in which case their initial superior returns are 'backfilled') or irrational investor behavior. While not dismissing the importance of these sources, we highlight how strategic behavior by fully rational agents alone delivers these predictions.

Our analysis of how an investor's investment horizon affects expected hedge fund returns offers and then resolves a similar paradox: the longer is the investor's horizon, the lower is the expected return of the hedge funds in which he invests. The resolution is that investors with longer horizons are, in effect, more patient, setting lower standards for continued reinvestment in the hedge fund because they value the embedded option of learning more about the hedge fund manager's ability—and they can only learn and continue to invest if the hedge fund survives. While very good hedge managers receive continued funds regardless of an investor's horizon, those with intermediate abilities who would receive funds in a full information setting, are more likely to obtain funding when their investors' are more long-sighted.

Our theoretical model indicates that lesser hedge fund managers would like to adopt investment strategies that place as much probability mass as possible on returns just above what an investor would demand for renewed funding—which, typically will be a little above the 'market' return. We have allowed for enormous discretion in the design of those investment strategies, but have remained silent about how they might construct those strategies in practice. One such practical vehicle are asset-backed securities (e.g., mortgage-backed securities) that pay off a little above the market return unless there is a large negative common shock, as happened to the U.S. housing market at the outset of the financial crisis. Arguably, the explosion of mediocre institutional investors over this period who pursued this investment strategy even drove down the price of this risk, a general equilibrium effect that we do not model (see Diamond and Rajan 2009 for an extensive discussion of this).

The Literature

In a companion paper, we analyze a two-period model where investments are partially scalable, so that an investor must now decide given a hedge fund's track record, how *much* to invest, rather than just decide *whether* to invest. We assume that the expected average payout from a hedge fund whose manager has ability ω and capital k is $\omega f(k)$, where f is strictly concave and increasing in k. Such partial scalability emerges naturally when the price impact of taking greater positions in an asset is positive. Investors put more money into better performing funds, which, with decreasing returns to scale, causes returns of better past performers to fall, as Berk and Green (2004) posit, and, for example, Fung et al. (2008) find. Now hedge fund managers must trade off investment scale versus likelihood of success in their decisions. A higher equilibrium payout means that the hedge fund is more likely to be run by an able manager, so that it will receive more funds. When investors set higher standards for substantial re-investments, they can more reliably identify good hedge fund managers, but higher standards also make it harder to unravel the quality of badly performing hedge funds, since more will be run by able managers who just happened to be unlucky. That is, higher standards reveal more information about winners, but less about losers.

Our companion paper shows that when $f(k) = k^{\alpha}$, and $\alpha > 1/2$, then in the best equilibrium that maximizes expected payoffs of those who invest in hedge funds, bad fund managers should adopt risky investment strategies that seek to mimic good hedge fund managers (who do not employ risky investment strategies), as total surplus is maximized when investors identify as many good hedge fund managers as possible. However, when a hedge fund's investments are less scalable, $\alpha < 1/2$, then in the best equilibrium, good fund managers employ risky investment strategies and try to separate themselves away from bad fund managers—with less scalability, it becomes more important to ensure that a hedge fund that receives significant investment is run by an able hedge fund manager.

Our framework adopts the premise that while a hedge fund manager can strategically design investment strategies to try to fool investors about their intrinsic investing skills, they cannot engage in fraud or misreport returns. As Stulz (2007) observes hedge funds often hold securities that are not traded on exchanges, and hence have significant discretion in pricing them as they see fit. A recent literature provides evidence of such hedge fund misreporting (see e.g., Agarwal et al. (2009, 2011), Bollen and Pool (2009, 2012), Cumming and Dai (2010), Dimmock and Gerkin (2012, 2013), and Jylha (2011)). While not dismissing the evidence of such misreporting and illegal behavior designed to conceal a lack of hedge fund investment skills, our paper focuses on the legal ways in which a hedge fund can do this, and the consequences for the dynamics of hedge fund returns.

A typical compensation contract for hedge fund managers—1-2 percent of the net asset value of the fund and 15-25 percent of asset returns above a specified hurdle rate—tends to invite risk taking (Stulz (2007)). Taylor (2003) examines a model where fund managers can invest in safe or risky assets, and finds that managers switch to risky strategies if their current performance lags a benchmark that must be achieved to receive new investment funds.

Agents in Degeorge et al. (1996, 2004) have private information regarding their quality. As in our model, low quality types may have an incentive to undertake a risky strategy or gamble so that they might be mistaken as a high type. In Degeorge et al. (2004), depending on model parameters, low quality types may also undertake less risky/sure thing strategies that perfectly reveal their type. Both Taylor (2003) and Degeorge et al. (1996, 2004) substantially restrict the possible investment alternatives (e.g., returns follow a normal distribution). But giving hedge fund managers little discretion in the design of their investment strategies, limits the insights about how they will design and shape those strategies in a world where hedge fund managers have significantly more discretion. We impose minimal structure on the investment strategies that fund managers can employ, beyond bounding fund payouts from below and requiring that they be "actuarily fair"; we then derive the precise forms that equilibrium investment strategies take, and how they evolve over time as investors learn more about a hedge fund manager's ability.

In our economy, fund managers employ risky investment strategies because the (endogenous) minimum return needed to obtain continued funding gives rise to a non-concave payoff structure for a fund manager. Those fund managers with abilities that lie in the non-concave portion of the payoff structure have strict incentives to gamble. In this regard, our motivation for gambling mirrors that in Ljungqvist (1994), where firm managers gamble on behalf of owners to exploit a non-concave payoff structure. Although one can, in principle, design compensation contracts to prevent excessive risk taking, Stulz (2007) points out they are not always successful. For example, many contracts stipulate that managers must recover past losses before they get a performance fee, the so-called "high-water" mark. But this constraint need not reign in excessive risky behavior if a manager can just close a fund after a large loss. DeMarzo et al. (2013) point out that optimal contracting can limit or rule out risky behavior, but in many cases the cost associated with eliminating this sort of behavior is too expensive. So in the end, principals have to live with the risky behavior that characterizes their agents.

2 The Basic Model

We consider a single hedge fund run by a manager with an $N \geq 2$ period investment horizon. In each period $n = 0, \ldots, N-1$, a type-ω hedge fund manager has potential access to investment strategies that require a unit of capital to implement, and have expected period payout ω. We refer to ω as the hedge fund manager's ability. It is common knowledge that ω is drawn from the cumulative distribution function $\Pi(\cdot)$ with support $[a, b]$, where $a > 0$. The manager's ability ω to identify good investment strategies is private information to the manager. Each period, the hedge fund manager must raise the unit of capital needed to implement his strategies from risk-neutral investors. We assume that neither the hedge fund manager nor the risk-neutral investors discount future payoffs.

The assumption that a hedge fund manager's investment strategies are not scalable—the fund manager needs a unit of capital to implement investment strategies, but additional capital is not productive—greatly facilitates analysis. It means that an investor's problem reduces to deciding whether or not to invest in the hedge fund given its track record, and not deciding how *much* to invest. Our companion paper allows for partial scalability in a two-period setting, showing the ways in which our findings qualitatively extend when we incorporate the realistic features that price impacts of large positions and a hedge fund manager's scarce human capital give rise to decreasing returns to scale in hedge fund investment strategies.

In practice, hedge fund managers have enormous discretion in their design of investment strategies. The period-n investment strategy chosen by a type-ω hedge fund manager is fully described by the distribution G_ω it induces over period payouts X_n. To capture the discretion that hedge fund managers have in designing investment strategies, we suppose that a type-ω hedge fund manager can employ the unit capital investment in *any* period-n investment strategy $G_\omega^n(\cdot)$ with the properties that $E_{G_\omega^n}(X_n) = \int_{X_n \geq z} X_n dG_\omega^n(X_n) \leq \omega$ and $X_n \geq z$, where $z < a$. That is, the expected period hedge fund payout is bounded from above by the fund manager's ability, and realized payouts are bounded from below by z. This lower bound z means that a fund manager does not have access to an investment strategy that does substantially better than ω in almost all states of the world, which would allow the manager to almost always succeed in mimicking the performance of a more able fund manager, offset by losing an arbitrarily large amount of money with a vanishingly small probability.

One can motivate z from a feasibility standpoint—a hedge fund manager might not be able to lose more than 100 percent of his investment capital, in which case the lower bound on payouts is $z = 0$. It may also be that losing vast amounts of money is inconsistent with the strategy style that a manager purports to adopt; or that a hedge fund manager who loses too much money risks running afoul of the law, and this deters fund managers from adopting such extreme investment strategies. Our analysis will focus on the limited liability bound of $z = 0$, but none of our qualitative characterizations hinge on this choice.

We first suppose that the hedge fund manager must rely on capital raised from a sequence of risk-neutral short-term investors who have one-period investment horizons. Later, we characterize how outcomes are affected when these investors have longer investment horizons. Investors observe a hedge fund's historical performance. Thus, while they do not observe a hedge fund manager's ability ω directly, a period-n investor knows the hedge fund manager's track record of past investment strategy payouts, $X^n = (X_0, X_1, \ldots, X_{n-1})$, and past capital inflows, $k^n = (k_0, k_1, \ldots, k_{n-1})$, where the non-scalability of investment strategies means that without loss of generality we can focus on $k_i \in \{0, 1\}$. A period-n investor uses this information to draw inferences about the fund manager's ability. The investor also has the option to invest unlimited amounts in an alternative asset with known gross expected return $\bar{R}$. For example, $\bar{R}$ could be the expected return on some broad market index. If investors provide the unit of capital to the hedge fund manager, they receive share $1 - \beta$ of the fund's end-of-period payout, with the hedge fund manager retaining share $\beta > 0$ as payment for his services. We take this share β as exogenous in order to focus on how the inferences investors make from fund payouts interact to influence the investment strategy choices of different hedge fund manager types.[1] Due to the non-scalability of a hedge fund manager's investment strategies, a period n investor's decision reduces to deciding, given X^n and k^n, whether or not to invest in the hedge fund.[2]

We assume that $(1-\beta)E(\omega) > \bar{R}$. This is a minimum requirement for a short-horizon investor to be willing to invest in a hedge fund that does not have a track record. When $(1-\beta)E(\omega) > \bar{R}$, a period-0 investor is willing to invest $k_0 = 1$, as long as hedge fund managers adopt *fair* investment strategies. An investment strategy G^n_ω is fair if $E_{G^n_\omega}(X_n) = \omega$. An investment strategy or investment gamble is unfair if $E_{G^n_\omega}(X_n) < \omega$ because, for example, the hedge fund manager needlessly wastes resources via some complicated, costly trading strategy. We use "investment strategy", and "investment gamble" interchangeably, and when an ω-type fund manager places probability one on the payout $X_n = \omega$, we say that the hedge fund manager does "not gamble" or adopts a "sure-thing" investment strategy.

[1]In practice, there is limited variation in hedge fund managers' compensation. Often, they receive two percent of the funds under management plus 20 percent of profits above some hurdle. If we pose our analysis in an infinite horizon setting, so there is no terminal period, then as long as hedge fund managers are sufficiently patient, our analysis qualitatively extends with this convex option-like feature to compensation.

[2]The analysis extends immediately if the scale of the required (fixed) investment varies over time.

An investment strategy for a period-n investor maps the hedge fund's track record and past capital inflows into a capital investment choice, $I_n(X^n, k^n) \to \tilde{k}_n \in [0,1]$, where $\tilde{k}_n$ is the probability the period-n investor invests in the hedge fund. A strategy for a hedge fund manager is a sequence of period investment functions mapping his type, track record, and past capital inflows into a feasible investment strategy when $k_n = 1$, $F_n(\omega, X^n, k^n) \to G^n_\omega(\cdot)$.

Investors form beliefs regarding the hedge fund manager's type, ω. The beliefs of a period-n investor about ω are described by a cumulative distribution function, μ, that depends on past capital inflows, k^n, and the manager's track record of past investment strategy payoffs, X^n. We omit dependence of beliefs on k^n in our notation, where it does not cause confusion.

The equilibrium concept is a perfect Bayesian equilibrium with refinements.

3 Two-Period Hedge Fund Investment Horizon

We first characterize equilibrium outcomes for hedge fund managers with a two-period investment horizon. Specifically, we consider a hedge fund manager whose investment horizon extends from period 0 to period 1, and a sequence of two short-term investors, one who cares about period-0 investments and one who cares about period-1 investments.

It turns out that there are many perfect Bayesian equilibria. We begin by describing a *simple cutoff-rule* equilibrium. We later show that this equilibrium is selected by multiple equilibrium refinements. In this simple cutoff-rule equilibrium, the period-0 investor always makes the unit capital investment in the hedge fund because $(1-\beta)E(\omega) > \bar{R}$ means that, ex-ante, the expected return from investing in a hedge fund manager who employs a fair investment strategy exceeds that on the alternative asset. The period-1 investor employs a simple cut-off rule strategy, providing capital to the hedge fund if and only if the hedge fund period-0 payout is sufficiently high, $X_0 \geq X^c$, where $(1-\beta)\, E_{\mu(X_0)}[\omega] = \bar{R}$ for $X_0 = X^c$. Here, $E_{\mu(X_0)}[\omega]$ is the expected hedge fund manager type when the period-0 payout is X_0, and investors update to hold belief function $\mu(X_0)$, over ω. If $X_0 < X^c$, the period-1 investor invests solely in the alternative asset.

In this equilibrium, in period 0, good hedge fund manager types $\omega \geq X^c$ adopt sure-thing investment strategies, as this guarantees that they will continue to receive capital from investors in period 1. In contrast, lesser hedge fund manager types $\omega < X^c$ cannot employ sure-thing investment strategies, else they would not receive investment capital in period 1. This cut-off for funding induces them to adopt risky investment strategies that sometimes succeed and payoff X^c, and sometimes fail and lose everything. To reduce notational clutter, we define $R \equiv \bar{R}/(1-\beta)$.

Proposition 1 *There exists a simple cutoff-rule equilibrium that is characterized by a critical value X^c that solves*

$$\frac{\int_a^{X^c} \omega^2 \Pi(d\omega)}{\int_a^{X^c} \omega \Pi(d\omega)} = R. \tag{1}$$

In period 0, hedge fund manager types $\omega < X^c$ adopt an investment strategy that places probability ω/X^c on the payout X^c, and remaining probability on 0, and better types $\omega \geq X^c$ employ a sure-thing investment strategy that delivers $X_0 = \omega$ with probability 1. The period-1 investor makes the unit capital investment in the hedge fund if $X_0 \geq X^c$; otherwise he invests only in the alternative asset. Each hedge fund manager type ω who receives continued funding in period 1 adopts the sure-thing investment strategy that delivers $X_1 = \omega$ with probability 1. The period-1 investor's beliefs satisfy

$$E_{\mu(X_0)}[\omega] = \begin{cases} \frac{\int_a^{X^c}(X^c-\omega)\omega\Pi(d\omega)}{\int_a^{X^c}(X^c-\omega)\Pi(d\omega)} & \text{if } X_0 = 0 \\ y(X_0) \in [a, R) & \text{if } 0 < X_0 < X^c \\ R & \text{if } X_0 = X^c \\ X_0 & \text{if } X_0 \in (X^c, b] \\ y(X_0) \in [a, b] & \text{if } X_0 > b \end{cases}.$$

If all hedge fund managers adopted sure-thing investment strategies, then a period-1 investor would provide the unit capital investment to a hedge fund manager if and only if the period payout X_0 exceeded R. But then, lesser-type hedge fund managers $\omega < R$ would not receive period-1 capital investment. Such less able hedge fund managers could do better if they adopted a risky investment strategy that stochastically concealed their lower abilities, placing positive probability on payoffs $X_0 \geq R$. In the simple cutoff-rule equilibrium, less able hedge fund managers $\omega < X^c$ do best to maximize the probability of receiving period-1 funding. They do this by not 'wasting' probability mass on unnecessarily high investment strategy payoffs, $X_0 > X^c$; and since any 'bad' investment payoff $X_0 < X^c$ results in not receiving funding, they also do not 'waste' probability mass on $X_0 \in (0, X^c)$.[3]

[3] More generally, they do not 'waste' probability mass on $X_0 \in (z, X^c)$.

A period-1 investor understands this strategic reasoning, and he also understands that some hedge fund managers whose investment strategies pay off $X_0 = X^c$ will be less able fund manager types $\omega < R$ who got lucky. Less able fund managers are less likely to have a successful investment outcome, but some will. To protect against investing in these less able, but lucky, hedge fund managers, a period-1 investor raises his cutoff for providing capital above R, to the level X^c that leaves him indifferent between investing in the hedge fund and investing solely in the alternative asset when all hedge fund managers with $\omega < X^c$ adopt the risky investment strategy that places as much probability as possible on X^c. Of course, in equilibrium, there will also be some unlucky hedge fund managers $\omega \in (R, X^c)$—those who realized $X_0 = 0$—in whom a period-1 investor would like to invest. But, not knowing their true types, a period-1 investor does not provide capital to failed hedge fund managers.

The equilibrium identified in Proposition 1 is a special case of a class of equilibria identified in Proposition 2, and we prove the result there. Proposition 1 identifies only one of a continuum of equilibria that we broadly index by the lowest hedge fund payout $\hat{X}^c > 0$ such that a hedge fund manager whose period-0 investment strategy pays off $X_0 = \hat{X}^c$ will receive capital from the period 1 investor. We index equilibria in this way because a fund manager with $\omega < \hat{X}^c$ will employ a fair investment strategy that pays off $\hat{X}^c$ with probability $\omega/\hat{X}^c$ and zero with residual probability. To describe these equilibria, it helps to define a particular cutoff, $\bar{X}$. Cutoff $\bar{X}$ has the feature that if: (a) all fund managers $\omega \in (a, \bar{X})$ adopt a period-0 investment strategy that places as much probability mass as possible on $\bar{X}$, and remaining probability on 0; and (b) all better fund manager types $\omega > \bar{X}$ do not have $X_0 \in \left[0, \bar{X}\right]$ in the support of their investment strategy, then $E_{\mu(X_0=0)}[\omega] = R$. This leaves a period-1 investor indifferent between investing in a fund manager whose investment strategy completely failed and investing only in the alternative asset. But then for $\hat{X}^c > \bar{X}$, if all fund managers $\omega \in (a, \hat{X}^c)$ place as much probability as possible on $X_0 = \hat{X}^c$, and the rest on $X_0 = 0$, and all fund managers $\omega > \hat{X}^c$ do not have $X_0 \in [0, \hat{X}^c]$ in the support of their investment strategy, a period-1 investor would want to invest even in hedge funds whose managers had unsuccessful investment outcomes $X_0 = 0$ because $E_{\mu(X_0=0)}[\omega] > R$. Thus, $\bar{X}$ represents the highest possible cutoff, or *standard*, that can be set on the period-0 hedge fund payout for providing capital to the hedge fund manager in period 1 such that a fund manager whose investment strategy pays out less than the standard is not funded. Since the probability that a hedge fund manager $\omega < \bar{X}$ places on $X_0 = 0$ is $1 - \omega/\bar{X}$, the critical value $\bar{X}$ solves

$$\frac{\int_a^{\bar{X}} \omega(\bar{X} - \omega)\Pi(d\omega)}{\int_a^{\bar{X}} (\bar{X} - \omega)\Pi(d\omega)} = R,$$

or

$$\bar{X} = \frac{\int_a^{\bar{X}} (\omega^2 - \omega R)\,\Pi(d\omega)}{\int_a^{\bar{X}} (\omega - R)\,\Pi(d\omega)}.$$

We now describe a broad class of equilibria that includes the simple cutoff-rule equilibrium identified in Proposition 1.

Proposition 2 *For each $\hat{X}^c \in (0, \bar{X})$, there exists an equilibrium in which all hedge fund managers with realization $X_0 = \hat{X}^c$ are funded in period 0. Hedge fund managers $\omega \leq \hat{X}^c$ adopt period-0 investment strategies that place probability $\omega/\hat{X}^c$ on payout $\hat{X}^c$, and residual probability on 0. They are funded if and only if $X_0 = \hat{X}^c$. Fund managers $\omega > \hat{X}^c$ are always funded, but they may adopt risky investment strategies in which they deliver realizations $X_0 \geq \hat{X}^c$ with probability one. In these equilibria, the expected period-1 payoff for a hedge fund manager of type ω takes the form $\beta\omega V(\omega)$, where $V(\omega)$ is the piecewise linear function,*

$$V(\omega) = \frac{\omega}{\hat{X}^c},\ \forall\omega < \hat{X}^c \quad \textit{and} \quad V(\omega) = 1,\ \forall\omega \geq \hat{X}^c.$$

There does not exist an equilibrium in which the lowest standard for reinvestment is $\hat{X}^c > \bar{X}$.

Proof. All proofs can be found in the Appendix. ■

Equilibria with $\hat{X}^c \in [X^c, \bar{X})$ can be supported by period-1 pessimistic out-of-equilibrium investor beliefs that $E_{\mu(X_0)}[\omega] \in [a, R)$ for $X_0 \in (0, \hat{X}^c)$. When $\hat{X}^c \in (X^c, \bar{X})$, we say that the standard for continued investment in the hedge fund is "excessively high."

One may wonder how equilibria with low standards $\hat{X}^c \in (0, X^c)$ for continued capital investment are supported. Such equilibria are supported by risky investment strategies taken by some hedge fund managers who are always financed. These hedge fund manager types ω, where $\hat{X}^c < \omega < \tilde{X}$, adopt fair investment strategies that place positive probability on both $\hat{X}^c$ and $\tilde{X}$. In turn, these investment strategies raise the expected quality of hedge fund managers who deliver $X_0 = \hat{X}^c$. Hedge fund managers $\hat{X}^c < \omega < \tilde{X}$ are indifferent among risky strategies, as well as sure thing strategies, that guarantee that they receive capital investment at period 1. One may think that this indifference can be broken by introducing arbitrarily small costs to adopting riskier investment strategies. But even with such costs, equilibria in which high ability fund managers employ risky investment strategies can be supported by (pessimistic) off-equilibrium investor beliefs that $E_{\mu(X_0)}[\omega] \in [a, R)$ for both $X_0 < \hat{X}^c$ *and* $X_0 \in (\hat{X}^c, \tilde{X})$. Hence, additional refinements are needed to eliminate these equilibria. In equilibria where fund managers $\omega > \hat{X}^c$ use risky investment strategies, investor beliefs are non-monotone in fund payouts, X_0. The non-monotonicity in beliefs induces fund manager types $\omega \in (\hat{X}^c, \tilde{X})$ to adopt fair investment strategies that place positive probability only on $\{\hat{X}^c, \tilde{X}\}$.

Most of the equilibria described in Proposition 2 seem unreasonable; for example, those with either excessively high cutoffs or non-monotone investor beliefs with risk taking by hedge fund managers who are sure to be funded. As well, there exist additional equilibria that are not detailed in Proposition 2 that seem odd. These additional equilibria feature "truly unnecessary" risk-taking by hedge fund managers $\omega > X^c$ when $\hat{X}^c \geq X^c$ or fund managers $\omega > \tilde{X}$ when $\hat{X}^c < X^c$. The risk taking is truly unnecessary in the sense that there exist equivalent equilibria to these additional equilibria where the above mentioned fund managers do not gamble and all other fund managers play the same strategies. However, these equilibria with unnecessary gambling cannot be eliminated by simply assuming small costs to adopting riskier investment strategies because they can be supported by non-monotone (pessimistic) investor beliefs.

There are even more implausible equilibria. For example, there exist equilibria exist in which good hedge fund managers employ *unfair* period 0 investment strategies, with fund managers $\omega > \hat{X}^c$ placing all probability mass on the outcome $\hat{X}^c$, supported by period-1 investor beliefs that $E_{\mu(X_0 \neq \hat{X}^c)}[\omega] \in [a, R)$, but $E_{\mu(X_0 = \hat{X}^c)}[\omega] \geq R$. Indeed, such unfair period-0 investment strategies can always support a "no-initial funding" equilibrium in which, even though $E[\omega] > R$, hedge fund managers do not receive capital investment in period 0, and only receive funding in period 1.[4] This no-funding equilibrium is supported by beliefs of period-0 investors that too many hedge fund manager types will pursue unfair period-0 investment strategies if they receive capital investment. Hedge fund managers would undertake unfair strategies for some perverse period-1 investor beliefs. For example, if capital were provided to fund managers at date 0, a period-1 investor's beliefs might be such that $E_{\mu(X_0)}[\omega] > R$ for $X_0 = R$, but $E_{\mu(X_0)}[\omega] < R$, for $X_0 \neq R$. Then, in a subgame in which cap-

[4]Fund managers always pursue fair,sure thing investment strategies in the last period of their investment horizons.

ital is provided in period 0, hedge fund managers $\omega > R$ would burn resources by undertaking unfair investment strategies, getting the sure payoff of R, while hedge fund managers $\omega < R$ adopt an investment strategy that places probability ω/R on R and residual probability on 0. But, since any hedge fund manager's investment strategy pays either R or zero in period 0, the expected payoff to a period-0 investor is strictly less than R. So he does not provide capital investment. Since the period-1 investor sees no track record and hedge fund managers pursue a fair investment strategy in period-1, the investor invests in the hedge fund.

Finally, as mentioned above, there is also an 'always-invest' equilibrium, in which the period-1 investor continues to invest in a hedge fund whose period-0 investment strategy lost everything. This is supported as an equilibrium when hedge fund managers pursue fair investment strategies that place positive probability only on the (unlikely) outcome $\hat{X}^c > \bar{X}$ and on 0, so that most investment strategies fail. By construction, the expected type of a hedge fund manager who pursued a failed investment strategy exceeds R.

We want to eliminate such implausible equilibria. To do this, we introduce a slight aversion of hedge fund managers to riskier investment strategies:

Assumption LC. *There is a lexicographical cost associated with riskier investment strategies. A type ω hedge fund manager strictly prefers investment strategy G^1_ω to investment strategy G^2_ω if G^1_ω has a higher expected lifetime payoff. If, however, G^1_ω and G^2_ω have the same expected payoff, then the fund manager strictly prefers the investment strategy with the smaller support on period investment payouts, as measured by the difference between the highest and lowest possible period investment payout.*

Assumption LC implies that a hedge fund manager first seeks to maximize expected lifetime profits. Then, from the set of investment strategies that maximize expected lifetime profits, the hedge fund manager selects an investment strategy with the least possible risk.[5]

We now provide two arguments for why the simple cutoff-rule equilibrium described in Proposition 1 is the "natural" one.

Proposition 3 *The ex-ante expected payoff of each period investor is at least as high in the simple cutoff-rule equilibrium as in any other equilibrium. Adding Assumption LC, among the set of equilibria that give period investors the highest expected payoff, hedge fund managers*

[5] The particular formulation of "a slight aversion to riskier investment strategies" is unimportant for our findings.

$\omega > X^c$ strictly prefer the simple cutoff rule equilibrium, and hedge fund managers $\omega \leq X^c$ are indifferent. Furthermore, under Assumption LC, the simple cutoff-rule equilibrium is the unique equilibrium to survive the Grossman-Perry refinement.

A period-0 investor is indifferent among all equilibria in which each hedge fund manager type adopts a fair investment strategy. From a period-1 investor's perspective, he is worse off in an equilibrium where the cutoff on investment strategy payouts X_0 for providing capital exceeds X^c, i.e., $\hat{X}^c > X^c$. This is because, relative to the simple cutoff rule equilibrium, the equilibrium with an excessive standard has good fund managers $\omega > X^c$ adopting risky investment strategies and, hence, some of these fund managers are needlessly denied funding in period 1 when their period-0 investment strategy fails. There are, however, no offsetting benefits associated with the higher cutoff $\hat{X}^c$. In particular, the composition of the pool of types $\omega < X^c$ is not improved with the higher standard $\hat{X}^c$, i.e., $E_{\mu\left(\hat{X}^c\right)}[\omega|\omega < X^c] = R$. Therefore, a period-1 investor's payoff will be increased if able hedge fund managers $\omega \in \left[X^c, \hat{X}^c\right]$ are funded in period 1, and this can be accomplished by adopting a lower cutoff, X^c.

One might think that a period-1 investor might be better off in an equilibrium in which risky investment strategies employed by types $\omega > X^c$ to support a cutoff $\hat{X}^c \in (R, X^c)$ because then some types $\omega \in [\hat{X}^c, X^c)$ are always funded, rather than just sometimes funded. However, the reduced weeding out of low ability hedge fund managers $\omega < R$ necessarily dominates. To see this, note that when the expected return conditional on $X_0 = \hat{X}^c$ is R, then it is a weighted average of two populations, where the expected return on the population of $\omega > X^c$ who deliver payout $\hat{X}^c$ exceeds R, implying that the expected return on the population of $\omega \leq X^c$ who deliver $\hat{X}^c$ is less than R. Thus, if the investor additionally knew that $\omega \leq X^c$, he would strictly prefer to invest instead in the alternative asset. (When $\hat{X}^c = X^c$, the investor knows that $\omega \leq X^c$ when $\hat{X}^c$ is delivered and does not strictly prefer to invest in the alternative asset.)

From an ex-ante perspective, prior to learning his type, a hedge fund manager always prefers an equilibrium with a lower cutoff since it raises the probability that he will receive capital if he turns out to be less able. When there are costs associated with riskier investment strategies, each hedge fund manager type prefers less risky investment strategies. In particular, given Assumption LC, all sufficiently able fund managers who can ensure period 1 capital inflows would like to adopt sure thing strategies. If they could adopt such a strategy, then equilibria with $\hat{X}^c < X^c$ cannot be supported, meaning that the "best" equilibrium from a sufficiently able hedge fund manager's perspective is the simple cutoff equilibrium. But, the costs associated with riskier investment strategies by themselves fail to eliminate (unreasonable) equilibria with cutoffs $\hat{X}^c < X^c$ since pessimistic out-of-equilibrium beliefs can be used to support them. However, since the Grossman-Perry refinement requires that out-of-equilibrium beliefs be updated in a consistent Bayesian fashion, it excludes such pessimistic beliefs that are used to support the unreasonable equilibria.

It is worth observing that the non-scalability of hedge fund investment strategies is important for the result that in the best equilibrium (from the perspective of investors) only the least able hedge fund managers adopt risky investment strategies. Our companion paper shows that when hedge fund investment strategies are partially scalable, the structure of the "best" possible equilibrium hedge fund manager investment strategies can take a more complicated form. Now the primary problem for an investor in a hedge fund becomes how *much* to invest, with more capital provided to hedge funds run by managers who are perceived as more able, rather than *whether* to invest. For example, it may be optimal to have better hedge fund managers adopt risky investment strategies to separate away from less able hedge fund managers whenever the extent of decreasing returns to scale in the investment technology are extreme enough to make it more important from a capital allocation perspective to identify which hedge fund managers are less able, rather than which ones are more able.

Who gains from unobservable ability? Compared to a full information environment in which investors know a hedge fund manager's ability, the simple cutoff-rule equilibrium has some hedge funds managers being made better off, some being made worse off and some being unaffected. All less able hedge fund manager types $\omega \in [a, R)$ benefit since, under full information, they never receive capital. These fund managers receive a strictly positive expected payoff in period 0 in the simple cutoff-rule equilibrium, and whenever they get lucky with their period-0 investment strategy, then they also receive a strictly positive payoff in period 1. Intermediate hedge fund manager types $\omega \in [R, X^c)$ are hurt since, under full information, they would always receive capital in period-1 and, therefore, receive positive payoffs in both periods. In the simple cutoff-equilibrium, these fund managers employ a risky period-0 investment strategy. While their *expected* period 0 payoff is the same as in a full information setting, their period 0 investment strategy fails with positive probability; and if this happens, their period 1 payoff is zero. Finally, sufficiently able hedge fund mangers types $\omega \in [X^c, b]$ receive the same payoffs in both information environments.

Empirical Regularities. We will show that this very simple model generates predictions that can reconcile many empirical regularities regarding hedge fund performance. We defer this presentation until after we solve for the equilibrium to the more general $N+1$-period hedge fund manager investment horizon.

Robustness. The analysis extends routinely if, after period-0 investors provide their capital, a hedge fund manager's period-0 expected payoff is subject to a period-0 shock $\eta_0 \geq -a$, so that a type ω hedge fund manager's investments have maximal expected period payoff $\omega + \eta_0$, as long as period-1 investors can infer η_0 before deciding whether to invest in the fund. Period-1 investors can obviously infer η_0 if it is directly observable. Investors may also be able to infer η_0 if it is a common shock that hits many hedge fund managers, so that investors can extract η_0 from the cross-section of hedge fund manager performances via the law of large numbers. *While we focus on a single hedge fund for simplicity, in practice, investors draw inferences about a fund manager's ability from his relative performance, and our model should be interpreted in that light.*

In Section 6 we examine how outcomes are affected when hedge fund managers abilities or payoffs are subject to idiosyncratic shocks. We examine two cases:

1. A type ω fund manager is hit with a mean-zero, idiosyncratic shock η_n that is independently distributed over time and privately observed *prior* to undertaking his investment strategy. This shock reflects the stochastic arrival of investment opportunities of different quality. Hence, a fund manager of type ω who is hit with shock η_n now has access to investment strategies with expected payoff $E_{G^n_\omega}(X) \leq \omega + \eta_n$.

2. Each fund manager is hit with an ex-post idiosyncratic (manager specific) shock to his payoff X_n *after* adopting an investment strategy. That is, the fund manager does not know the shock when selecting his investment strategy.

In each case, we characterize how the shocks affect equilibrium designs of a hedge fund manager's investment strategy.

One should caution that an investor's ability to infer η_n from the cross-section presumes that the distribution of hedge fund manager abilities is common knowledge. One could imagine a setting in which investors also learn about the distribution of hedge fund manager abilities,[6] in effect, learning about the expected over-all worth of investing in hedge funds rather than more standard assets. In this case, some of a positive common η_n shock (or ex-post shock) might be attributed to a better over-all distribution of hedge funds, so that only "some" of it is accounted for by investors in their decision making.

Our analysis also presumes that a given hedge fund's prospects do not systematically improve or worsen over time from the ex-ante perspective of an investor contemplating providing capital to a hedge fund. Such perturbations would not dramatically alter our qualitative predictions,[7] but they can substantially complicate analysis, especially when a hedge fund has a longer investment horizon. We next analyze precisely this case.

4 Multiperiod hedge fund investment horizon

We now consider a hedge fund manager who has an N-period investment horizon, where $N \geq 3$. The hedge fund manager must rely on capital raised from a sequence of risk neutral investors who have one-period investment horizons. Thus, a period-n investor only cares about investment payoffs in period n. Because these investors have only one-period investment horizons, there are no dynamic learning considerations. In particular, short-horizon investors have no incentive to set lower cutoffs on hedge fund period payouts for providing continued capital inflows because they do not attach a positive option value to being able to switch subsequently to the alternative asset. As a result, a hedge fund manager will receive capital in period n if and only if $E_{\mu_n(X^{n-1})}[\omega] \geq R$, where X^{n-1} is the history of payoffs from the fund manager's period investment strategies, i.e., $X^{n-1} = \{X_0, ..., X_{n-1}\}$.

Once again, there are many perfect Bayesian equilibria. However, we focus on the multi-period analogue of the simple cutoff-rule equilibrium, where the minimum payout that the hedge fund must achieve in period $n-1$ in order to receive capital in period n, X^c_{n-1}, satisfies $E_{\mu_n\left(X^{n-1}=X^{(n-1)c}\right)}[\omega] = R$, where $X^{(n-1)c} = \{X^c_0, \ldots, X^c_{n-1}\}$ is the history where, in each period, the hedge fund pays out the minimum amount required to receive continued capital

[6] The explosion in the total dollars invested in hedge funds and other institutional investors is consistent with this. For example, in the 1990s and early 2000s, many universities (e.g., Harvard) began to allocate substantial portions of their portfolios to hedge funds and other institutional investors. The subsequently relatively poor performance of hedge funds (see The Economist Dec 22, 2012) suggests that they may have over-estimated the average ability of these institutional investors to provide superior returns.

[7] This would lead to correlated entry and closure of hedge funds found in the data (Grecu et al. 2007) See also http://online.wsj.com/article/SB10001424127887324640104578163251346728208.html.

investment. One can show that the Grossman-Perry refinement together with Assumption LC eliminate all other equilibria. The next proposition characterizes the evolution of the sequence of cutoffs set by investors for continued infusion of capital into the hedge fund.

Proposition 4 *The multi-period simple cutoff-rule equilibrium is characterized by a sequence of cutoffs for re-investment* $\left\{X_0^c, \cdots, X_{N-1}^c\right\}$ *implicitly defined by*

$$\frac{\int_a^{X_n^c} \omega^{n+2} \Pi(d\omega)}{\int_a^{X_n^c} \omega^{n+1} \Pi(d\omega)} = R, \ \text{for } n \in \{0, \ldots, N-1\}. \tag{2}$$

A period 0 investor makes the unit capital investment in the hedge fund. Subsequent period n *investors make the unit capital hedge fund investment if and only if* $X_n \geq X_n^c$ *for* $n = 0, \ldots N-1$. *The cutoffs for re-investment strictly decline for older hedge funds, i.e.,* $X_0^c > X_1^c > \cdots > X_{N-1}^c$. *In each period* $n < N$, *fund manager types* $\omega \in (0, X_n^c)$ *that receive capital employ fair investment strategies that place probability* ω / X_n^c *on* X_n^c *and residual probability on 0, and fund manager types* $\omega \geq X_n^c$ *adopt sure-thing investment strategies that deliver* $X_n = \omega$ *with probability 1. In period* N *all hedge fund managers who receive funding employ sure-thing investment strategies.*

Intuitively, over time, investors become more confident that a hedge fund manager with the successful track record X^{nc} is "good": Less able fund managers are stochastically weeded out. As a consequence, investors do not require such a high payout by more established hedge funds in order to be willing to provide capital. That is, the cutoffs for period hedge fund payouts for continued capital investment fall over time. As a result, over time, some hedge fund manager types $\omega \in [X_n^c, X_{n-1}^c)$ who initially had to employ risky investment strategies in periods $0, \ldots n-1$ in order to have a chance of receiving continued capital inflows can switch to sure-thing investment strategies in period n. Of course, such fund managers only reach this point if their initially risky period investment strategies realized lucky outcomes.

To gain insights into (a) the extent to which period cutoffs for reinvestment exceed R (which measures how many good hedge fund manager types must adopt risky investment strategies), and (b) the rate at which these cutoffs decline over time as investors learn more about a fund manager's likely ability, it is useful to consider an explicit parameterization. Accordingly, suppose that the initial distribution over fund manager types $\Pi(\cdot)$ is uniform, and write the lower support as $a = \alpha R$, where $\alpha < 1$ measures the percent by which the worst fund manager provides a lower expected period return than the alternative asset. Then, from (2), the cutoffs for reinvestment solve:

$$\frac{(X_n^c)^{n+3} - (\alpha R)^{n+3}}{(X_n^c)^{n+2} - (\alpha R)^{n+2}} = (1 + \frac{1}{n+2})R.$$

Rearranging yields

$$(X_n^c)^{n+2}[(1 + \frac{1}{n+2})R - X_n^c] = (\alpha R)^{n+2} R(1 + \frac{1}{n+2} - \alpha).$$

If the worst hedge fund manager is completely inept, $\alpha \approx 0$, then $X_n^c \approx (1 + \frac{1}{n+2})R$. This implies that investors set a very high standard for a hedge fund manager who is just starting out, $X_0^c \approx 1.5R$. However, the standards for continued re-investment then drop sharply to $X_1^c \approx 1.33R$ and $X_2^c = 1.25R$, then slowly tailing off so that $X_{98}^c = 1.01R$. Conversely, when almost no hedge fund managers are bad, so that $\alpha \approx 1$, then $X_n^c \sim R$.

Quite generally, one can solve for $X_0^c = R\frac{3-2\alpha+\sqrt{(3+2\alpha)^2-(4\alpha)^2}}{4}$. Table 1 details different period cutoffs for various levels of α when $R = 1$. The table reveals that the re-investment

Table 1: Period cutoffs

α	X_0^c	X_1^c	X_2^c	X_3^c	X_4^c
0.0	1.50	1.333	1.25	1.20	1.167
0.5	1.366	1.284	1.229	1.191	1.162
0.8	1.176	1.157	1.141	1.128	1.117
1.0	1.00	1.00	1.00	1.00	1.00

cutoff for more experienced hedge fund managers becomes relatively insensitive to the quality of the worst fund manager type, as long as a non-trivial fraction lack ability, and declines slowly for more experienced hedge fund managers. Table 2 presents the corresponding hazards into liquidation for the hedge fund when the best fund manager type has ability $\omega = 2$. For plausible lower bounds on fund manager abilities ($\alpha = 0.5, 0.8$), hazards are initially high, initially decline quickly both due to the reduced standard for re-investment, and the stochastic weeding out of less able hedge fund managers, but then slowly tail off.[8]

[8] We will show that when investors have longer investment horizons, the hazards into liquidation have lower peaks, but remain elevated for longer.

Table 2: Hazard Rates

α	H_0	H_1	H_2	H_3	H_4
0.0	0.375	0.158	0.078	0.043	0.012
0.5	0.137	0.103	0.062	0.038	0.025
0.8	0.050	0.038	0.029	0.022	0.017
1.0	0.000	0.000	0.000	0.000	0.000

Empirical Implications and Discussion

The simple cutoff equilibrium that we describe has only three sources of uncertainty: uncertainty due to the unobserved heterogeneity in fund manager abilities, uncertainty due to commonly-observed shocks that investors can identify and "subtract out" in their calculations of hedge fund performance, and endogenous uncertainty introduced by less able fund managers who seek to mimic the performance of moderately-skilled hedge fund managers. Nonetheless, the dynamics of hedge fund returns in our simple model can reconcile many features regarding hedge fund returns and survival such as:

- Hedge funds must initially do "quite well" to receive continued capital inflows—most hedge funds that succeed initially required some luck in the sense that their initial returns must exceed their long-run returns. More established hedge funds receive capital inflows even if their returns slightly decline over time. This reflects that the rate of learning about hedge fund manager ability is very high initially, but drops off sharply once hedge funds have sufficient track records (roughly at rate $1/N$).

- Failure rates of hedge funds are initially very high, but fall sharply for more established (older) hedge funds. Howell (2001) finds that the probability a hedge fund fails in its first year is 7.4%. The more relevant failure rate is the second year failure rate of 20.3% in the second year—in practice, hedge funds do not fail "right away" due to lock-in and restriction periods.[9] As a result, hazards are initially flatter than our simple model predicts. Moreover, failure rates of younger hedge funds are substantially under-measured, because many do not survive long enough to enter hedge fund data bases, and there is selective backfilling of past (earlier) returns of hedge funds in the data bases

[9] Agarwal et al. (2009) document a median lock-in of one year, and there is often a restriction period, typically of four months, over which, after giving notice, one cannot withdraw funds.

that initially did well (Malkiel and Saha, 2005). Liquidation rates in 2008-09 were 15 percent per year and in 20 percent in 2010.[10] http://www.bloomberg.com/news/2010-09-21/hedge-fund-closure-rate-may-rise-to-20-on-lack-of-capital-merrill-says.html

- Identifying failure from the data is tricky as hedge funds self report, and some of the best hedge fund managers may cease reporting because they may not be actively seek new money, or choose to opt out of the data bases for other reporting reasons, or they may have merged with another hedge fund, and so on, (Gregoriou (2002)). Malkiel and Saha (2005) report a survivorship bias in hedge fund returns of 4.5% due to failing to account for exit; Brown, Goetzman and Ibbotson, 1999 report 3 percent bias (exit was higher post 2000).

- Nonetheless, even after accounting for these potential upward biases, average returns of funds that exit database are very poor in the few months preceding their exit, with highly negative Sharpe ratios. More generally their returns and Sharpe ratios are far worse at the end of their reporting lives. For example, the average monthly return for exiting firms in their final 3 months is -0.61% compared to 0.49% during their earlier lives, while the average Sharpe index is -1.859 in their final 3 months compared to 0.102 during their earlier lives (Grecu et al. 2007). The poor performance of 'true' exiting firms would be even worse to the extent that some firms that cease reporting do so because they merge or cease to advertise because they do not want additional investment.

- Many established hedge funds deliver returns that marginally exceed the return on alternative assets. This reflects that as investors observe longer histories of hedge fund success, they update to conclude that the survivors are likely better, making the investors more willing to provide them capital, even if their subsequent performance is slightly less good.

- A hedge fund's returns tend to fall over time. This reflects: (1) many hedge fund managers must initially take risky gambles, and succeed, generating payoffs that exceed their long-run average expected payoffs, and; (2) over time, as investors set lower cutoffs for continued reinvestment, these hedge fund managers employ less risky gambles, generating slightly lower returns when they succeed (and better hedge fund managers

[10] See http://www.bloomberg.com/news/2010-09-21/hedge-fund-closure-rate-may-rise-to-20-on-lack-of-capital-merrill-says.html

who initially had to employ risky gambles may be able to adopt sure thing investment strategies). In hedge fund databases, both of these effects are reinforced to the extent that there is backfill bias, so that the sample is differentially comprised of hedge funds that were initially successful. We show in Section 6.1 that the average decline over time in a hedge fund's return is reinforced if hedge fund investment opportunities have an idiosyncratic component, so that in some periods a hedge fund can identify better investment opportunities than in other periods—i.e., they are subject to pre-investment shocks to their investment opportunities. This means that successful hedge funds initially tend to be those that received positive idiosyncratic shocks, and hence had better investment opportunities than they will typically be able to deliver in the future.

- Better-performing funds have more persistence in their returns (Jagannathan et al. 2010, Jame 2013). In our simple model, this reflects that good hedge fund managers can adopt sure thing strategies, while less able hedge fund managers must employ risky strategies, and their expected returns decline conditional on their risky strategies succeeding.

- Returns of more established hedge funds are less volatile, and more predictable.

- Even though returns for a (surviving) hedge fund tend to decline with time, in the cross-section, the returns for older funds exceed those on less established funds if one accounts for the survivorship bias.[11]

Some of these predictions have important implications for interpretation of empirical findings. In particular, they indicate that regressions of hedge fund returns on a laundry list of regressors that include fixed hedge fund manager effects (to identify the manager's individual alpha) must be interpreted carefully. One can contemplate a researcher running such a regression, designed to isolate the impact of the hedge fund manager's experience on his performance, and concluding that more established hedge funds offer inferior returns on average. Rather than conclude that the hedge fund manager grows senile over time, one could imagine the researcher making conclusions regarding backfill bias, or strategic choices of safer, lower mean investment strategies, or appeal to a Berk and Green-style explanation that revolves around successful hedge funds receiving more capital investment, which

[11] This cross-sectional result can be 'reversed' in the data if there is sufficiently greater under-reporting of failure by younger hedge funds (backfill bias).

reduces their marginal returns. While recognizing that these features can contribute to the observed empirical relationship, appeal to such factors is unnecessary. In fact, this predicted relationship emerges in our extremely stripped-down, stylized model of strategic investment by hedge fund managers.

One can also imagine an investment advisor counseling investors on the basis of this empirical relationship to put their money in a pool of less-established hedge funds in order to generate superior returns. In fact, in our model, the relevant measure is the pooled cross-section of hedge fund returns. Further, such regressions, which include manager experience as a regressor, would reveal a *positive* coefficient on experience (absent backfill biases). This reflects that, over time, less able hedge fund managers are stochastically weeded out over time when their period investment strategies fail—on average, the expected ability of surviving hedge fund managers rises with experience in the cross-section.

While our model predicts that many institutional investors will adopt risky investment strategies that place significant weight on returns that slightly exceed those on alternative assets, and residual weight on very poor returns, it is silent on how institutional investors should implement those risky investment strategies in practice. In fact, in the early 2000s, one easy way to implement such a strategy in practice was to invest in tranches of mortgage-backed securities, where there was a risk of correlated default that we eventually experienced in the second half of the decade. Such assets realized slightly better returns than alternative assets, in those early years. Most of the focus on the sources of the crash of the housing market has been on the moral hazard of lenders extending loans to households that were unlikely to repay, or the small down payments that put many households under water on their mortgages, once housing prices started to fall. To the extent that securitization of these mortgages has received attention, it has only been because lenders needed to repackage and sell most of their mortgages in order to have enough capital to lend at such high levels. What has received far less attention is the question: Why were institutional traders willing to buy these repackaged loans, and why were they willing to do so at such low prices, driving down the implicit price of the risk of correlated default? One might posit that these assets offered simple ways in which to implement the risky investment strategies that we describe, and the explosive growth in the numbers of institutional investors may have sharply bid down the prices, as the institutional investors who lacked ability sought to deliver returns that slightly beat those on alternative assets. Our simple model, of course abstracts from these general equilibrium effects.

5 Longer Investor Horizons

To ease presentation, we have focused on investors with one-period horizons. It is, however, important to understand how longer investment horizons for potential investors affect equilibrium outcomes. Investors with longer horizons value the learning associated with observing a hedge fund's performance for longer since they have the option to switch to the alternative asset in the future.

To illustrate the qualitative impacts associated with longer investor horizons, we suppose that both the investor and hedge fund manager have three-period horizons, caring about payoffs in periods 0, 1 and 2. In each period, the investor chooses to make the unit capital investment in the hedge fund or to invest solely in the alternative asset. The investor sets two cutoffs for continued investment, $\hat{X}_0^c$ and $\hat{X}_1^c$, where $\hat{X}_n^c$ represents the minimum period n payout that results in the investor providing capital to the hedge fund in period $n+1$. We distinguish these critical cutoffs from their analogues, (X_0^c, X_1^c), for investors with one-period investment horizons described in Proposition 4. We focus on the simple cutoff-rule equilibrium—the analogue to that in Proposition 4—where the investor earns an expected future return equal to that on the alternative asset from a hedge fund whose payout just meets the cutoff for continued capital investment. Now, however, the relevant return for the investor is his expected future *lifetime* return, where, after observing X_0, the investor takes into account that if he provides capital to the hedge fund, then he can switch to the alternative asset in period 2 if he is dissatisfied with the period 1 payout.

We now show that investors with longer investment horizons initially set lower cutoffs for providing continued capital investments in order to have the opportunity to learn more about the hedge fund manager's ability. This reduces the probability that they prematurely cease to provide capital to a fund run by an able manager. In turn, some hedge fund managers with intermediate abilities adopt less risky investment strategies.

Proposition 5 *In equilibrium, a long-horizon investor sets a lower initial cutoff on period-0 payouts for continued capital investments than a short-horizon investor, $\hat{X}_0^c < X_0^c$. However, they set the same final period cutoff, $\hat{X}_1^c = X_1^c$, where $\hat{X}_1^c < \hat{X}_0^c$.*

Because $\hat{X}_1^c = X_1^c \leq \min\left\{\hat{X}_0^c, X_0^c\right\}$, the period-1 distributions of hedge fund manager types conditional on $X_1 = X_1^c$ are the same for period-0 cutoffs X_0^c and $\hat{X}_0^c$. This reflects that a higher cutoff on period 0 payouts reduces the fraction of *all* types of hedge fund managers *that are below the period-1 cutoff* by the same proportion. To see this, note that if the period-0 cutoff is X^c, then the fraction of all types $\omega < X^c$ with $X_0 = X^c$ is ω/X^c. A higher period-0 cutoff reduces the fraction of all types below the period-1 cutoff by a factor of $\hat{X}_0^c/X_0^c$, compared to the lower period-0 cutoff. As a result, the short-horizon and long-horizon investors set the same period 1 cutoff for providing capital in the terminal period 2.

In contrast, a long-horizon investor sets a lower cutoff on period 0 payouts for providing capital in period 1. A long-horizon investor places a positive value on basing period 2 capital investment decisions on information revealed in period 1, whereas a short-term investor does not. Most obviously, because the cutoff for continuing to provide capital falls over time, a long-term investor attaches a positive value to learning in period 1 that all $\omega \in [\hat{X}_1^c, \hat{X}_0^c]$ are valuable hedge fund managers, and should receive capital. This leads a long-horizon investor to set a lower period-0 cutoff in order to avoid prematurely ceasing to provide capital to good hedge fund managers.

These qualitative insights extend: The longer is an investor's horizon, the lower are the cutoffs that the investor sets early in the investment horizon for providing capital to the hedge fund. In turn, this implies that the hazard into liquidation does not have as high a peak (less able hedge fund managers are weeded out more slowly), but also that the hazard remains high for longer (because more less able hedge fund managers remain).

Setting a lower period-0 cutoff means that more hedge fund managers $\omega \in (X_1^c, X_0^c)$ receive capital in all three periods, which is socially beneficial since $X_1^c > R$, and only a fraction ω/X_1^c of remaining ω-type managers with $\omega < X_1^c$, are funded in the final period.

The lower cutoffs for continued capital provision set by longer-horizon investors have implications for hedge-fund performance that may initially seem paradoxical:

Proposition 6 *Expected hedge fund payouts in periods 1 and 2 are lower with long-horizon investors than short-horizon investors.*

One could imagine a researcher isolating two groups of investors, one with long horizons, and the other with short horizons, calculating their hedge fund returns, and concluding from the lower average measured hedge fund returns in the portfolio of the long-horizon investors that these investors are "worse" than the short-horizon investors, or that short-horizon investors are better off than their long-horizon counterparts. Obviously, such conclusions are misplaced.

The expected performance of a hedge fund is lower in both periods 1 and 2 with long-horizon investors than with short-horizon investors *precisely* because the former has more “lower ability” hedge fund managers than the latter. That is, they both fund all able hedge fund manager types $\omega > X_0^c$, but a short-horizon investor is less likely to fund types $\omega < X_0^c$. But some of these lower ability hedge fund managers are able to offer an expected payoff that exceeds that on the alternative asset since $R < \hat{X}_0^c < X_0^c$ and $R < \hat{X}_1^c = X_1^c$. The correct comparison is not of these lower ability hedge fund managers with higher ability ones, but rather the comparison of their expected return with the alternative asset. Thus, it would be incorrect to compare the lower expected payouts from *surviving* hedge funds that receive capital from long-horizon investor with those associated with a short-horizon investor. The correct conclusion to draw is that too many short-horizon investors prematurely switch to investing in the alternative asset in period 1 because of their high period-0 cutoff—too few short-horizon investors provide capital to hedge funds.

6 Pre- and Post-Investment Shocks

So far we have characterized equilibrium outcomes in settings where any common shocks that hit the payoffs of all hedge fund managers are filtered out via comparisons of relative performances. Hence, the only remaining uncertainty that a hedge fund manager faces is the endogenous uncertainty created by his choice of investment strategy. In reality, hedge funds receive exogenous *idiosyncratic* shocks outside of their control that affect their payouts. Because investors in hedge funds do not observe these shocks, they complicate their efforts to extract information about a hedge fund manager’s ability from observed hedge fund payouts.

One must distinguish the impacts of idiosyncratic shocks that occur prior to a hedge fund manager’s investment choices from those that occur afterwards. Pre-investment, a hedge fund manager may receive an idiosyncratic shock that affects the quality of his investment strategy opportunity—in some periods a fund manager may be able to uncover better investment possibilities than in others. From a forecasting perspective, a potential investor wants to isolate the permanent component that describes a fund manager’s ability, as it is this long-term ability that drives expected future hedge fund performance. However, since the hedge fund manager observes the pre-investment shock prior to selecting an investment strategy, he can directly condition his investment strategy choice on both his ability and the shock realization.

After a hedge fund manager chooses an investment strategy, his portfolio may be hit with a (post-investment) shock that affects the ultimate hedge fund payoff. While a hedge fund manager will take account of the possibility of post-investment shocks when choosing his investment strategy, he is unable to condition his investment strategy choice on a particular shock realization since he does not know it at that time.

To highlight the qualitative effects of such shocks we return to our two-period economy with short-horizon investors. We focus on the simple cut-off equilibrium, where a period 1 investor provides capital to a hedge fund if and only if its period 0 total payout meets or exceeds the cutoff X^c. In this cutoff equilibrium, the expected ability of the hedge fund manager who generates a payout of X^c is equal to R. We now explore how these two sources of uncertainty, which are outside the control of a hedge fund manager, affect investor cutoffs for continued capital investment, and the design of a fund manager's investment strategies.

6.1 Pre-Investment Shocks

To ease presentation, we assume that a hedge fund manager's ability ω is drawn from a uniform distribution on $[a, b]$, with associated pdf, $\pi(\cdot)$. With pre-investment shocks, In both periods 0 and 1, the hedge fund manager receives an idiosyncratic shock η_n prior to selecting an investment strategy that determines the quality of the investment opportunities to which he has access in the period. Thus, a fund manager with ability ω who is hit with a shock η_n can implement any investment strategy that delivers a non-negative payoff with an expected payoff that does not exceed $\omega + \eta_n$. We assume that η_n is identically and independently distributed over time on $[-\bar{y}, \bar{y}]$ according to a pdf $h(\eta_n)$. We suppose that $h(\eta_n)$ is symmetric, so that $h(\eta_n) = h(-\eta_n)$. We also assume that $\bar{y}$ is small enough that the solutions characterized below are interior, i.e., we assume that $\bar{y} < a$ and $b > X^c + \bar{y}$.

In period 0, if $\omega + \eta_0 < X^c$, then it is optimal for the hedge fund manager to undertake the risky investment strategy $\{0, X^c\}$, i.e., the hedge fund manager adopts an investment strategy that pays out X^c with probability $(\omega + \eta_0)/X^c$, and pays out zero otherwise. If, instead $\omega + \eta_0 > X^c$, then it is optimal for the hedge fund manager to pursue the sure-thing investment strategy. In effect, in period 0 the hedge fund manager behaves as *if* his permanent type is $\omega +$

η_0. Interestingly, even though the expected value of the pre-investment shock is zero, we have,

Proposition 7 *Pre-investment idiosyncratic shocks reduce the equilibrium cutoff X^c set by investors for continued capital investment in the hedge fund.*

The intuition underlying Proposition 7 is that achieving a given cut-off provides the investor with better news when there are non-trivial idiosyncratic shocks η_0 compared to when there are not. The news is better because some high ω hedge fund manager types happen to get unlucky (negative) draws of η—and must pursue a risky investment strategy—and some low ω types happen to get lucky draws of η—and pursue a sure-thing investment strategy. With the symmetric density, $h(\eta)$, lucky low ω types are "replaced" by unlucky high ω types, and this raises the expected ability of the hedge fund manager whose payoff hits the cutoff.

Thus, pre-investment shocks have three important qualitative empirical implications: (1) they reduce the spread between the return that less able hedge fund managers seek to achieve and the return on standard assets; (2) hazards into liquidation have a lower peak, but remain high for longer; and (3) because surviving hedge funds are more likely to have received positive idiosyncratic shocks, their future expected performance will fall to reflect that their period-expectation is always zero.

6.2 Post-Investment Shocks

With post-investment shocks, after each hedge fund manager type ω implements his period-0 investment strategy $G^0_\omega(X_0)$, the fund's payout is hit by a (post-investment) shock η_0. We examine a multiplicative shock structure, where the "final" total payout in period 0 is $\tilde{X}_0 = \eta_0 X_0$, where $E[\eta_0] = 1$, and $\eta_0 > 0$. The period 1 investor only observes $\tilde{X}_0$ and not its separate components, X_0 and η_0. We focus on a multiplicative shock structure because it delivers a non-negative fund payout. One can formulate additive shock structures in a variety of ways (e.g., with $E[\eta_0] = 0$ and allowing for $\tilde{X}_0 = \eta_0 + X_0 < 0$ or imposing a high "enough" bound on X_0) that yield qualitatively identical results.

To ease presentation, we suppose that the post-investment shock η_0 is drawn from a uniform distribution on $[1-\delta, 1+\delta]$, where $0 < \delta < 1$. The associated cdf, $F(\eta_0)$, is

$$F(\eta_0) = \begin{cases} 0 & \text{for } \eta_0 \leq 1-\delta \\ -\frac{1-\delta}{2\delta} + \frac{1}{2\delta}\eta_0 & \text{for } \eta_0 \in [1-\delta, 1+\delta] \\ 1 & \text{for } \eta_0 \geq 1+\delta \end{cases} . \tag{3}$$

Our more general presentation is designed to highlight how the results extend.

Denote the date 0 cutoff as X^c. In equilibrium, the period 1 investor provides capital to the hedge fund if and only if $\tilde{X}_0 = X_0\eta_0 \geq \underline{X}^c$. Given a cutoff $\underline{X}^c$, a hedge fund manager with ability ω seeks to maximize the probability that $\tilde{X}_0 \geq \underline{X}^c$ by choosing an appropriate investment strategy, $G^0_\omega(X_0)$. The hedge fund manager must receive a shock of $\eta_0 \geq \underline{X}^c/X_0$ to obtain investment capital in period 1. This occurs with probability $1 - F(\underline{X}^c/X_0)$, where

$$1 - F\left(\underline{X}^c/X_0\right) = \begin{cases} 0 & \text{for } X_0 \leq \underline{X}^c/\left(1+\delta\right) \\ \frac{1+\delta}{2\delta} - \frac{1}{2\delta}\frac{X^c}{X_0} & \text{for } X_0 \in \left[\underline{X}^c/\left(1+\delta\right), X^c\left(1-\delta\right)\right] \\ 1 & \text{for } X_0 \geq \underline{X}^c/\left(1-\delta\right) \end{cases} . \tag{4}$$

Note that the probability of success, $1 - F\left(\underline{X}^c/X_0\right)$, is strictly concave in X_0 for $X_0 \in \left[\underline{X}^c/\left(1+\delta\right), \underline{X}^c\left(1-\delta\right)\right]$.

The hedge fund manager's period 0 optimization problem is,

$$\max_{G^0_\omega} \int_{X_0 \geq 0} \left[1 - F(\underline{X}^c/X_0)\right] dG^0_\omega\left(X_0\right) \tag{5}$$

$$\text{subject to} \int_{X_0 \geq 0} dG^0_\omega\left(X_0\right) = 1 \text{ and } \int_{X_0 \geq 0} X_0 dG^0_\omega\left(X_0\right) = \omega,$$

where the constraints reflect that the manager chooses a fair investment strategy.

To understand the nature of the period 0 strategy $G^0_\omega\left(X\right)$ that hedge fund manager ω employs, consider the concave hull of $1 - F\left(\underline{X}^c/X\right)$. It is either strictly concave for some $X \in \left(X^u, \underline{X}^c/\left(1-\delta\right)\right)$—as $0AB$ in Figure 1—or piecewise linear for all $X > 0$—as $0AB$ in Figure 2. In either case, this implies that all sufficiently weak hedge fund managers pursue a risky strategy that places probability ω/X^u on $X_0 = X^u$, and residual probability on $X_0 = 0$, while hedge fund managers with $\omega > X^u$ employ sure-thing investment strategies. Thus, for hedge fund manager types, $\omega < X^u$, the period-0 optimization problem (5) simplifies to

$$\max_{X^u} \left[1 - F\left(\underline{X}^c/X^u\right)\right]/X^u. \tag{6}$$

If X^u is less than $\underline{X}^c/\left(1-\delta\right)$—as in Figure 1—then X^u is given by the implicit solution to the first-order condition,[12]

$$\frac{f\left(\underline{X}^c/X^u\right) X^c}{\left(X^u\right)^3} - \frac{\left[1 - F\left(\underline{X}^c/X^u\right)\right]}{\left(X^u\right)^2} = 0.$$

[12] With uniformly-distributed shocks, second-order conditions hold.

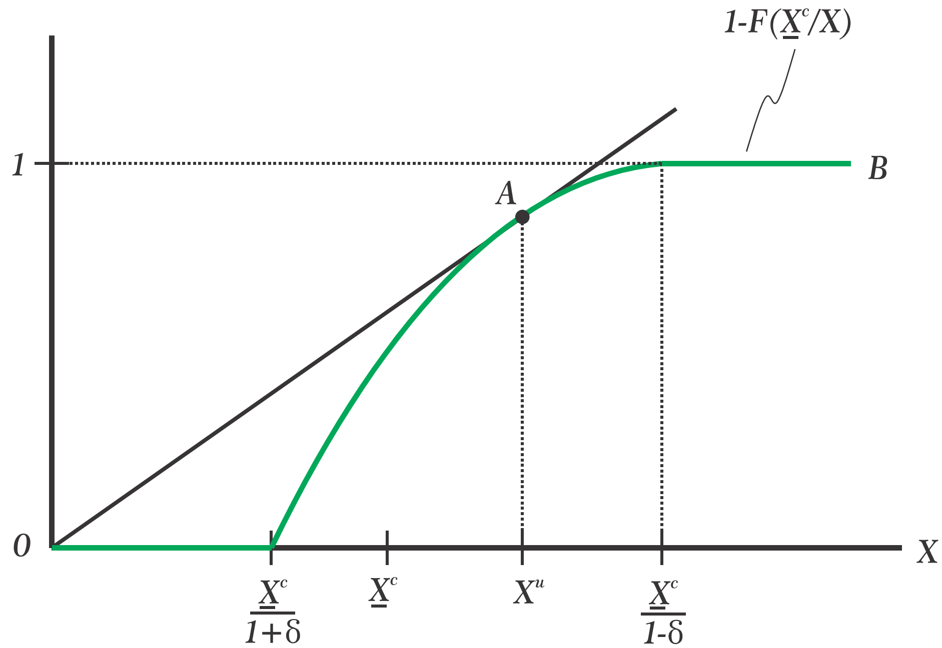

Figure 1: "Large" δ

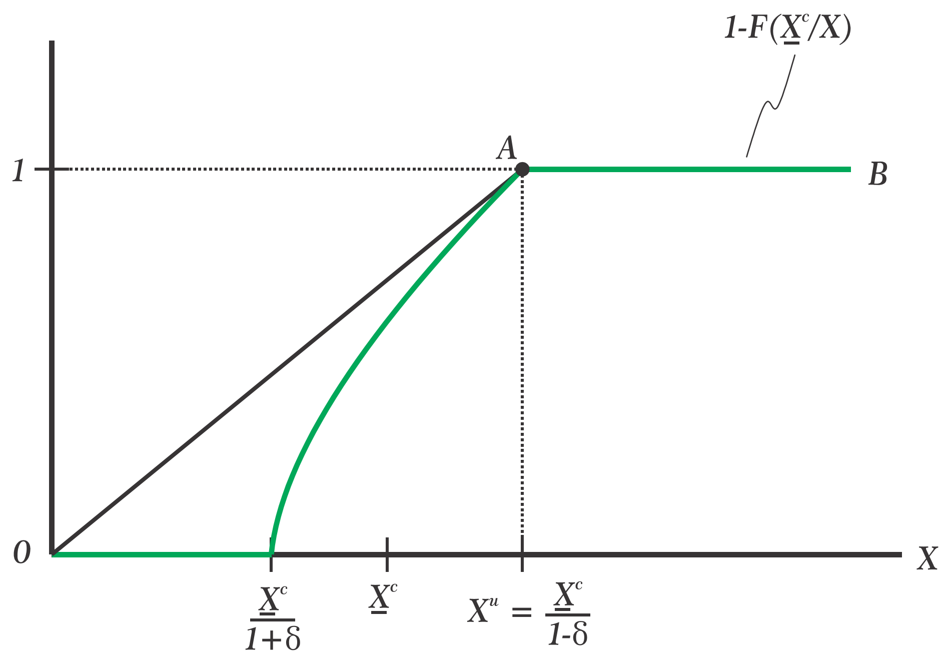

Figure 2: "Small" δ

We can re-arrange this first-order condition to solve for X^u in terms of the hazard,

$$X^u = \underline{X}^c \frac{f\left(\underline{X}^c/X^u\right)}{1 - F\left(\underline{X}^c/X^u\right)}. \tag{7}$$

Adding the assumption that η has a uniform distribution, one can solve explicitly for

$$X^u = \frac{2\underline{X}^c}{1+\delta}, \tag{8}$$

when X^u is interior.

When there is little "ex post" uncertainty, i.e., when δ is sufficiently small, then—as in Figure 2—the hedge fund manager resolves all funding uncertainty whenever his risky strategy succeeds by choosing $X^u = \underline{X}^c/\left(1-\delta\right)$. When $X^u = \underline{X}^c/(1-\delta)$ and the hedge fund manager's risky strategy succeeds, he is always funded in period 1 since his lowest possible final payout is $(1-\delta)\underline{X}^c/(1-\delta) = \underline{X}^c$. Thus, the solution to (6) is given by $X^u = \min\left\{\frac{X^c}{1-\delta}, \frac{2X^c}{1+\delta}\right\}$, so that $X^u = 2\underline{X}^c/\left(1+\delta\right)$ for $\delta \geq 1/3$ and $X^u = \underline{X}^c/\left(1-\delta\right)$ for $\delta \leq 1/3$.

Consider now a cut-off equilibrium characterized by: (1) a cutoff, $\underline{X}^c$; (2) the fair risky investment strategy $\{0, X^u\}$ adopted by hedge fund managers $\omega \in [a, X^u)$, with $\omega > X^u$ pursuing the sure-thing investment strategy; and (3) investor beliefs $\mu_{\underline{X}^c}$, which are consistent with the distribution of hedge fund manager abilities conditional on the hedge fund period 0 payoff $\tilde{X}_0$ and $E_{\mu(\underline{X}^c)}[\omega] = R$. Suppose further that the lowest type $\hat{b} < b$ that pursues the sure-thing investment strategy is always funded. This implies that $\delta \leq \frac{1}{3}$, i.e., ex-post shocks are small. Thus, only types $\omega < X^u$ who are hit by $\eta_0 = 1-\delta$ realize X^c implying that

$$E_{\mu(\underline{X}^c)}[\omega] = \frac{\int_a^{X^u} \frac{\omega^2}{X^u}\Pi(d\omega)}{\int_a^{X^u} \frac{\omega}{X^u}\Pi(d\omega)} = \frac{\int_a^{X^u} \omega^2\Pi(d\omega)}{\int_a^{X^u} \omega\Pi(d\omega)} = R. \tag{9}$$

It follows that when possible ex-post shocks are small, X^u equals the cutoff X^c for continued investment when there are no ex-post shocks (compare (9) with (1) in Section 3). Thus, small ex-post shocks do not alter the investment *strategies* chosen by hedge fund managers. However, such ex-post shocks do reduce the *cutoff* for continued funding to $\underline{X}^c = (1-\delta)X^c$. Further, while there is clustering of hedge fund payouts, the cluster is now spread over $[(1-\delta)X^c, (1+\delta)X^c]$, and, in fact, peaks with the uniform distribution at $(1+\delta)X^c$, as more types $\omega > X^c$ who do not adopt risky investment strategies still receive unlucky η_0 draws and hence lower ex-post payouts. Importantly, when the extent of ex-post uncertainty is small, i.e., when $\delta \leq 1/3$, the probability with which hedge fund managers secure continued re-investment is unaffected, as only the probability with which the risky investment strategies of bad hedge fund managers $\omega < X^c$ succeed at delivering $X^u = X^c$ determines whether they are funded.

Suppose now that the lowest type $\hat{b} < b$ that pursues a sure-thing investment strategy is not always funded. This implies that $\delta > \frac{1}{3}$, i.e., ex-post shocks can be large. When this is so, $X^u = \frac{2\underline{X}^c}{1+\delta}$. Consequently, some hedge fund managers whose 'initial' investment strategies succeed still receive unlucky draws that cause them not to be funded. We can solve for the critical η_0^* that a hedge fund manager $\omega < X^u$ requires to receive period 1 funding: $X^u \eta_0^* = \frac{2\underline{X}^c}{1+\delta}\eta_0^* = \underline{X}^c \Rightarrow \eta_0^* = \frac{1+\delta}{2}$. Furthermore, all types $\omega \in (\frac{2\underline{X}^c}{1+\delta}, \frac{\underline{X}^c}{1-\delta})$ sometimes fail to receive funding. Equilibrium demands that

$$R = E_{\mu(\underline{X}^c)}[\omega] = \frac{\int_a^{X^u} \frac{\omega^2}{X^u} f(\underline{X}^c/X^u)\Pi(d\omega) + \int_{X^u}^{\frac{X^u(1+\delta)}{2(1-\delta)}} \omega f(\underline{X}^c/\omega)\Pi(d\omega)}{\int_a^{X^u} \frac{\omega}{X^u} f(\underline{X}^c/X^u)\Pi(d\omega) + \int_{X^u}^{\frac{X^u(1+\delta)}{2(1-\delta)}} f(\underline{X}^c/\omega)\Pi(d\omega)}.$$

Adding the assumption that η_0 has a uniform distribution yields:

$$R = E_{\mu(\underline{X}^c)}[\omega] = \frac{\int_a^{X^u} \frac{\omega^2}{X^u}\Pi(d\omega) + \int_{X^u}^{\frac{X^u(1+\delta)}{2(1-\delta)}} \omega\Pi(d\omega)}{\int_a^{X^u} \frac{\omega}{X^u}\Pi(d\omega) + \int_{X^u}^{\frac{X^u(1+\delta)}{2(1-\delta)}} \Pi(d\omega)}. \tag{10}$$

It follows directly that at $X^u = X^c$, the right-hand side of (10) exceeds R, and indeed for a fixed X^u, the right-hand side is increasing in δ. To retrieve equality, it must be that as ex-post shocks grow larger, i.e., as δ increases, then X^u falls. That is, greater ex-post uncertainty causes hedge fund managers to introduce less endogenous strategic investment uncertainty. These ex-post shocks impair an investor's ability to learn about a hedge fund manager's ability—some relatively poor performers may be skilled but unlucky hedge fund managers. In turn, this causes the investor to be more willing to extend funding.

Gathering the observations from these extensions of our basic model reveals that enriching our base stylized model to allow for investors with longer investment horizons or pre- or post-investment shocks to hedge fund payoffs all qualitatively serve to induce increased patience in hedge fund investors. In equilibrium, investors become more willing to tolerate lower initial investment returns, which causes less able hedge fund managers to pursue less risky investment strategies, thereby slowing the exit rate of hedge funds.

One can also show that ex-post shocks serve as a refinement of sorts, reducing the set of equilibria that can be supported by perverse investor beliefs in the absence of those shocks (i.e., relative to the basic no-shock model in Section 3). While such shocks do not eliminate the no fund equilibrium, they help reduce the set of equilibria with excessively high minimum standards for continued investment. For example, with large ex-post shocks, i.e., $\delta > 1/3$, and lexicographic costs associated with riskier investment strategies, a unique funding equilibrium obtains without appeal to refinements. This is because the lexicographic costs ensure that good hedge fund manager types $\omega > 2\underline{X}^c/(1+\delta)$ employ sure thing investment strategies;[13] and the shocks are large enough that, in equilibrium, some hedge funds realize "successful initial" payout X^u, but bad post-investment shocks result in a 'final period' payout $\tilde{X}_0 = X^u\eta_0 < \underline{X}^c$, so that they do not receive continued capital investment. Therefore, we must have, $R = E_{\mu(\underline{X}^c)}[\omega]$: pessimistic off-equilibrium beliefs given $\tilde{X}_0 \in (0, \underline{X}^c)$ cannot be used to support other equilibria, making additional refinements unnecessary.

7 Concluding Remarks

Our paper begins with the observation that hedge fund managers zealously conceal investment strategies. This complicates the inference problem of investors who only see the returns of those investment strategies on a periodic basis and must forecast what future returns will be. Hedge fund managers know their "true abilities" and understand that investors will shift investments away from poorly-performing funds toward better performers. We consider a simple setting in which the problem of a potential investor in a hedge fund is *whether* to provide capital and not how much capital to provide. This allows us to model the enormous discretion hedge fund managers have in the design of their investment strategies. We allow hedge fund managers to tailor their investments however they see fit, requiring only that the investment strategy have a payoff that is bounded from below and that, in expectation, returns reflect the hedge fund manager's ability. Given these modest restrictions, we characterize the different investment strategies that can emerge in equilibrium, together with the criteria set by investors for providing continued capital to the hedge funds.

[13] That is, $b > X^u$. To see this, suppose instead that $b \leq X^u$. Then all hedge fund managers pursue the risky strategy $\{0, X^u\}$, and the expected quality of a hedge fund manager with payout $\underline{X}^c$ is $\int_a^b \omega^2 \Pi(d\omega)/\int_a^b \omega\Pi(d\omega)$. But, since $\int_a^b \omega\Pi(d\omega) > R$, a contradiction obtains:

$$R = E_{\mu(\underline{X}^c)}[\omega] = \frac{\int_a^b \omega^2 \Pi(d\omega)}{\int_a^b \omega\Pi(d\omega)} > \int_a^b \omega\Pi(d\omega) > R.$$

We then show that equilibrium is uniquely pinned down under a set of different equilibrium refinements. In this equilibrium, investors set simple cutoff standards for re-investment that slightly exceed the expected return on alternative investments. Facing such re-investment standards, less able fund managers employ risky investment strategies that maximize the probability of meeting the re-investment standards at the cost of a positive probability of producing disastrously low returns, while more able fund managers choose not to introduce extra risk to their investment strategies. Re-investment standards decline for more experienced hedge fund managers. This reflects that investors have longer track records on which to assess performance, and that less able fund managers are stochastically weeded out over time.

We are not the first to recognize a manager's incentive to gamble. The notion that the unobservability of investment strategies can induce hedge fund managers to "employ" "unnecessarily" risky strategies dates back at least to Degeorge et al. (1996). Related incentives show up in Ljungqvist (1994). The key contributions of our paper are first to derive the structure of those risky strategies in an environment with minimal restrictions on their structure. Unlike most of the literature, we do not require that managers choose from a set of narrowly specified investment alternatives. Instead, we allow hedge fund managers to freely design their own investment payoff structure. We do this not for the sake of generality, but rather because this is what we observe in practice, and because this minimalist approach to investment selection allows us to match a set of empirical regularities regarding hedge fund performance and to resolve a set of seeming paradoxes.

We derive a host of implications for the equilibrium dynamics of hedge fund returns and survival. For example, we predict that in a regression with fixed hedge fund manager effects the returns of more experienced hedge fund managers should decline, even though the average profits of investors in hedge funds rise with hedge fund manager experience due to learning. We show how idiosyncratic shocks to a hedge fund manager's investment opportunities or idiosyncratic ex post shocks to fund payouts strengthen this relationship. So, too, we show that the longer is an investor's horizon, the lower is the expected return of the hedge funds in which he invests. We predict that more experienced hedge funds deliver less volatile returns; that persistence of returns is greater for more able hedge fund managers; hedge fund failure rates are initially very high, but fall sharply with hedge fund manager experience due both to the improved selection and the declining reinvestment standards; that returns of exiting hedge funds will be far worse than historical returns; and so on.

Although our model provides new insights and helps us understand many qualitative empirical features of hedge fund performance, it is restricted in an important regard: the size of a hedge fund is either zero or one. This means that we cannot analyze the relationship between *incremental* flows into and out of hedge funds and hedge fund performance. We address this in our companion paper.

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9 Proofs

Proof to Proposition 2

When $\hat{X}^c \in (X^c, \bar{X})$, where X^c is defined in (1), hedge fund managers $\omega \in [a, \hat{X}^c)$ use the risky fair investment strategy $\{0, \hat{X}^c\}$ and hedge fund managers $\omega \in [\hat{X}^c, b]$ use the sure-thing investment strategy. Period-1 investor beliefs are summarized by:

$$E_{\mu(X_0)}[\omega] = \begin{cases} \frac{\int_a^{\hat{X}^c} \omega(X^c-\omega)\Pi(d\omega)}{\int_a^{\hat{X}^c}(X^c-\omega)\Pi(d\omega)} < R & \text{if } X_0 = 0 \\ y(X_0) \in [a, R) & \text{if } X_0 \in (0, \hat{X}^c) \\ \frac{\int_a^{\hat{X}^c} \omega^2\Pi(d\omega)}{\int_a^{\hat{X}^c} \omega\Pi(d\omega)} \geq R & \text{if } X_0 = \hat{X}^c \\ X_0 & \text{if } X_0 \in [\hat{X}^c, b] \\ y(X_0) \in [a, b] & X_0 > max\{b, \hat{X}^c\} \end{cases}.$$

The beliefs of investors are consistent with hedge fund manager behavior. Given the strategy of hedge fund managers, investors optimally provide a unit capital investment to hedge fund managers with $X_0 \geq \hat{X}^c$ in period 1; and given the beliefs and strategies of investors, the best response for hedge fund managers is the strategy described above.

When $\hat{X}^c \in (0, X^c)$, hedge fund managers $\omega \in [a, \hat{X}^c)$ use the risky fair investment strategy $\{0, \hat{X}^c\}$, hedge fund managers $\omega \in (\hat{X}^c, \tilde{X})$ use the risky fair investment strategy $\{\hat{X}^c, \tilde{X}\}$ (where the notation $\{x, y\}$ denotes the fair investment strategy in which the hedge fund manager places as much probability as possible on the higher fund payout, y and the remaining probability on the lower fund payout x), and if $b > \tilde{X}$, then hedge fund managers

$\omega > [\tilde{X}, b]$ use the sure-thing investment strategy. Period-1 investor beliefs are given by:

$$E_{\mu(X_0)}[\omega] = \begin{cases} \frac{\int_a^{\hat{X}^c} \omega(X^c - \omega)\Pi(d\omega)}{\int_a^{\hat{X}^c} (X^c - \omega)\Pi(d\omega)} < R & \text{if } X_0 = 0 \\ y(X_0) \in [a, R) & \text{if } X_0 \in \left(0, \hat{X}^c\right) \\ \frac{\int_a^{\hat{X}^c} \omega(\omega/\hat{X})\Pi(d\omega) + \int_{\hat{X}^c}^{\tilde{X}} \omega(\tilde{X} - \omega)/(\tilde{X} - \hat{X}^c)\Pi(d\omega)}{\int_a^{\hat{X}^c} (\omega/\hat{X})\Pi(d\omega) + \int_{\hat{X}^c}^{\tilde{X}} (\tilde{X} - \omega)/(\tilde{X} - \hat{X}^c)\Pi(d\omega)} \geq R & \text{if } X_0 = \hat{X}^c \\ y(X_0) \in [a, R) & \text{if } X_0 \in (\hat{X}^c, \tilde{X}) \\ \frac{\int_{\hat{X}^c}^{\tilde{X}} \omega\left(\omega - \hat{X}^c\right)\Pi(d\omega)}{\int_{\hat{X}^c}^{\tilde{X}} \left(\omega - \hat{X}^c\right)\Pi(d\omega)} > R & \text{if } X_0 = \tilde{X} \\ X_0 & \text{if } \tilde{X} < X_0 \leq b \\ y(X_0) \in [a, b] & \begin{array}{l} \text{if } \tilde{X} \leq b < X_0 \text{ or} \\ \text{if } b \leq \tilde{X} < X_0 \end{array} \end{cases},$$

for $\tilde{X}$ sufficiently large. Such an $\tilde{X}$ exists given the assumption that $E[\omega] > R$. These beliefs are consistent with hedge fund manager behavior, and it is optimal for the period-1 investor to provide a unit capital investment to the hedge fund manager if and only if $X_0 \in \{\hat{X}^c\} \cup \{\tilde{X}\} \cup (\tilde{X}, b)$, where $(\tilde{X}, b) = \emptyset$ if $b < \tilde{X}$. In turn, it is a best response for hedge fund managers to use the risky far investment strategy $\{0, \hat{X}^c\}$ if $\omega \in (a, \hat{X}^c]$, to use the risky fair investment strategy $\{\hat{X}^c, \tilde{X}\}$ if $\omega \in (\hat{X}^c, \tilde{X})$, and to use the sure-thing investment strategy if $\omega > [\tilde{X}, b]$ when $b > \tilde{X}$.

The form of $V(\omega)$ is immediate from the construction.

A cutoff equilibrium does not exist if $\hat{X}^c > \bar{X}$ because the date-1 investor would invest in a manager that has a date-0 realization of $X_0 = 0$ since $E_{\mu(0)}[\omega] > R$.

Proof to Proposition 3

Any equilibrium in which hedge fund managers receive the unit capital investment at date 0 and adopt fair investment strategies are optimal from the perspective of the period 0 investor since $E[\omega] > R$.

We first focus on equilibria in which the probability the period 1 investor funds a period 1 project is either zero or one, i.e., $k(X_0) \in \{0, 1\}$, for all X_0. We then solve for the set of ω that the period 1 investor should always fund in equilibrium given that lesser hedge fund manager types choose their investment strategies optimally. We then extend the analysis to deal with $k(X_0) \in (0, 1)$.

Let $\hat{\omega}$ be the worst type that is always funded, i.e., $\hat{\omega} = min\{\omega | k(X_0 = \omega) = 1$, where the assumption that $a < R$ implies that it is not optimal to fund all hedge fund manager types. Then the optimal investment strategies of hedge fund managers $\omega < \hat{\omega}$ place probability $\omega/\hat{\omega}$ on $X_0 = \hat{\omega}$, and residual probability on 0. Since $k(X_0)$ for $X_0 > \hat{\omega}$ does not affect the investment strategies of hedge fund managers $\omega < \hat{\omega}$, it follows that it is (weakly) optimal for the investor to set $k(X_0) = 1$ for all $X_0 > \hat{\omega}$. Thus, $\hat{\omega}$ solves

$$\max_{\hat{\omega}} \int_a^{\hat{\omega}} \left\{ (1-\beta)\left(\omega + \frac{\omega^2}{\hat{\omega}}\right) + \left(1 - \frac{\omega}{\hat{\omega}}\right)\bar{R} \right\} \Pi(d\omega) + \int_{\hat{X}}^b (1-\beta)2\omega\Pi(d\omega).$$

The Leibnitz terms drop out of the associated first-order condition for the above maximization problem yielding,

$$\int_a^{\hat{\omega}} \left[-(1-\beta)\frac{\omega^2}{\hat{\omega}^2} + \frac{\omega}{\hat{\omega}^2}\bar{R} \right] \Pi(d\omega) = 0.$$

Substituting for $\bar{R} = (1-\beta)R$, multiplying by $\hat{\omega}^2/(1-\beta)$, and rearranging yields

$$\frac{\int_a^{\hat{\omega}} \omega^2 \Pi(d\omega)}{\int_a^{\hat{\omega}_1} \omega \Pi(d\omega)} = R,$$

which implies that $\hat{\omega} = X^c$.

We now extend the analysis to deal with the possibility of equilibria with $k(X) \in (0,1)$. In this analysis, rather than exhaustively delineate all possible equilibria, we work with the concave hull of the equilibrium funding probabilities, $\bar{k}(\cdot)$. Thus, $\bar{k}(X_0)$ is the probability with which someone with type $\omega = X_0$ expects to be funded in equilibrium. Optimization by hedge fund managers together with the fact that an investor who mixes between funding and not must be indifferent implies that the concave hull of the equilibrium funding probabilities must be a concave piecewise linear function of X_0 with at most two kinks. Importantly, both hedge fund managers and period investors are indifferent between the equilibrium and a setting in which a hedge fund manager with type $\omega = X_0$ does not adopt a risky investment strategy and is funded with probability $\bar{k}(X_0)$. Obviously, the latter is not typically an equilibrium (investors would update, and want to either fund for sure or never fund), but both deliver the same funding probabilities for any given hedge fund type (e.g., in the simple cutoff equilibrium, a type $\omega = X^c/2$ delivers X^c with probability one-half, and zero with the remaining probability one-half, and hence is funded with probability one-half, and $\bar{k}(X^c/2) = 1/2$), and hence are payoff equivalent for all parties.

To see that the concave hull is linear with at most two kinks, note that $k(X_0) \in (0,1)$ implies $E_{\mu(X_0)}[\omega] = R$. Denote the smaller kink (when it exists) by X_A and the larger (when it exists) by X_B. Then $E_{\mu(X_0)}[\omega] = R$ for any $X_0 \in [X_A, X_B)$ in the support of an equilibrium investment strategy, and $k(X_0) = 1$ for $X_0 \geq X_B$ is in the support of an equilibrium investment strategy (presuming there is a larger kink)), and, if there is a kink, then all $\omega < X_A$ adopt the investment strategy $\{0, X_A\}$. Note that types $\omega > X_B$ place probability 0 on $X_0 < X_B$.

There are 3 cases to analyze:

Case 1: No kinks on $[0, b]$. Suppose there is an equilibrium in which $\bar{k}(X)$ is linear for all $X \in [0, b]$, where $\bar{k}(0) = 0$ and $\bar{k}(b) < 1$. The equilibrium is payoff equivalent to one where all hedge fund managers $\omega \in [a, b]$ employ fair investment strategy $\{0, b\}$ and the investor provides period 1 funding when $X = b$ with probability $\bar{k}(b) < 1$. For these strategies, $E_{\mu(X_0)}[\omega] > R$, which means that the expected payoff to the investor will increase if he provides period 1 funding when $X = b$ with probability one. Hence, an equilibrium in which all hedge fund manager types employ fair investment strategy $\{0, b\}$ and the investor provides period 1 funding with probability one if and only if $X \geq b$ generates a higher expected payoff to the investor than any equilibrium in which $\bar{k}(X)$ is linear for all $X \in [0, b]$ and $k(b) < 1$. Therefore, an equilibrium in which $\bar{k}(X)$ that is linear for all $X \in [0, b]$, where $\bar{k}(0) = 0$ and $\bar{k}(b) < 1$ cannot maximize the investor's payoff.

Case 2: One kink on $[0, b]$*, no kinks on* $X > b$. Suppose there is an equilibrium in which $\bar{k}(X)$ is linear for all $X \in [0, \hat{X}]$, where $\bar{k}(0) = 0$, $\hat{X} \leq b$ and $\bar{k}(X) = 1$ for all $X \geq \hat{X}$. The equilibrium payoffs are equivalent to strategies that have hedge fund managers $\omega \in [a, \hat{X})$ employing fair investment strategies $\{0, \hat{X}\}$, hedge fund managers $\omega \in [\hat{X}, b]$ play sure thing strategies and investors providing period 1 funding with probability 1 for $X \geq \hat{X}$, and with zero probability otherwise. From above, the investors' payoffs are maximized by $\hat{X} = X^c$. Therefore, the simple cutoff equilibrium provides the highest payoff among all equilibria in which $\bar{k}(X)$ is linear for all $X \in [0, \hat{X}]$, where $\bar{k}(0) = 0$, $\hat{X} \leq b$ and $\bar{k}(X) = 1$ for all $X \geq \hat{X}$.

Case 3: Two kinks. Suppose there is an equilibrium in which $\bar{k}(X)$ is piecewise linear over $X > 0$ with kinks at X_A and X_B, where $a < X_A < b$, $X_A \leq X_B$, $\bar{k}(0) = 0$, $0 < \bar{k}(X_A) \leq 1$, $X_B \gtreqless b$, and $\bar{k}(X) = 1$ for all $X \geq X_B$. The equilibrium payoffs are equivalent one in which hedge fund managers $\omega \in [a, X_A)$ employ fair investment strategies $\{0, X_A\}$, hedge fund managers $\omega \in [X_A, X_B]$ employ fair investment strategies $\{X_A, X_B\}$, hedge fund managers $\omega \in [X_b, b]$ play sure thing strategies and investors providing period 1 funding with probability $\bar{k}(X_A)$ for $X = X_A$, with probability one for $X \geq X_B$, and with zero probability otherwise. For these (equilibrium) strategies, consider the choices of X_A, X_B and $k(X_A)$ that maximize the expected payoff of the investor. In such an equilibrium, the critical values, X_A and X_B, that maximize the investor's expected payoff solve

$$\begin{aligned}
&\max_{X_A, X_B} \int_a^{X_A} \left\{(1-\beta)\left[k(X_A)\frac{\omega}{X_A}(X_A+\omega)+(1-k(X_1))\frac{\omega}{X_A}(X_A+R)\right]\right. \\
&\qquad\qquad \left. +\left(1-\frac{\omega}{X_A}\right)\bar{R}\right\} d\Pi(\omega) \\
&+(1-\beta)\int_{X_A}^{\max\{X_B,b\}}\left[\frac{\omega-X_A}{X_B-X_A}(X_B+\omega)+k(X_1)\frac{X_B-\omega}{X_B-X_A}(X_A+\omega)+\right. \\
&\qquad\qquad \left. +(1-k(X_1))(X_A+R)\right] d\Pi(\omega) \\
&+(1-\beta)\int_{\min\{X_B,b\}}^{b} 2\omega d\Pi(\omega),
\end{aligned}$$

where $k(X_A)$ is the probability that the period-1 investor provides a unit capital investment to a hedge fund manager that has a period-0 realization equal to X_A. Note that by construction, the period-1 investor will provide funds to a hedge fund manager whose period-0 realization is X_B with probability one. Since $0 < k(X_A) < 1$, investor indifference implies that

$$\int_a^{X_A}\frac{\omega^2}{X_A}d\Pi(\omega)+\int_{X_A}^{\max\{X_B,b\}}\omega\frac{X_B-\omega}{X_B-X_A}d\Pi(\omega)=R\left[\int_a^{X_A}\frac{\omega}{X_A}d\Pi(\omega)+\int_{X_A}^{\max\{X_B,b\}}\frac{X_B-\omega}{X_B-X_A}d\Pi(\omega)\right].$$

Substituting this into the above maximization problem and simplifying, we get[14]

$$\begin{aligned}
&\max_{X_A, X_B}\int_a^{X_A}\left\{(1-\beta)\frac{\omega}{X_A}(X_A+\omega)+\left(1-\frac{\omega}{X_A}\right)\bar{R}\right\}d\Pi(\omega) \\
&+(1-\beta)\int_{X_A}^{\max\{X_B,b\}}\left\{\frac{\omega-X_A}{X_B-X_A}(X_B+\omega)+\frac{X_B-\omega}{X_B-X_A}(X_A+\omega)\right\}d\Pi(\omega) \\
&+(1-\beta)\int_{\min\{X_B,b\}}^{b}2\omega d\Pi(\omega) \\
=\ &\max_{X_A}\int_a^{X_A}\left\{(1-\beta)\left(\omega+\frac{\omega^2}{X_A}\right)+\left(1-\frac{\omega}{X_A}\right)\bar{R}\right\}\Pi(d\omega)+\int_{X_A}^{b}(1-\beta)2\omega\Pi(d\omega).
\end{aligned}$$

Note that X_B does not enter the objective function. The associated first-order condition for the above maximization problem is,

$$\int_a^{X_A}\left[-(1-\beta)\frac{\omega^2}{X_A}+\frac{\omega}{X_A}\bar{R}\right]\Pi(d\omega)=0,$$

[14] It is important to note that the final expression that follows is also valid if $k(X_A)=1$. This is because, as we will see, the solution to the maximization problem has $X_A = X^c$ and for the simple cutoff equilibrium we know that $E_{\mu(X^c)}[\omega]=R$.

and can be simplified to read

$$\frac{\int_a^{X_A} \omega^2 \Pi(d\omega)}{\int_a^{X_A} \omega \Pi(d\omega)} = R,$$

which implies that $X_A = X^c$, where X^c is defined in equation (1). The solution to the above problem delivers a concave hull $\bar{k}(X)$ that is identical to that of the simple cutoff equilibrium, i.e., $X_A = X^c$, $X_A \leq X_B$, and $\bar{k}(X_A) = \bar{k}(X_B) = 1$.

Hence, the simple cutoff equilibrium provides the investor with the highest possible expected payoff.

The proof to the Grossman-Perry part of the proposition is in 8 steps.

Step 1: We first prove that there cannot exist an equilibrium where the period-0 investor provides investment capital to the hedge fund manager and the period-1 investor never provides investment to the hedge fund manager. Suppose that such an equilibrium exists. Then, because of assumption LC, all hedge fund managers ω must be employing sure-thing investment strategies in period 0. But then for $X_0 > R$, must reflect $\omega > R$. But then it is strictly optimal for the period-1 investor to provide the hedge fund manager a unit of investment capital at date 1 (as they employ sure thing strategies at date 2), a contradiction.

Step 2: We can rule out an equilibrium where continued capital investment requires that the period 0 realization of the hedge fund be *exactly* $\hat{X}$, i.e., if $X_0 \neq \hat{X}$, then the probability of continued capital investment is zero. In such an equilibrium, a hedge fund managers $\omega \in [a, \hat{X}]$ use the risky fair investment strategy $\{0, \hat{X}\}$ and hedge fund managers $\omega \in (\hat{X}, b]$ use unfair strategies that effectively destroy $\omega - \hat{X}$ units period-0 output. Such an equilibrium does not survive the Grossman-Perry refinement. To see this, suppose that a period-0 realization equal to $\tilde{X} > \hat{X}$ is observed. If outcome $\tilde{X}$ receives continued investment with probability one, then hedge fund managers $\omega \in (\hat{X}, \tilde{X})$ have an incentive to use the risky fair investment strategy $\{\hat{X}, \tilde{X}\}$—as this increases their period-0 expected payoff from $\hat{X}$ to ω—and hedge fund managers $\omega \in \left[\tilde{X}, b\right]$ have an incentive to provide the payout $\tilde{X}$—as this would increase their period-0 payoff by $\tilde{X} - \hat{X}$. Further, $\tilde{X}$ would not be in the support of the investment strategies of hedge fund managers $\omega \in [a, \hat{X})$ as this would simply lower the probability of receiving capital investment in period 1. Therefore, $E_{\mu(\tilde{X})}[\omega] > R$, and the period-1 investor will provide a unit of investment capital to a hedge fund manager that has a period-0 realization of $\tilde{X}$, a contradiction. (Note that this step rules out the equilibrium described in the text where period-0 investors do not provide capital investment to the hedge fund manager. Recall that perverse period-1 investor beliefs were required to support such an equilibrium. If, however, the period-0 investor provided investment capital to the hedge fund manager, then the period-1 investor would only provide investment capital to the hedge fund manager if $X_0 = \hat{X} = R$. The above argument demonstrates that the beliefs required to support such an outcome do not survive the Grossman-Perry refinement.)

Step 3: We now show that there can be at most one value of X, say X_A with $k_1(X_A) \in (0,1)$ that is observed along the equilibrium path, where $k_1(X_A) \in (0,1)$ should be interpreted as a mixed capital investment strategy by the investor. If $k_1(X_A) \in (0,1)$, then $E_{\mu(X_A)}[\omega] = R$. Suppose there exists an equilibrium characterized by X_A, X_B and X_C such that $0 < k_1(X_A), k_1(X_B) < 1$ and $k_1(X_C) = 1$, where $X_A < X_B < X_C$. Since assumption LC implies that any given hedge fund manager will employ a risky investment strategy with most two points of support, in the equilibrium, hedge fund managers $\omega \in (a, X_A)$ gamble on $\{0, X_A\}$, hedge fund managers $\omega \in (X_A, X_B)$ gamble on $\{0, X_B\}$ and hedge fund managers $\omega \in (X_B, X_C)$ employ fair investment strategy $\{0, X_C\}$. Since hedge fund managers with X_B in the support of their investment strategy are uniformly better than those with X_A, it must be that $E_{\mu(X_A)}[\omega] < E_{\mu(X_B)}[\omega]$, a contradiction. Hence, in any equilibrium there can be at most one value of X, say X_A, such that $k_1(X_A) \in (0,1)$.

Step 4: Step 3 does not rule out the possibility that there exists X_A and X_B such that $k_1(X_A) \in (0,1)$ and $k_1(X_B) = 1$, as this is consistent with $\mu(X_A) = R$ and $\mu(X_B) > R$ for $X_B > X_A$. We now show that the Grossman-Perry refinement precludes this possibility. Suppose there is an equilibrium characterized by hedge fund managers $\omega \in (a, X_A)$ using the risky fair investment strategy $\{0, X_A\}$ and $\omega \in (X_A, X_B)$ using the risky fair investment strategy $\{X_A, X_B\}$, where $E_{\mu(X_A)}[\omega] = R$ and $k_1(X_A) \in (0,1)$. Clearly, $E_{\mu(X_B)}[\omega] > R$ and $k_1(X_B) = 1$. Suppose first that hedge fund managers $\omega \in (a, X_A)$ strictly prefer the risky fair investment strategy $\{0, X_A\}$ to $\{0, X_B\}$, i.e., $k_1(X_A)/X_A > 1/X_B$. Now suppose that $\hat{X} < X_B$ is observed, where $X_B - \hat{X}$ is arbitrarily small. *If* $k_1(\hat{X}) = 1$, then hedge fund managers $\omega \in (X_A, \hat{X})$ prefer the risky fair strategy $\{X_A, \hat{X}\}$, $\omega \in (\hat{X}, X_B)$ prefer the risky fair strategy $\{\hat{X}, X_B\}$, and $\omega = \hat{X}$ prefers the sure-thing investment strategy to the equilibrium strategy because, in each case, the expect payoff to the hedge fund manager exceeds that from equilibrium risky strategy $\{X_A, X_B\}$. For $X_B - \hat{X}$ sufficiently small, no hedge fund manager $\omega \in (0, X_A)$ has an incentive to use the risky fair investment strategy $\{0, \hat{X}\}$ even if $\tilde{k}_1(\hat{X}) = 1$. Hence, if the period 1 investor observes $\hat{X}$, then, using the Grossman-Perry logic, he will conclude that $E_{\mu(\hat{X})}[\omega] > R$, and provide the hedge fund manager with period-0 payout $\hat{X}$ a unit of capital in period 1, a contradiction. Hence, there cannot exist an equilibrium such that there exists X_A and X_B with $\tilde{k}_1(X_A) \in (0,1)$, $k_1(X_B) = 1$, and $k_1(X_A)/X_A > 1/X_B$. Now consider $k_1(X_A)/X_A = 1/X_B$ (if $k_1(X_A)/X_A < 1/X_B$ then X_A is not in the support of any hedge fund manager's investment strategy). First consider $X_B > X^c$, and suppose that $\hat{X}$ is observed, where $X^c < \hat{X} < X_B$. If $k_1(\hat{X}) = 1$, then hedge fund managers $\omega \in (a, X_A)$ prefer the risky fair strategy $\{0, \hat{X}\}$, hedge fund managers $\omega \in (X_A, \hat{X})$ prefer the risky fair strategy $\{0, \hat{X}\}$, hedge fund managers $\omega \in (\hat{X}, X_B)$ prefer the risky fair strategy $\{\hat{X}, X_B\}$, and $\omega = \hat{X}$ prefers the sure-thing investment strategy to the equilibrium strategy because, in all cases, the associated expected payoff exceeds that associated with the proposed equilibrium. Since $\hat{X} > X^c$, $E_{\mu(\hat{X})}[\omega] > R$, and the period 1 investor will provide capital to a hedge fund manager who generates a payout of $\hat{X}$ in period 0, a contradiction.

Finally, posit an equilibrium in which there exists X_A and X_B such that $k_1(X_A) \in (0,1)$, $k_1(X_B) = 1$, $k_1(X_A)/X_A = k_1(X_B)/X_B$ and $X_B \leq X^c$. Then,

$$E_{\mu(X_A)}[\omega] = \frac{\int_a^{X_A} \omega(\omega/X_A)\Pi(d\omega) + \int_{X_A}^{X_B} \omega(X_B - \omega)/(X_B - X_A)\Pi(d\omega)}{\int_a^{X_A}(\omega/X_A)\Pi(d\omega) + \int_{X_A}^{X_B}(X_B - \omega)/(X_B - X_A)\Pi(d\omega)} \equiv \frac{N}{D}.$$

We now sign of the derivative of $E_{\mu(X_A)}[\omega]$ with respect to X_A. Note that

$$\begin{aligned}
sign\frac{\partial E_{\mu(X_A)}[\omega]}{\partial X_A} &= sign[-\int_a^{X_A} \omega(\omega/X_A^2)\Pi(d\omega) + \int_{X_A}^{X_B} \omega(X_B - \omega)/(X_B - X_A)^2\Pi(d\omega)]D \\
&\quad -[\int_a^{X_A} (\omega/X_A^2)\Pi(d\omega) + \int_{X_A}^{X_B} (X_B - \omega)/(X_B - X_A)^2\Pi(d\omega)]N \\
&= sign(\frac{1}{X_A} + \frac{1}{X_B - X_A})[\int_a^{X_A} \omega\Pi(d\omega) \int_{X_A}^{X_B} \omega(X_B - \omega)\Pi(d\omega) \\
&\quad - \int_a^{X_A} \omega^2\Pi(d\omega) \int_{X_A}^{X_B} (X_B - \omega)\Pi(d\omega)] \\
&= sign[\frac{\int_a^{X_A} \omega\Pi(d\omega) \int_{X_A}^{X_B} \omega(X_B - \omega)\Pi(d\omega)}{\int_a^{X_A} \omega\Pi(d\omega) \int_{X_A}^{X_B}(X_B - \omega)\Pi(d\omega)} - \frac{\int_a^{X_A} \omega^2\Pi(d\omega) \int_{X_A}^{X_B}(X_B - \omega)\Pi(d\omega)]}{\int_a^{X_A} \omega\Pi(d\omega) \int_{X_A}^{X_B}(X_B - \omega)\Pi(d\omega)}] \\
&= sign[\frac{\int_{X_A}^{X_B} \omega(X_B - \omega)/(X_B - X_A)\Pi(d\omega)}{\int_{X_A}^{X_B}(X_B - \omega)/(X_B - X_A)\Pi(d\omega)} - \frac{\int_a^{X_A} \omega(\omega/X_A)\Pi(d\omega)}{\int_a^{X_A}(\omega/X_A)\Pi(d\omega)}] > 0,
\end{aligned}$$

since this first term represents the expected fund manager type who gets payout X_A from the population of hedge fund managers $\omega \in (X_A, X_B)$ with fair investment strategy $\{X_A, X_B\}$ and the second term represents the expected fund manager type who gets payout X_A from the population of hedge fund managers $\omega \in [a, X_A)$ who employ fair investment strategy $\{0, X_A\}$. Since $X_A < X_B \leq X^c$, $\partial E_{\mu(X_A)}[\omega]/\partial X_A > 0$ means increasing X_A all the way to X_B raises the expectation, but at $X_A = X_B$, $E_{\mu(X_A)}[\omega] \leq R$, a contradiction (i.e., there cannot be an equilibrium where $k_1(X_A) > 0$).

Step 5: The first four steps allows us to focus on cutoff equilibria, i.e., equilibria characterized by a hedge fund manager below some critical value using risky investment strategies and, as a result, receiving continued capital investment with probability less than one, and by a hedge fund manager above some critical value that receives continued capital investment with probability one. Consider now equilibria that are characterized by an excessively high standard $\hat{X}^c > X^c$, where X^c is given by equation (1). In such an equilibrium, $E_{\mu\left(\hat{X}^c\right)}[\omega] > R$. In the equilibrium, hedge fund managers $\omega < \hat{X}^c$ use the risky fair investment strategy $\{0, \hat{X}^c\}$ and hedge fund managers $\omega \geq \hat{X}^c$ use the sure-thing investment strategy. Such an equilibrium is supported by period-1 investor beliefs $E_{\mu(X_0)}[\omega] = y(X_0) \in [a, R)$ for all $X_0 \in (0, \hat{X}^c)$. Given these beliefs, hedge fund managers will use any outcome in $(0, \hat{X}^c)$ as a support in the risky investment strategy. Such beliefs, however, do not survive the Grossman-Perry equilibrium refinement. Suppose that $\tilde{X}_0 \in [X^c, \hat{X}^c)$ is observed and the period-1 investor will invest a unit of capital with the hedge fund manager if $X_0 = \tilde{X}_0$. Then, hedge fund managers $\omega \in \left(0, \tilde{X}_0\right)$ will prefer to use the risky fair investment strategy $\left\{0, \tilde{X}_0\right\}$ to $\{0, \hat{X}^c\}$; hedge fund manager $\omega = \tilde{X}_0$ will prefer to use the sure-thing investment strategy to the risky fair investment strategy $\{0, \hat{X}^c\}$; and hedge fund managers $\omega \in (\tilde{X}_0, \hat{X}^c)$ will prefer to use the risky fair investment strategy $\{\tilde{X}_0, \hat{X}^c\}$ to $\{0, \hat{X}^c\}$.Therefore, any hedge fund manager $\omega \in (0, \hat{X}^c)$ has an incentive to defect from proposed play. Using the Grossman-Perry logic, the period-1 investor beliefs associated with outcome $\tilde{X}_0 \in [X^c, \hat{X}^c)$ is

$$E_{\mu\left(\tilde{X}_0\right)}[\omega] = \frac{\int_a^{\tilde{X}_0} \omega(\omega/\tilde{X}_0)\Pi\,(d\omega) + \int_{\tilde{X}_0}^{\hat{X}} \omega(\omega - \tilde{X}_0)/(\hat{X} - \tilde{X}_0)\Pi\,(d\omega)}{\int_a^{\tilde{X}_0}(\omega/\tilde{X}_0)\Pi\,(d\omega) + \int_{\tilde{X}_0}^{\hat{X}}(\omega - \tilde{X}_0)/(\hat{X} - \tilde{X}_0)\Pi\,(d\omega)} > R.$$

Hence, the period-1 investor will provide a unit capital of investment to a hedge fund manager that produces a payout of $\tilde{X}_0 \in [X^c, \hat{X}^c)$ in period 0. Therefore, there does not exist an equilibrium characterized by an excessively high standard.

Step 6: To rule out equilibria that have a non-monotone standard, i.e., $\hat{X}^c < X^c$, note that such a standard can be only be supported by having hedge fund managers $\omega \in (\hat{X}^c, \tilde{X})$ use the risky fair investment strategy $\{\hat{X}^c, \tilde{X}\}$, which raises $E_{\mu\left(\hat{X}^c\right)}[\omega]$ to R. Suppose that the period-1 investor observes a period-0 payout equal to $\hat{X} \in (\hat{X}^c, \tilde{X})$ and invests a unit of capital with the hedge fund manager with probability one. Then, hedge fund managers $\omega \in \left(\hat{X}^c, \hat{X}\right)$ have an incentive use the risky fair investment strategy $\left\{\hat{X}^c, \hat{X}\right\}$; hedge fund manager $\omega = \hat{X}$ has an incentive to use the sure-thing investment strategy; and hedge fund managers $\omega \in (\hat{X}, \tilde{X})$ have an incentive to use the risky fair investment strategy $\{\hat{X}, \tilde{X}\}$ since, in all cases, the "defecting" risky strategies have smaller payout supports compared to the equilibrium strategy, and provide the same expected payoff. Clearly, hedge fund managers $\omega \in (0, \hat{X}^c)$ have no incentive to use $\hat{X}$ as a support for their risky strategies. The Grossman-Perry refinement implies that the investor will, in fact, invest a unit of capital with a hedge fund manager that has a date-0 realization $X_0 = \hat{X}$ since $E_{\mu(\hat{X})}[\omega] > R$. Therefore, there cannot be an equilibrium that has a non-monotone standard.

Step 7: We can rule out equilibria where there is a single cutoff equal to X^c, hedge fund managers $\omega \in (0, X^c)$ use the risky fair investment strategy $\{0, X^c\}$, hedge fund managers $\omega \geq X^c$ use the sure-thing investment strategy, the investor invests a unit of capital with probability one if $X_0 > X^c$, and invests a unit of capital with probability less than one if $X_0 = X^c$, i.e., $\tilde{k}(X^c) < 1$. To see this, note that fund manager $\omega \in (0, X^c)$ would prefer to use the risky fair investment strategy $\{0, X^c + \varepsilon\}$, where $\varepsilon > 0$ is arbitrarily small, instead of $\{0, X^c\}$ because the probability of achieving $X^c + \varepsilon$ is approximately equal to that of achieving X^c but the former receives a unit of capital investment in period 1 with probably 1.

Step 8: Finally, it is straight forward to rule out equilibria that are characterized by "unnecessary gambling," since smaller gambles are preferred to larger ones.

Steps 1-8 imply that the equilibrium identified in Proposition 1 is only equilibrium to survive the Grossman-Perry refinement and assumption LC.

Proof to Proposition 4

We first show that the set of cutoff values is unique in the sense that there do not exist equilibria where the cutoff for continued capital investment in the hedge fund depends non-trivially on the entire track record of fund performance. That is, one might contemplate distinct hedge fund payout paths, where if a hedge fund has a payout $\tilde{X}_n > \hat{X}_n$ at period n, then it might require a payout of only $\tilde{X}_\tau < \hat{X}_\tau$ at period $\tau > n$ to continue to receive a unit of capital investment from the investor, while another hedge fund that had a payout of $\hat{X}_n$ at period n might require a payout $\hat{X}_\tau > \tilde{X}_\tau$ at period τ to continue to receive a unit of capital investment from the investor. Let $X_{n-1}^c\left(X^{n-1}\right)$ be the minimum payout in period $n-1$ such that an investor will make a unit capital investment in the hedge fund in period n, given that the hedge fund's payout history is X^{n-1}. For concreteness, assume that $N=3$. The expected payoff to hedge fund manager $\omega \leq \min\left\{X_1^c\left(X_0\right), X_0^c\right\}$ in period 0 is proportional to

$$\frac{\omega}{X_0^c}\left\{X_0^c+\frac{\omega}{X_1^c\left(X_0^c\right)}\left[X_1^c\left(X_0^c\right)+\omega\right]\right\}.$$

Suppose there are nontrivial cutoffs at periods 0 and 1: $\tilde{X}_0^c$ and $\hat{X}_0^c$ in period 0, where $\tilde{X}_0^c<\hat{X}_0^c$, and $X_1^c(\hat{X}_0^c)$ and $X_1^c(\tilde{X}_0^c)$ in period 1, where $X_1^c(\hat{X}_0^c)<X_1^c(\tilde{X}_0^c)$. If some hedge fund managers prefer the lower period 0 cutoff and others prefer the higher period cutoff, then there will exist a fund manager, say $\tilde{\omega}$, that is indifferent between using the risky fair investment strategy $\{0, \tilde{X}_0^c\}$ and $\{0, \hat{X}_0^c\}$. This indifference implies that

$$\frac{\tilde{\omega}}{\tilde{X}_0^c}\left\{\bar{X}_0^c+\frac{\tilde{\omega}}{X_1^c\left(\tilde{X}_0^c\right)}\left[X_1^c\left(\tilde{X}_0^c\right)+\tilde{\omega}\right]\right\}-\frac{\tilde{\omega}}{\hat{X}_0^c}\left\{\hat{X}_0^c+\frac{\tilde{\omega}}{X_1^c\left(\hat{X}_0^c\right)}\left[X_1^c\left(\hat{X}_0^c\right)+\tilde{\omega}\right]\right\}=0. \quad (11)$$

If we view the left-hand side of (11) as a function of $\tilde{\omega}$ and differentiate with respect to $\tilde{\omega}$, we get

$$1+\frac{1}{\bar{X}_0^c}\left[2\tilde{\omega}+\frac{3\tilde{\omega}^2}{X_1^c\left(\bar{X}_0^c\right)}\right]-\left\{1+\frac{1}{\hat{X}_0^c}\left[2\tilde{\omega}+\frac{3\tilde{\omega}}{X_1^c\left(\hat{X}_0^c\right)}\right]\right\}. \quad (12)$$

To sign this derivative, note that (11) can be rewritten as,

$$1+\frac{1}{\bar{X}_0^c}\left[\tilde{\omega}+\frac{\tilde{\omega}^2}{X_1^c\left(\bar{X}_0^c\right)}\right]-\left\{1+\frac{1}{\hat{X}_0^c}\left[\tilde{\omega}+\frac{\tilde{\omega}^2}{X_1^c\left(\hat{X}_0^c\right)}\right]\right\}=0. \quad (13)$$

Subtracting (13) from (12) yields

$$\frac{1}{\bar{X}_0^c}\left[\tilde{\omega}+\frac{2\tilde{\omega}^2}{X_1^c\left(\bar{X}_0^c\right)}\right]-\frac{1}{\hat{X}_0^c}\left[\tilde{\omega}+\frac{2\tilde{\omega}^2}{X_1^c\left(\hat{X}_0^c\right)}\right]. \quad (14)$$

Comparing (13) to (14), and noting that $\tilde{X}_0^c<\hat{X}_0^c$ and $X_1^c(\hat{X}_0^c)<X_1^c(\tilde{X}_0^c)$, reveals that (14) is negative, which, in turn, implies that (12) is negative. Hence, all hedge fund managers $\omega>\tilde{\omega}$ prefer the risky fair gamble strategy $\{0, \hat{X}_0^c\}$ and all fund managers $\omega<\tilde{\omega}$ prefer the risky fair investment strategy $\{0, \tilde{X}_0^c\}$. This, however, implies that $E_{\mu(\hat{X}_0^c)}[\omega]>E_{\mu(\tilde{X}_0^c)}[\omega] \geq R$. But equilibria where $E_{\mu(\hat{X}_0^c)}[\omega]>R$ are precluded by the Grossman-Perry equilibrium refinement and assumption LC. This reasoning extends to $N>3$.

We now characterize the unique equilibrium that survives the Grossman-Perry refinement. The equilibrium is characterized by a declining set of cutoffs for continued capital investment. Hedge fund managers $\omega \geq X_n^c$ use the sure-thing investment strategy in period n and in future periods. So over time, some "intermediate-type fund manager," for example $X_{n+1}^c < \omega < X_n^c$, use the risky fair investment strategy $\{0, X_n^c\}$, for the first n periods and then switch to the sure-thing investment strategy thereafter. Let $\Pi_n(\cdot)$ be the endogenous cumulative distribution function of fund manager types remaining at the beginning of period n, given the equilibrium outcome that fund managers $\omega \in (0, X_n^c)$ use the risky fair investment strategy $\{0, X_n^c\}$. Then X_n^c solves

$$\frac{\int_a^{X_n^c} \omega(\omega/X_n^c)\Pi_n\,(d\omega)}{\int_a^{X_n^c}(\omega/X_n^c)\Pi_n\,(d\omega)} = R.$$

Note that

$$\Pi_1\,(\omega) = \int_a^{\omega} \frac{\omega}{X_0^c}\Pi\,(d\omega)\,,$$

and an induction argument establishes that,

$$\Pi_n\,(\omega) = \int_a^{\omega} \frac{\omega^n}{X_0^c \cdot X_1^c \cdots X_{n-1}^c}\Pi(d\omega).$$

Hence, the cutoff X_n^c is given implicitly by the solution to

$$\frac{\int_a^{X_n^c} \omega(\omega/X_n^c)\Pi_n\,(d\omega)}{\int_a^{X_n^c}(\omega/X_n^c)\Pi_n\,(d\omega)} = \frac{\int_a^{X_n^c} \omega^{n+2}\Pi\,(d\omega)}{\int_a^{X_n^c} \omega^{n+1}\Pi\,(d\omega)} = R. \tag{15}$$

Up to this point we have only asserted that X_n^c is strictly decreasing in n. We now prove that this is, in fact, the case. Define $E_{\mu_n(X_0^c)}[\omega]$ as the expected value of ω conditional on observing X_0^c each period $n, n-1, \ldots, 0$ when all hedge fund managers $\omega < X_0^c$ use the risky fair investment strategy $\{0, X_0^c\}$ in every period $n \geq 0$ and hedge fund managers $\omega \geq X_0^c$ always use the sure-thing investment strategy. Hence,

$$E_{\mu_n(X_0^c)}[\omega] = \frac{\int_a^{X_0^c} \omega^{n+2}\Pi(d\omega)}{\int_a^{X_0^c} \omega^{n+1}\Pi(d\omega)}.$$

Consider now what happens to the value of $E_{\mu_n(X_0^c)}[\omega]$ as n increases. Note that X_n^c is strictly decreasing in n if and only if $E_{\mu_n(X_0^c)}[\omega]$ is strictly increasing in n. If $E_{\mu_n(X_0^c)}[\omega] > E_{\mu_{n-1}(X_0^c)}[\omega]$ for $n \geq 1$, then

$$\frac{\int_a^{X_0^c} \omega^{n+2}\Pi\,(d\omega)}{\int_a^{X_0^c} \omega^{n+1}\Pi\,(d\omega)} > \frac{\int_a^{X_0^c} \omega^{n+1}\Pi\,(d\omega)}{\int_a^{X_0^c} \omega^{n}\Pi\,(d\omega)},$$

or

$$\int_a^{X^c} \omega^{n+2}\Pi(d\omega) \int_a^{X^c} \omega^{n}\Pi(d\omega) > \left(\int_a^{X^c} \omega^{n+1}\Pi(d\omega)\right)^2 .$$

If we define $F(d\omega) = \left[\omega^{n+2}\Pi(d\omega)\right]^{.5}$ and $G(d\omega) = \left[\omega^{n}\Pi(d\omega)\right]^{.5}$, we can rewrite the above inequality as

$$\int_a^{X^c} F(d\omega)^2 \int_a^{X^c} G(d\omega)^2 > \left(\int_a^{X^c} F(d\omega)\, G(d\omega)\right)^2 ,$$

which is simply a statement of Schwartz's inequality. Therefore, $E_{\mu_n(X_0^c)}[\omega] > E_{\mu_{n-1}(X_0^c)}[\omega]$, which implies that X_n^c is strictly decreasing in n.

Proof to Proposition 5

Let $V_3\left(\hat{X}_0^c, \hat{X}_1^c\right)$ represent the long-horizon investor's expected payoff in period 2 given the hedge fund's payout was $\hat{X}_0^c$ in period 0 and $\hat{X}_1^c$ in period 1. The critical values, $\hat{X}_0^c$ and $\hat{X}_1^c$, solve $V_3\left(\hat{X}_0^c, \hat{X}_1^c\right) = \bar{R}$ or $V_3\left(\hat{X}_0^c, \hat{X}_1^c\right)/(1-\beta) = R$. Therefore,

$$\begin{aligned}
\frac{V_3\left(\hat{X}_0^c, \hat{X}_1^c\right)}{1-\beta} &= \frac{\int_a^{\hat{X}_1^c} \omega \left\{\frac{\omega}{\hat{X}_1^c}\left[\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)\right]\right\}}{\int_a^{\hat{X}_1^c} \frac{\omega}{\hat{X}_1^c}\left[\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)\right]} \\
&= \frac{\int_a^{\hat{X}_1^c} \omega^3\Pi(d\omega)}{\int_a^{\hat{X}_1^c} \omega^2\Pi(d\omega)} = R,
\end{aligned} \tag{16}$$

which implies that $\hat{X}_1^c = X_1^c$, see (15). Hence, the cutoff set on the period 1 hedge fund payout for continued period 2 capital investment does not vary with the investor's horizon.

Now consider the cutoff set by the long-horizon investor on the period 0 hedge fund payout for continued period 1 capital investment. The expected lifetime payoff to this investor when it achieves a payout $\hat{X}_0^c$ in period 0 is

$$\begin{aligned}
V_3\left(\hat{X}_0^c\right) = &\frac{\int_a^{\hat{X}_1^c}\left\{\frac{\omega}{\hat{X}_1^c}\left[(1-\beta)\hat{X}_1^c\bar{R} + \omega(1-\beta)\right] + \left(1-\frac{\omega}{\hat{X}_1^c}\right)\bar{R}\right\}\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)}{\int_a^{\hat{X}_0^c}\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)} + \\
&\frac{\int_{\hat{X}_1^c}^{X_0^c}\left[\omega(1-\beta)\bar{R} + \omega(1-\beta)\right]\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)}{\int_a^{\hat{X}_0^c}\frac{\omega}{\hat{X}_0^c}\Pi(d\omega)}.
\end{aligned}$$

Recognizing that $\int_a^{\hat{X}_1^c} \omega^3 \Pi(d\omega) = R \int_a^{\hat{X}_1^c} \omega^2 \Pi(d\omega)$ from (16), this expression simplifies to

$$V_3\left(\hat{X}_0^c\right) = \frac{\int_a^{\hat{X}_1^c} \left[(1-\beta)\,\omega^2 \bar{R} + \omega \bar{R}\right] \Pi(d\omega) + \int_{\hat{X}_1^c}^{\hat{X}_0^c} (1-\beta)\,\omega^2 \left(1+\bar{R}\right) \Pi(d\omega)}{\int_a^{\hat{X}_0^c} \omega \Pi(d\omega)}. \tag{17}$$

Note that efficiency requires that $V_3\left(\hat{X}_0^c\right) = \bar{R}^2 + \bar{R}$, i.e., the critical value at date 0 is such that the investor is indifferent between investing in the fund and the alternative asset.

Consider now the payoffs to short-horizon investors. In particular, we are interested in the expected payoff to an investor in date 1, given that the date 0 fund payout was X_0^c. Denote this expected payoff by $V_1(X_0^c)$. To facilitate comparison between the long-horizon and short-horizon investors, note that $V_1(X_0^c)\bar{R} + \bar{R} = \bar{R}^2 + \bar{R}$, since $V_1(X_0^c) = \bar{R}$. Therefore,

$$\begin{aligned} V_1\left(X_0^c\right)\bar{R} + \bar{R} &= \frac{\int_a^{X_0^c} \left[\frac{\omega}{X_1^c}(1-\beta)\,X_1^c \bar{R}\right] \frac{\omega}{X_0^c} \Pi(d\omega) + \int_a^{X_0^c} \bar{R} \frac{\omega}{X_0^c} \Pi(d\omega)}{\int_a^{X_0^c} \frac{\omega}{X_0^c} \Pi(d\omega)} \\ &= \frac{\int_a^{X_1^c} \left[(1-\beta)\,\omega^2 \bar{R} + \bar{R}\omega\right] \Pi(d\omega) + \int_{X_1^c}^{X_0^c} \left[(1-\beta)\,\omega^2 \bar{R} + \bar{R}\omega\right] \Pi(d\omega)}{\int_a^{X_0^c} \omega \Pi(d\omega)} \end{aligned} \tag{18}$$

Suppose that $\hat{X}_0^c = X_0^c$. Now, recalling that $\hat{X}_1^c = X_1^c$ and subtracting the numerator of (18) from that of (17), we get

$$\begin{aligned} &\int_{X_1^c}^{X_0^c} (1-\beta)\,\omega^2 - \int_{X_1^c}^{X_0^c} \bar{R}\omega \Pi(d\omega) \\ = &\int_{X_1^c}^{X_0^c} \left[(1-\beta)\,\omega - \bar{R}\right] \omega \Pi(d\omega). \end{aligned} \tag{19}$$

But since, $(1-\beta)\,\omega - \bar{R} > 0$ for all $\omega \in [X_1^c, X_0^c]$, expression (19) is strictly positive, a contradiction. Since (17) exceeds (18) when it is assumed that $\hat{X}_0^c = X_0^c$, it must be the case that $\hat{X}_0^c < X_0^c$.

Proof to Proposition 6

The expected performance of the hedge fund in period 1 is,

$$\frac{\int_a^X \frac{\omega^2}{X} \Pi(d\omega) + \int_X^b \omega \Pi(d\omega)}{\int_a^X \frac{\omega}{X} \Pi(d\omega) + \int_X^b \Pi(d\omega)}, \tag{20}$$

where X represents the period-0 cutoff and $X \in [\hat{X}_0, X_0)$. The sign of the derivative of (20) with respect to X is

$$sign\left[-\int_a^X \frac{\omega^2}{X^2}\Pi(d\omega)\int_X^b \Pi(d\omega)+\int_a^X \frac{\omega}{X^2}\Pi(d\omega)\int_X^b \omega\Pi(d\omega)\right].$$

$$= sign\left[-\frac{\int_a^X \omega^2\Pi(d\omega)}{\int_a^X \omega\Pi(d\omega)}+\frac{\int_X^b \omega\Pi(d\omega)}{\int_X^b \Pi(d\omega)}\right]$$

The absolute value of the first term is less than X and the value of the second term exceeds X. Hence, this expression is positive, which means that an increase in the period-0 standard increases the period-1 expected fund performance.

The expected performance of the hedge fund in period 2 is

$$\frac{\int_a^{X_1^c} \frac{\omega^3}{XX_1}\Pi(d\omega)+\int_{X_1^c}^X \frac{\omega^2}{X}\Pi(d\omega)+\int_X^b \omega\Pi(d\omega)}{\int_a^{X_1^c} \frac{\omega^2}{XX_1}\Pi(d\omega)+\int_{X_1^c}^X \frac{\omega}{X}\Pi(d\omega)+\int_X^b \Pi(d\omega)}. \tag{21}$$

The sign of the derivative of (21) can be written as

$$\begin{aligned} sign\Bigg[&-\int_{X_1^c}^X \omega^2\Pi(d\omega)\int_X^b \Pi(d\omega)+\int_{X_1^c}^X \omega\Pi(d\omega)\int_X^b \omega\Pi(d\omega) \\ &-\int_a^{X_1^c} \frac{\omega^3}{X_1}\Pi(d\omega)\int_X^b \Pi(d\omega)+\int_a^{X_1^c} \frac{\omega^2}{X_1}\Pi(d\omega)\int_X^b \omega\Pi(d\omega)\Bigg]. \end{aligned} \tag{22}$$

The first line can be rearranged as

$$= sign\left[-\frac{\int_{X_1^c}^X \omega^2\Pi(d\omega)}{\int_{X_1^c}^X \omega\Pi(d\omega)}+\frac{\int_X^b \omega\Pi(d\omega)}{\int_X^b \Pi(d\omega)}\right].$$

The first term is less than X and the second term exceeds X, meaning that the sign is positive. The second line of (22) can be rearranged as

$$sign\left[-\frac{\int_a^{X_1^c} \omega^3\Pi(d\omega)}{\int_a^{X_1^c} \omega^2\Pi(d\omega)}+\frac{\int_X^b \omega\Pi(d\omega)}{\int_X^b \Pi(d\omega)}\right].$$

The first term is less than $-X_1^c$ and the second term exceeds X_1^c. All of this implies that (22) is positive, which means that an increase in the period-0 standard increases the period-2 expected hedge fund performance.

Proof to Proposition 7

Since the expected ability of a fund manager whose fund pays out $X_0 = X^c$ is R, X^c solves

$$R = E_{\mu(X^c)}[\omega] = \frac{\int_{\omega+\eta_0<X^c} [(\omega+\eta_0)\omega/X^c]\, \pi(\omega)\, h(\eta_0) d\eta_0 d\omega}{\int_{\omega+\eta_0<X^c} [(\omega+\eta_0)/X^c]\, \pi(\omega)\, h(\eta_0) d\eta_0 d\omega}$$
$$= \frac{\int_{\omega+\eta_0<X^c} (\omega+\eta_0)\omega h(\eta_0) d\eta_0 d\omega}{\int_{\omega+\eta_0<X^c} (\omega+\eta_0) h(\eta_0) d\eta_0 d\omega},$$

where the simplification reflects the uniform uncertainty of fund managers' type ω. The conditional expectation can be expanded to read,

$$\frac{\int_a^{X^c-\bar{y}} \omega^2 d\omega + \int_{X^c-\bar{y}}^{X^c} [\int_{-\bar{y}}^{X^c-\omega} \omega(\omega+\eta_0) h(\eta_0) d\eta_0 + \int_{X^c-\omega}^{\bar{y}} A(\omega, h)\, h(\eta_0) d\eta_0] d\omega}{\int_a^{X^c-\bar{y}} \omega d\omega + \int_{X^c-\bar{y}}^{X^c} [\int_{-\bar{y}}^{X^c-\omega} (\omega+\eta_0) h(\eta_0) d\eta_0 + \int_{X^c-\omega}^{\bar{y}} A'(\omega, h)\, h(\eta_0) d\eta_0] d\omega}, \tag{23}$$

where $A(\omega, \eta_0) = [\omega + 2(X^c - \omega)][\omega + 2(X^c - \omega) - \eta_0]$ and $A'(\omega, \eta_0) = (\omega + 2(X^c - \omega) - \eta_0)$. To understand the construction of (23), consider each of the three terms in the numerator:

1. The first integral is over hedge fund manager types $\omega < X^c$ such that $\omega + \bar{y} < X^c$. Even if they receive the highest possible idiosyncratic shock, these hedge fund managers must use a risky investment strategy in order to achieve X^c.

2. The second integral is over hedge fund manager types $\omega < X^c$ who do not receive a sufficiently large η_0 shock, and hence must adopt a risky investment to reach X^c. It does not include those hedge fund manager types who receive big positive η_0 shocks, $\eta_0 > X^c - \omega$, and hence adopt sure thing investment strategies.

3. The last integral is over hedge fund manager types $\omega > X^c$ who receive sufficiently large negative η_0 shocks that push them below X^c, and hence must choose a risky investment strategy in order to achieve X^c. This integral term exploits the symmetry of η_0, $h(\eta_0) = h(-\eta_0)$, to use the bounds of integration that apply to hedge fund manager types $\omega < X^c$. This facilitates comparisons of (23) with the base-case where η_0 always equals zero. Thus, for any hedge fund manager type $\omega = X^c - \epsilon$ for $\epsilon = X^c - \omega \in (0, \bar{y})$ the corresponding actual hedge fund manager type is $\hat{\omega} = X^c + \epsilon$, which we write as $X^c - \epsilon + 2(X^c - (X^c - \epsilon)) = X^c - \epsilon + 2\epsilon = X^c + \epsilon$; and the counterpart got unlucky receiving a $-\eta_0 < 0$ shock rather than a positive η_0 shock. Thus, the true type,

$\hat{\omega} = \omega + 2(X^c - \omega)$, enters the expectation (of period 1 ability) in the first bracketed term in $A(\omega, \eta_0)$, and the hedge fund manager's period 0 resources $\hat{\omega} - \eta_0$, which determine the probability of achieving X^c, are the second bracketed term in $A(\omega, \eta_0)$.

The denominator can be interpreted analogously. Now, substitute

$$A(\omega, \eta_0) = (2X^c - \omega - \eta_0)(2X^c - \omega) = \omega + \eta_0 + 2X^c(2(X^c - \omega) - \eta_0)$$

in the numerator of (23), and

$$A'(\omega, \eta_0) = 2X^c - \omega - \eta_0 = \omega + \eta_0 + 2(X^c - (\omega + \eta_0))$$

in the denominator, to rewrite the conditional expectation as

$$\begin{aligned}
& \frac{\int_a^{X^c} \omega^2 d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2X^c(2(X^c - \omega) - \eta_0)h(\eta_0)d\eta_0 d\omega}{\int_a^{X^c} \omega d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2(X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega} \\
= \; & [\int_a^{X^c} \omega^2 d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2X^c((X^c - \omega)h(\eta_0)d\eta_0 d\omega + \\
& \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2X^c((X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega] \times \\
& [\int_a^{X^c} \omega d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2(X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega]^{-1} \\
> \; & \frac{\int_a^{X^c} \omega^2 d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2X^c((X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega}{\int_a^{X^c} \omega d\omega + \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2(X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega} \\
= \; & \frac{\int_a^{X^c} \omega^2 d\omega + \alpha X^c}{\int_a^{X^c} \omega d\omega + \alpha} > \frac{\int_a^{X^c} \omega^2 d\omega}{\int_a^{X^c} \omega d\omega},
\end{aligned}$$

where $\alpha = \int_{X^c-\bar{y}}^{X^c} \int_{X^c-\omega}^{\bar{y}} 2(X^c - (\omega + \eta_0))h(\eta_0)d\eta_0 d\omega$. The final inequality follows (by cross-multiplying the inequality) since,

$$\int_a^{X^c} \omega d\omega (\int_a^{X^c} \omega^2 d\omega + \alpha X^c) - \int_a^{X^c} \omega^2 d\omega (\int_a^{X^c} \omega d\omega + \alpha) = \alpha \int_a^{X^c} (X^c - \omega)\omega d\omega > 0.$$

Thus, for any given cutoff X^c, pre-investment shocks raise the expected hedge fund type that pays out X^c in the following sense. Conditional on achieving the cutoff (1) in Section 3 (where there were no pre-investment shocks), the expected ability of a hedge fund manager exceeds R. To retrieve equality with R, it must be that the cutoff X^c for continued investment in the hedge fund is reduced.

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