Industry Dynamics and the Minimum Wage: A Putty-Clay Approach

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Abstract

We document two new findings about the industry-level response to minimum wage hikes. First, restaurant exit and entry both rise following a hike. Second, there is no change in employment among continuing restaurants. We develop a model of industry dynamics based on putty-clay technology that is consistent with these findings. In the model, continuing restaurants cannot change employment, and thus industry-level adjustment occurs gradually through exit of labor-intensive restaurants and entry of capital-intensive restaurants. We also show that the putty-clay model matches many other findings in the empirical minimum wage literature, including a small short run disemployment effect of the minimum wage and complete price pass-through. Interestingly, however, the putty-clay model produces a larger long run disemployment effect of the minimum wage.

Keywords: minimum wage, employment, putty-clay, industry dynamics

JEL codes: L11, E24, J36

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1 Introduction

This paper presents new evidence on how the restaurant industry, the largest U.S. employer of low-wage labor, responds to minimum wage hikes. We document two new empirical findings. First, exit and entry among limited service (i.e. fast food) restaurants rise after a minimum wage hike. Second, there is no change in employment among continuing limited service restaurants. Together, these results imply an economically small impact on employment two years after a minimum wage hike. We show that an augmented putty-clay model explains these responses. To the best of our knowledge, we are the first to provide micro-level evidence supportive of the importance of putty-clay relative to competing models of firm dynamics.

Our empirical findings are derived from the Census of Employment and Wages (QCEW), a database used to compile unemployment insurance payroll records collected by each state's employment office. The QCEW provides detailed information on each establishment's name, location, and employment level at a monthly frequency. We follow Card and Krueger (1994), Addison, Blackburn, and Cotti (2009), and Dube, Lester, and Reich (2010), among others, and compare restaurants that reside in counties near state borders where the minimum wage has risen on one side of the border but not the other. Our results suggest that exit and entry, particularly among chains, increases in the year following a minimum wage hike. By contrast, we find no comparable exit or entry effect among full service restaurants and mixed evidence among other accommodation and food service industries, both of which make less use of low-wage labor.

To interpret these findings, we describe a model of industry dynamics that extends the putty-clay framework of Sorkin (2015) and Gourio (2011) to incorporate endogenous exit as in Campbell (1998). In the model, new entrants can choose from a menu of capital-labor intensities but, once the establishment is built, output is Leontief between capital and labor. In this environment, adjusting the capital-labor mix in response to higher wages requires shutting down labor-intensive establishments and opening capital-intensive establishments. Hence, the model predicts that, given reasonable parameters, both entry and exit rise in response to a minimum wage hike.

Not only does the putty-clay model match our new empirical findings on exit and entry, but it generates three other predictions that appear consistent with the minimum wage literature. First, the model implies that the cost of higher minimum wages are fully passed onto consumers in the form of higher prices (Aaronson (2001), Aaronson, French, and MacDonald (2008), and Harasztosi and Lindner (2015)). Second, despite the pass-through, profits and firm value among incumbent restaurants falls, as in Draca, Machin, and Reenen (2011) and Bell and Machin (2016). Third, because continuing restaurants cannot adjust their employment levels, the putty-clay model generates a small short-run employment response, consistent with much of the literature.

A key implication of the model is that the short- and long-run effects of minimum wage hikes are different. In the putty-clay model, the disemployment effect of the minimum wage hike grows over time, as labor intensive incumbent restaurants are slowly replaced with more capital intensive entrants. Thus, the empirical assessment in the literature that the *short-run* disemployment effects of minimum wage hikes are small may provide an imperfect guide to the longer run effects of minimum wage hikes. Specifically, relative to what is typically inferred from existing work, alternative minimum wage policies may have more negative employment consequences and be a less effective redistributive tool.

This paper is organized as follows. In section 2, we briefly review the relevant theoretical and empirical literatures and argue that benchmark models of industry dynamics, as well as models incorporating imperfect competition in labor markets, are unable to fully explain the facts that we present on exit, entry, and employment after a minimum wage hike. Sections 3 to 5 describe the QCEW data, the estimation strategy, and the empirical results. In section 6, we present the putty-clay model, which is used in section 7 to show how a minimum wage hike impacts exit and entry. A calibration of the model is presented in section 8, which we use to discuss the plausibility of the model and the long-term consequences of minimum wage hikes. Section 9 concludes.

2 Literature Review

Putty-clay models have been effective at matching aggregate business cycle (Gilchrist and Williams (2000) and Atkeson and Kehoe (1999)) and financial market (Gourio (2011)) facts in a number of settings.[1] Our results complement earlier research by documenting that firm

[1]Adjustment cost and job search models can match many of the same facts. But putty-clay has been able to better match both short- and long-run responses to cost shocks such as energy price shocks, whereas adjustment cost models that match short-run movements tend to overstate responses in the long-run (Atkeson and Kehoe (1999)).

entry and exit decisions are consistent with the predictions of putty-clay models. As such, we believe we are the first to provide establishment-level empirical evidence supportive of the relevance of putty-clay technology.

The key feature of the putty-clay model – that potential entrants are able to pick a capital-labor ratio that is well-suited to the minimum wage while incumbents are not – is not present in several benchmark models typically used to describe industry dynamics or the impact of minimum wage hikes. For example, Hopenhayn (1992) assumes that factor proportions can freely change. Thus his model predicts an increase in exit and a fall in entry after a minimum wage hike.

Search models contain a mechanism that can potentially match our entry and exit results. In Flinn (2006), a minimum wage hike causes the lowest productivity matches to break up, generating a spike in firm exit. Additional exit increases the number of job searchers, potentially raising the return to posting a vacancy and thus potentially causing a spike in firm entry. That said, because Flinn (2006) is solved in steady-state, as is standard in the literature, his model does not distinguish between entry and exit. Furthermore, Flinn's model does not speak to our continuing firm results because it is a model of a firm vacancy and a single potential worker. Models with multiple worker firms, as in Elsby and Michaels (2013) or Acemoglu and Hawkins (2014), could likely match our exit results but not the lack of employment change among continuing firms.[2]

Thus we believe that putty-clay is a key part of any explanation of the industry dynamics that we empirically document.[3]

Our paper also adds to the voluminous literature on the employment effects of the minimum wage, surveyed by Neumark and Wascher (2008).[4] In particular, we believe we are among the first (see also Rohlin (2011)) to estimate the firm entry and exit responses to

[2]In a multi-worker firm model, the minimum wage hike would have heterogeneous effects among firms. High-paying firms benefit from the increased ease of finding workers and therefore might expand. Low-paying firms for which the minimum wage hike is binding might contract.

[3]Like us, Jovanovich and Tse (2010) document evidence of a simultaneous spike in entry and exit in response to industry-level technology shocks. They develop a vintage capital model to describe these facts. However, their model still allows firms to freely adjust their capital-labor ratio, and thus would not predict a simultaneous spike in entry and exit after a minimum wage, or other factor price, change.

An important aspect of the putty-clay model is the decision of when to scrap. In this sense we also contribute to the optimal scrapping and replacement literature (Adda and Cooper (2000)) by aggregating to the industry level.

[4]A sampling of papers since 2008 includes Dube, Lester, and Reich (2010), Clemens and Wither (2014), Neumark, Salas, and Wascher (2014), Aaronson and Phelan (2015), Aaronson, Agarwal, and French (2012), and Allegretto et al. (2016). Some of these recent papers use panel data methods.

minimum wage hikes.[5] Estimation of these responses provides clearer tests of models of low-wage labor market structure, which is critical for evaluating labor market policies to help the poor. Moreover, we show that the putty-clay model is consistent with other market responses to minimum wage hikes that have been studied in the literature, including higher price levels (e.g. Aaronson (2001), Aaronson, French, and MacDonald (2008), Basker and Khan (2013), and Harasztosi and Lindner (2015)), lower profits (Draca, Machin, and Reenen (2011)) and firm values (Bell and Machin (2016)), and larger disemployment in the long-run than the short-run (Baker, Benjamin, and Stanger (1999), Meer and West (2015), and Sorkin (2015)). Our findings also complement recent work that finds a reduction in hiring and separations after a minimum wage hike (Brochu and Green (2013), Dube, Lester, and Reich (2015), and Gittings and Schmutte (2014)). Our results imply minimum wage hikes increase firm turnover, while their results suggest worker turnover declines among firms that neither enter nor exit following a minimum wage hike. Nevertheless, we see these results as potentially complementary to ours in that each suggest important dynamic dimensions, either within or across establishments, in which there are responses to a labor cost shock.[6]

The only paper we are aware of that simultaneously studies exit and entry in response to a minimum wage increase in the U.S. is Rohlin (2011). Using detailed firm locations derived from the Dun and Bradstreet Marketplace data files, he finds that state minimum wages hikes instituted between 2003 and 2006 discouraged firm entry but had little impact on the exit and employment of establishments in existence at least 4 years prior. Rohlin identifies exit, entry, and employment effects within miles of state borders, rather than at the coarser county level that we use. However, his main results are reported at the 1 digit (6 industries) SIC level, far too aggregated to distinguish heavy minimum wage users. Strikingly, the largest negative entry appears in manufacturing, where only 3 and 10 percent of its workforce is paid within 110 and 150 percent of the minimum wage and where previous work (e.g. Dube, Lester, and Reich (2010)) has found no earnings or employment effects of minimum wage hikes. In contrast, our study concentrates on the restaurant industry, where just over half of workers are paid within 150 percent of the minimum wage. Given Rohlin's detailed geographic precision, sample sizes get quite small when results are reported at the more relevant 2 digit

[5]Mayneris, Poncet, and Zhang (2015) study exit responses to minimum wage hikes in China. See also Huang, Loungani, and Wang (2015).

[6]Similarly, Brochu et al. (2015) emphasize different responses among continuing, beginning and ending employment matches, which is analogous to our distinctions among continuing, entering, and exiting firms.

industry level.

3 Data

We study the restaurant industry (NAICS 722) because it is the largest employer of workers at or near the minimum wage, accounting for roughly 16 percent of such employees between 2003 and 2006 according to the Current Population Survey's Outgoing Rotation Groups.[7] Moreover, the intensity of use of minimum wage workers in the restaurant industry is amongst the highest of the industrial sectors (Aaronson and French (2007)). Like many studies before this one (e.g. Katz and Krueger (1992), Card and Krueger (1995), Card and Krueger (2000), Neumark and Wascher (2000), Aaronson and French (2007) and Aaronson, French, and MacDonald (2008)), we concentrate specifically on limited service establishments, which are especially strong users of minimum wage labor.[8]

Under an agreement with the Bureau of Labor Statistics (BLS), we were granted access to the establishment-level employment data provided in the Quarterly Census of Employment and Wages (QCEW).[9] The QCEW program compiles unemployment insurance payroll records collected by each state's employment office. The records contain the number of UI-covered employees on the 12^{th} of each month. The main advantages of the QCEW are that it covers virtually all firms, and has very little measurement error. But as is typical of administrative datasets, information about establishments is sparse. In particular, the key variables are establishment ID, employment, location, and trade and legal name. The former three are used to measure exit, entry, and employment changes by geographic location. The trade/legal name allows us to identify establishments that are part of large chains.[10]

Our results are derived from five state-level minimum wage hikes – a 17 percent increase in California phased in between January 2001 and January 2002, a 26 percent increase in

[7]The next largest employer, retail grocery stores, employs just under 5 percent of minimum or near minimum wage workers.

[8]In "limited service" (LS) outlets, meals are served for on or off premises consumption and patrons typically place orders and pay at the counter before they eat. In "full service" (FS) outlets, wait-service is provided, food is sold primarily for on-premises consumption, orders are taken while patrons are seated at a table, booth or counter, and patrons typically pay after eating. Unfortunately, prior to 2001, industry codes were unable to differentiate limited service and full service outlets. This is one reason why we concentrate on minimum wage changes in the 2000s. Another reason is that there is significant concern about the accuracy of single establishment reporting prior to 2001. We describe this problem below.

[9]We also use data prior to 2003 when the QCEW was referred to as the ES-202.

[10]However, the BLS' confidentiality restrictions do not allow us to disclose the chain names nor how we developed our list.

Table 1: State minimum wage increases

		Minimum wage			
Year	State	Old	New	% Change	Comparison states
Jan. 2001	California	5.75	6.25	8.7	OR, NE, AZ
Jan. 2002	California	6.25	6.75	8	OR, NE, AZ
Jan. 2003	Oregon*	6.50	6.90	6.2	ID
Jan. 2004	Illinois	5.15	5.50	6.8	IN, IA, KY, MO
Jan. 2005	Illinois	5.50	6.50	18.2	IN, IA, KY, MO
Aug. 2005	Minnesota	5.15	6.15	19.4	IA, ND, SD
Jan. 2005	DC	6.15	6.60	7.3	MD, VA
Jan. 2006	DC	6.60	7.00	6.1	MD, VA

Note: *Oregon simultaneously instituted an automatic inflation adjustment. Source: Monthly Labor Review, January issues.

Illinois phased in between January 2004 and January 2005, a 19 percent increase in Minnesota in August 2005, a 6 percent increase in Oregon in January 2003 that also included the introduction of an annual Consumer Price Index adjustment, and a 14 percent increase in Washington DC phased in during January 2005 and January 2006 – and their adjacent neighboring states in the early- to mid-2000s (see Table 1).[11] While a number of other states passed minimum wage changes during the 2000s, we exclude them because either a) the state QCEW data was not accessible (e.g. Pennsylvania, Massachusetts, New York), b) the change is small (e.g. Consumer Price Index adjustments), or c) bordering states also raised their minimum wage.[12] Nevertheless, these five states and their neighbors contain significant numbers of restaurants along the borders.

We face three measurement issues with regard to creating a consistent panel of QCEW restaurant employment, entry, and exit.

First, many small restaurants appear to exit and then re-enter within a year. These look like seasonal businesses that are open, for example, only in the summer or the winter. To address this concern, we define an entrant after the hike as an establishment without employment in the year before the hike but with average monthly employment above 15 in each of the two six-month periods starting a year after the hike.[13] Likewise, we define an

[11] Other than Oregon, the hikes are of comparable size; our results are robust to dropping the Oregon hike.

[12] We also excluded a) Wisconsin as a comparison state to Illinois and Minnesota and b) California as a comparison state to Oregon because of their own minimum wage activity.

[13] To take a concrete example, an entrant after the August 2005 hike in Minnesota is an establishment with no employment in August 2004- July 2005 and average employment above 15 in both the August 2006-January 2007 and February 2007-July 2007 periods. For measurement of entry, exit and employment in the

exit after the hike as an establishment having average employment above 15 employees in each of two six month periods prior to the minimum wage hike and no employment starting a year after the hike. We document how our results vary when we alter the size requirement between 1 and 20 employees.

Second, the BLS did not collect industry NAICS codes until 2001. Therefore, we must use BLS imputations of industry for establishments that exit prior to 2001. This problem is only relevant for one of the five state-level minimum wage hikes that we exploit (the 2001-02 California hike); the other four state-level minimum wage hikes that we study take place well after 2000, and thus imputed data is not needed.

Third, firms sometimes group establishments together for reporting purposes. In a multi-establishment firm, an individual establishment's birth or death may look instead like growth or contraction of a larger continuing firm. Moreover, reporting arrangements can switch between multi-unit and individual establishment reporting over time. Switches from multi-unit to individual establishment reporting ("breakouts") will appear in our data as multiple births with the possibility of a death. Switches from individual to multi-unit reporting ("consolidations") will appear as multiple deaths with the possibility of a birth. Fortunately, using the QCEW Breakout and Consolidations Link (BCL) file, we can identify and drop establishments that were ever involved in a breakout or consolidation. Furthermore, we consider the robustness of our results to imposing an upper bound of 100 employees on establishment size. Like Card and Krueger (2000), we find that our results are robust to changes in this upper-bound.

Appendix Tables A4 and A5 provides more details on sample construction and summary statistics.

4 Empirical Strategy

States might be more likely to raise the minimum wage in good times. Thus, standard state-level difference-in-difference regressions may confound the impact of the minimum wage with the economic conditions that allowed minimum wage legislation to move forward.

period prior to the hike (the "pre-period"), as our difference-in-differences estimator requires,we shift our definitions of entry, exit, and employment back two years to avoid using post-minimum wage hike data. Thus, for the August 2005 Minnesota minimum wage hike, we define a pre-period entrant as having no employment in August 2002- July 2003, and average employment above 15 in both the August 2004-January 2005 and February 2005-July 2005 periods.

To circumvent this problem, we focus on restaurants in counties near state borders, as in Dube, Lester, and Reich (2010). Geographically nearby restaurants in different states with different minimum wages likely face similar economic environments (other than having a different minimum wage). This comparison then allows us to flexibly control for time-varying shocks.

In particular, we consider the following specification:

$$Y_{ispt} = \beta \log w_{ist} + a_{pt} + \alpha_s + \epsilon_{ispt} \tag{1}$$

where Y_{ispt} is the outcome of interest, w_{ist} is the minimum wage faced by restaurant i in state s at time t, a_{pt} is a full set of border segment-time dummies (e.g., northern California-southern Oregon in Jan. 2013), α_s is a state dummy, and ϵ_{ispt} is a residual that is assumed uncorrelated with the minimum wage. We concentrate on three measures of Y_{ispt}: entry, exit, and the log of employment among continuously-operating establishments. Entry is an indicator variable of whether restaurant i existed at time t and not at time $t-1$. Similarly, exit is an indicator for whether restaurant i existed at time $t-1$ and not time t. Our sample includes restaurants that are in counties on the state border or adjacent to a county on the state border among the states listed in Table 1.

Equation (1) is a generalization of the usual difference-in-difference approach. To see this comparison, define t_{np} as the time of the first minimum wage hike for border segment p. Differencing equation (1) yields:

$$(Y_{ispt_{np+1}} - Y_{ispt_{np-1}}) = \beta(\log w_{ist_{np+1}} - \log w_{ist_{np-1}}) + (a_{pt_{np+1}} - a_{pt_{np-1}}) + (\epsilon_{ispt_{np+1}} - \epsilon_{ispt_{np-1}}). \tag{2}$$

Next, define the state that raised the minimum wage hike as state s and the comparison state that borders state s but did not have the hike as state ς. Differencing equation (2) across states s and ς yields

$$(Y_{ispt_{np+1}} - Y_{ispt_{np-1}}) - (Y_{i\varsigma pt_{np+1}} - Y_{i\varsigma pt_{np-1}}) = \beta(\log w_{ist_{np+1}} - \log w_{ist_{np-1}}) + residual \tag{3}$$

where $residual = (\epsilon_{ispt_{np+1}} - \epsilon_{ispt_{np-1}}) - (\epsilon_{ispt_{np+1}} - \epsilon_{ispt_{np-1}})$. As with difference-in-differences, all of the dummy variables related to time and geography vanish. To account for minimum wage hikes that are phased in over multiple years, we define the difference between the pre-hike period $t-1$ and the post-hike period t to be two years. The coefficient β is converted to an exit and entry elasticity using pre-hike sample means (β is already an elasticity in the log employment regression). The elasticity should be interpreted as the estimated percent change in an outcome (exit or entry probability or employment among continuing establishments) in the treated counties relative to the control counties in response to a 1 percent minimum wage hike.

There are three differences between equation (3) and the usual difference-in-differences specification. First, by using the log of the minimum wage and taking differences we effectively use the percent change in the minimum wage rather than a dummy for whether the minimum wage increased.[14] Second, instead of comparing just two states, equation (3) allows us to pool multiple state level minimum wage hikes. Third, our approach does not compare changes across states, but across border segments. This allows us to compare, for example, a change in entry in Northern California to a change in entry in southern Oregon.

While this county border discontinuity approach is appealing for the reasons mentioned above, it is imperfect if there are spillovers or if border counties are not similar. Spillovers could occur through either product or labor markets. An example of a product market spillover would be consumers crossing the border in response to the minimum wage hike. Similarly, an example of a labor market spillover would be workers crossing the border either to avoid or pursue the minimum wage hike. While spillovers are certainly plausible, we do not know of any empirical evidence of their existence, let alone quantitative importance. The other criticism of the border design is that neighboring counties across the border may not form a good "control" group (Neumark, Salas, and Wascher (2014)). Dube, Lester, and Reich (2015) present evidence that nearby counties are more similar than distant counties in terms of levels and trends of covariates, which provides some evidence that they would be similar in terms of time-varying shocks as well.

While the border discontinuity approach leverages variation due to minimum wage hikes, it does not leverage variation in how binding the minimum wage hike might be. For example, some border counties that experience minimum wage hikes have more low-wage workers, and we would expect to see larger effects in those places. Unfortunately, we are not able to

[14] We also experimented with using an indicator for a minimum wage hike (i.e. define $w_{st-\tau}$ as 1 if there was a minimum change and 0 otherwise), rather than the magnitude of the increase and found similar results.

exploit this heterogeneity since our data does not contain individual wages or other relevant characteristics such as establishment-level wages, profits, prices, or output. Moreover, we are working with a limited number of minimum wage hikes. We believe, however, that exploiting richer establishment-level data is a promising direction for future work, especially as more cities and states consider or have already instituted historically high minimum wage hikes.

5 Results

Table 2 reports the impact of a minimum wage increase on the likelihood of exit (row A), entry (row B), and change in employment of continuing firms (row C). Results for limited service restaurants are presented in columns (1) to (3), full service restaurants in column (4), and establishments that are not restaurants but in the NAICS 72 hospitality and food services industry in columns (5) and (6). Bootstrapped standard errors are in parentheses. All estimates are reported as elasticities evaluated at sample means.

We find that exit of limited service establishments unambiguously rises in the year after a minimum wage increase. A 1 percent increase in the minimum wage causes exit rates in limited service establishments to go up 2.40 percent (standard error of 0.86 percent). This estimate implies a 10 percent increase in the minimum wage would increase, on average, the limited service annual exit rate from its sample mean of 5.7 percent prior to the minimum wage increase to approximately 7.1 percent after the hike. Our exit estimates are statistically significant at conventional levels.[15] Exits rise faster among chains than non-chains (row A in column 2 versus column 3).

By sharp contrast, there is no impact of a minimum wage increase on the exit of full service restaurants (column 4, row A), nor on other NAICS72 establishments other than restaurants and hotels and motels (column 6, row A), where minimum wage labor share is lower (Aaronson and French (2007)). We do find a large impact on hotels and motels (column 5, row A). Hotels are fairly intensive users of minimum wage labor, although the magnitude of the estimated effect is still surprising.[16]

[15] Results are also statistically significant at the 5 percent level for all establishments when standard errors are clustered at the state-border or state-border-segment level.

[16] We should note, however, that those results are particularly sensitive to the choice of standard error. When we cluster-correct at the state-level, the standard errors from the border state specification rise to 5.30 (from 2.11 with bootstrapping), suggesting the hotel and motel results are highly influenced by a small number of areas. For the other exit estimates, clustering and bootstrapping produce roughly similar estimated standard errors. The cluster-corrected standard error for limited service restaurant exit is somewhat higher as well: 1.24

Table 2: Elasticity of exit, entry, and employment among continuing firms

	Limited service restaurants			Full service	Hotels/	Other
	All	Chains	Non-chains	restaurants	motels	NAICS72
	(1)	(2)	(3)	(4)	(5)	(6)
A. Exit	2.40	5.27	1.58	-0.75	8.00	-1.98
	(0.86)	(2.14)	(0.91)	(0.75)	(2.11)	(1.12)
	16,191	6,961	9,230	18,184	3,634	4,210
B. Entry	1.37	2.64	0.78	0.14	0.34	1.21
	(0.61)	(1.02)	(0.74)	(0.62)	(1.51)	(1.26)
	16,513	7,188	9,325	18,529	3,606	4,259
C. Change in employment	-0.05	-0.08	-0.04	-0.12	0.35	0.22
among continuing	(0.07)	(0.08)	(0.10)	(0.07)	(0.52)	(0.19)
establishments	14,993	6,555	8,438	16,825	3,324	3,827

Note: Each cell is from a separate regression. For each regression, we report elasticities evaluated at sample means, bootstrapped standard errors (in parentheses), and sample sizes. All estimates include state border-time dummies.

Row B reports results on entry rates. Similar to exit, entry also increases in the year after a minimum wage hike. We find a one percent increase in the minimum wage leads to 1.37 (standard error of 0.61) percent increase in the entry rate in the year after the hike relative to the two years prior.[17] Given these estimates, a 10 percent increase in the minimum wage would increase the limited service annual entry rate from its sample mean of 8.7 percent prior to the minimum wage increase to approximately 9.9 percent after the hike. The estimated entry effect is larger in establishments affiliated with a chain; entry rises by 2.64 (1.02) percent among chains but 0.78 (0.74) percent among non-chains. Notably, there is again no impact on the entry of full service restaurants, hotel and motels, or other non-restaurant NAICS72 establishments.

Row C reports results on employment changes among continuing firms. We find little evidence of a significant change in employment among any NAICS72 industries, including limited service restaurants, after a minimum wage increase.

(cluster) versus 0.86 (bootstrap, column (1)). For full service restaurants, the clustered-corrected standard error is somewhat lower 0.57 (cluster) versus 0.75 (bootstrap, column (4)).

[17] The cluster-corrected standard error for limited service restaurant entry is 1.02, implying a t-statistic of 1.34.

Table 3 provides a number of robustness checks of our benchmark specification (shown again for convenience in row A). Rows (B) to (D) vary the minimum employee size required to be in our sample from 1 to 20 employees. Of particular note, the aggregate limited service exit and entry elasticities are economically small and statistically indistinguishable from zero when the smallest establishments are included (row B). Yet even within this sample, we find economically meaningful, albeit not always statistically significant, differences between chains and non-chains. That is especially the case for entry, where the elasticity for chains is 2.09 (0.76) and is -0.18 (0.37) for non-chains. Once we drop the smallest restaurants (rows C and D), the exit and entry results become larger, although entry remains concentrated among chains regardless of establishment size. This pattern by size may indicate the difficulty of measuring exit and entry among the smallest establishments or, plausibly, that the minimum wage shocks apply in particular to establishments with a sizable workforce, which typically are chains.

Our benchmark specification allows for state border segment dummies. This implies that, for example, the Illinois-Indiana border is part of one labor market. To allow for more flexibility, we also split each border into four equal-length segments (what we call state-border-segments) based on air distance from the southern or eastern-most point of that border and include the state-border segment a_{pt} as a control. Although these results, reported in row (E), are a bit weaker overall, we view their general tenor as again supportive of the benchmark results – exit is fairly broad-based but entry is concentrated among chains.

Other reasonable perturbations, including excluding 100+ employee establishments to avoid concern that there are multi-establishments in the sample (row F) and excluding LA, Orange, and San Diego counties (row G),[18] have little impact on our inferences. Indeed, as a whole, entry, and in some cases exit, differences between chains and non-chains are, if anything, more apparent in some of these cases.

Finally, we also computed results for restaurants that are within 25 miles of another restaurant on the other side of the state border. For this sample, we do not have the statistical power to differentiate chains and non-chains; indeed we lose around three-quarters of our baseline sample. However, for limited service restaurants in total, the exit elasticity is 2.23 (1.53), the entry elasticity is 1.16 (1.04), and the continuous employment elasticity is -0.135

[18]Los Angeles, Orange, and San Diego counties are next to counties bordering Nevada or Arizona, although the vast majority of population in these three counties is far from the border.

Table 3: Robustness of exit and entry responses, limited service (LS) establishments

	Exit			Entry			Employment at continuing firms		
	All LS	Chains	Non-chains	All LS	Chains	Non-chains	All LS	Chains	Non-chains
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A. Baseline (table 2)	2.40	5.27	1.58	1.37	2.64	0.78	-0.05	-0.08	-0.04
	(0.86)	(2.14)	(0.91)	(0.61)	(1.02)	(0.74)	(0.07)	(0.08)	(0.10)
	16,191	6,961	9,230	16,513	7,188	9,325	14,993	6,555	8,438
B. Minimum	0.05	1.24	-0.12	0.28	2.09	-0.18	-0.10	0.11	-0.17
employee size	(0.75)	(2.06)	(0.68)	(0.34)	(0.76)	(0.37)	(0.11)	(0.16)	(0.14)
is 1	40,739	9,558	31,181	39,769	9,839	29,930	35,684	8,824	26,860
C. Minimum	1.19	3.46	0.57	0.69	2.35	0.00	-0.08	-0.01	-0.11
employee size	(0.63)	(1.72)	(0.71)	(0.49)	(0.92)	(0.59)	(0.06)	(0.09)	(0.09)
is 10	21,354	7,920	13,434	21,700	8,219	13,481	19,571	7,411	12,160
D. Minimum	3.93	6.70	3.14	1.05	2.52	0.38	-0.04	-0.07	-0.02
employee size	(1.06)	(2.52)	(1.09)	(0.75)	(1.34)	(0.93)	(0.09)	(0.09)	(0.13)
is 20	11,928	5,634	6,294	12,019	5,740	6,279	11,093	5,357	5,736
E. State border	2.37	3.64	2.03	0.97	1.85	0.52	-0.06	-0.19	-0.03
segments	(0.93)	(2.44)	(0.98)	(0.70)	(1.12)	(0.85)	(0.09)	(0.09)	(0.14)
	16,100	6,925	9,175	16,452	7,162	9,290	14,942	6,532	8,410
F. Exclude 100+	2.73	5.64	1.84	1.51	2.70	0.94	-0.05	-0.06	-0.04
employee	(0.88)	(2.02)	(0.91)	(0.62)	(1.01)	(0.74)	(0.07)	(0.08)	(0.10)
establishments	15,961	6,899	9,062	16,257	7,120	9,137	14,781	6,497	8,284
G. Exclude LA,	2.09	4.22	1.57	0.95	1.85	0.55	-0.09	-0.15	-0.05
Orange, SD	(0.93)	(2.17)	(0.96)	(0.62)	(1.09)	(0.72)	(0.06)	(0.07)	(0.09)
counties	11,091	4,659	6,432	11,366	4,863	6,503	10,366	4,428	5,938

Note: LS=limited service. Each cell is from a separate regression with elasticities evaluated at sample means and bootstrapped standard errors in parentheses.. Regressions control for state-border fixed effects except row (E) which controls for state border segments. See text for details.

(0.088), all similar in magnitude and statistically indistinguishable to our baseline estimates.

Together, the exit and entry results have roughly offsetting effects on net employment. Combined with the economically small impact on the employment of continuing restaurants, we estimate a disemployment elasticity of -0.1, suggesting that a 10 percent increase to the minimum wage reduces employment about 1 percent, although that estimate is highly imprecise. Precision aside, the point estimate is squarely in the range of previous estimates in the literature, especially those that use a border discontinuity design (e.g. Dube, Lester, and Reich (2010) and Addison, Blackburn, and Cotti (2009)).

Overall, we read the results as suggesting that restaurant exit and entry rise in response to a minimum wage hike.[19] Employment barely changes among establishments that remain open throughout the period.

6 The Putty-Clay Model

The previous section of the paper showed that restaurant entry and exit rise, and employment at existing restaurants changes very little, following a minimum wage hike.

As we argued in section 2, these findings are inconsistent with benchmark models of industry dynamics which allow incumbent restaurants to freely substitute across factors in response to a minimum wage hike. In these models, minimum wage hikes affect incumbents and potential entrants indistinguishably and therefore do not generate a simultaneous spike in exit and entry. Indeed, our calibration exercise in section 8.2 illustrates that if restaurants can freely substitute across factors, a minimum wage hike generates an increase in exit and a *decrease* in entry.

Our first goal in developing a model is to illustrate a mechanism that generates a simultaneous spike in exit and entry. We have purposefully kept the model simple to transparently highlight this mechanism – the differential impact of minimum wage hikes on incumbents and potential entrants. This difference generates "excess" exit relative to what would happen if incumbents and potential entrants were affected in the same way, and additionally clears space in the market for a spike in entry.

[19] We have also used the Census' Statistics of U.S. Businesses (SUSB), which collects industry-state-year level information on exit, entry, and employment changes among continuing firms. We find qualitatively similar although quantitatively smaller effects on exit and entry in the SUSB. In particular, we find that restaurant entry and exit both rise within two years of a minimum wage change. No such effect is observed among non-restaurant NAICS72 establishments.

We formalize this mechanism in a model of industry dynamics based on putty-clay technology. When a restaurant enters, it can freely choose its input mix, so its technology is flexible like putty. The novel feature of the putty-clay model is that, after entry, the technology hardens to clay and the input level and mix is fixed for the life of the restaurant. This puts incumbent restaurants at a cost disadvantage following a minimum wage hike. Indeed, some exiting incumbents would remain open if they could adjust their input mix. This displacement of incumbents by more capital-intensive entrants generates a spike in entry following the hike. The fixity of the capital-labor ratio after entry also implies—consistent with the empirical evidence—that there is on average no change in employment among continuing firms *in response to* a minimum wage hike. We acknowledge that employment fluctuates within restaurants for reasons, such as seasonality or labor turnover, that the model does not capture. But we view the model as being useful for understanding the effect of a long lived cost shock, such as a minimum wage hike. Moreover, we think it notable that employment among continuing firms does not appear to respond to minimum wage hikes.

Our second goal in developing the model is to have our empirical estimates of employment, entry, and exit tightly inform our calibrated long run disemployment effect and other potential responses to the minimum wage. Because our data only includes information on employment, we cannot estimate the production function for restaurants. Hence, by sparsely parameterizing the model, there is a clear mapping from our empirical estimates to the model. That said, we believe that a richer model would deliver similar answers to the questions we ask of our calibration exercise.

In particular, we ask three questions. First, are the implied parameter values plausible? Second, are the results from the model quantitatively consistent with other findings in the minimum wage literature? Third, is the calibrated short run disemployment effect different than the long run disemployment effect, and how quickly does the long run disemployment effect emerge? We view the last question as central, since Sorkin (2015) demonstrates that standard empirical techniques do not recover structural long-run employment elasticities.

This section sketches the key features and results of the model. Further details and proofs are in the appendix.

6.1 Production

Restaurants produce food using four inputs: capital, high-skill labor, low-skill labor, and materials. Capital includes land, structures, and machinery. Low-skilled labor is paid the minimum wage. A restaurant bundles inputs to produce initial output y_0.

Ex-ante, restaurants can flexibly substitute between inputs. Restaurants face a CES production function so that output at time 0 (the birth of the restaurant) is

$$y_0 = A_0(\alpha^k k^{\frac{\sigma-1}{\sigma}} + \alpha^m m^{\frac{\sigma-1}{\sigma}} + \alpha^h h^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\alpha = \alpha^k + \alpha^m + \alpha^h$ implies constant returns to scale, A_0 is the productivity of an entering restaurant, σ is the elasticity of substitution, k is capital, m is materials, h is high-skill labor, and l is low-skill labor.

Ex-post, the production function is Leontief and restaurants cannot substitute between inputs. Let k', m', h' and l' denote the initial input choices. In subsequent periods, restaurant optimization and constant returns to scale imply that the restaurant either operates with its original proportions at full capacity, or does not operate:

$$y_j = \begin{cases} A_j(\alpha^k k'^{\frac{\sigma-1}{\sigma}} + \alpha^m m'^{\frac{\sigma-1}{\sigma}} + \alpha^h h'^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l'^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} & \text{if } k \geq k', l \geq l', h \geq h', m \geq m' \\ 0 & \text{otherwise.} \end{cases}$$

This equation emphasizes two features of the restaurant's technology.

First, a restaurant's productivity is time-varying. Specifically, once it enters, a restaurant becomes deterministically less productive over time. A restaurant that is age j has total factor productivity (TFP) $A_j = A_0 e^{-\delta j}$, where δ is the deterministic TFP depreciation term. While this assumption is somewhat stark, it allows a transparent mapping from the empirical estimates to the model. As we discuss further in section 8, the exit elasticity pins down the depreciation parameter, δ. The reason is that δ determines how responsive exit is to the minimum wage hike. In particular, δ governs how many "marginal" restaurants there are. A high δ implies that there are few marginal restaurants and so a low exit elasticity, while a low δ implies that there are many marginal restaurants and so a high exit elasticity.

Second, given the Leontief assumption, a continuing restaurant either produces with its original factor mix ($\{k', m', h', l'\}$) or does not operate. While the notation allows the restaurant to use more of a factor than it did in its initial input mix, this alteration is never optimal (we discuss the exit decision below). Combining the rate of technology depreciation and the Leontief assumption, the output of an incumbent restaurant aged j is $y_j = y_0 e^{-\delta j}$.

6.2 Prices

Restaurants assume all prices remain constant over the life of the restaurant. We denote the price of the output good by P. We denote the rental prices of materials, high-skill labor, and low-skill labor (i.e., minimum wage) by p^m, w^h, and w, respectively.

We model capital as a partially irreversible investment, which generates an interesting exit decision. The restaurant purchases capital at price p^k and can resell the capital at price ηp^k, where $\eta < 1$. This resale discount means that immediately after spending $p^k k$ on capital, an amount $(1-\eta)p^k k$ is sunk. Because of this partial irreversibility, restaurants do not immediately shut down after their capital becomes less productive. We borrow this modeling device from Campbell (1998). The substantive assumption of partial irreversibility of capital investments has widespread empirical support; see, for example, Ramey and Shapiro (2001) and the cites therein.

6.3 Factor Demands

A restaurant makes two decisions at entry. First, it decides its input mix which is then fixed once capital is installed. This is a forward-looking decision that therefore considers the effective factor prices over the life of the restaurant. Second, it decides what exit rule to follow. For the moment take J, the age of restaurant exit, as given. We endogenize J in section 6.4.

Assuming an interest rate r, discounted payments over the life of a new restaurant for materials, high-skill labor, and low-skill labor are $q^m \equiv (\int_0^J e^{-rj} p^m dj)$, $q^h \equiv (\int_0^J e^{-rj} w^h dj)$, and $q^w \equiv (\int_0^J e^{-rj} w dj)$, respectively. Recall that capital can be purchased at price p^k and re-sold at price ηp^k. Therefore, discounted payments to a unit of capital are $q^k \equiv p^k(1 - e^{-rJ}\eta)$. Because a restaurant that initially produces y_0 at time 0 will produce $y_j = y_0 e^{-\delta j}$ at time j, total present discounted value of revenue over the life of the restaurant is $q^p y_0 = (\int_0^J e^{-(r+\delta)j} P dj) y_0$. Thus a restaurant's profit over its lifetime is:

$$\pi \equiv q^p A_0 (\alpha^k k^{\frac{\sigma-1}{\sigma}} + \alpha^m m^{\frac{\sigma-1}{\sigma}} + \alpha^h h^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} - q^w l - q^m m - q^h h - q^k k. \quad (5)$$

Consequently, an entering restaurant solves the following maximization problem:

$$max_{\{k,m,h,l,J\}}\pi \tag{6}$$

subject to equation (4), which implies the conditional factor demands, given the exit age J are:

$$\begin{aligned} l &= \frac{y_0}{\left[\alpha^k\left(\frac{\alpha^k}{1-\alpha}\frac{q^w}{q^k}\right)^{\sigma-1}+\alpha^m\left(\frac{\alpha^m}{1-\alpha}\frac{q^w}{q^m}\right)^{\sigma-1}+\alpha^h\left(\frac{\alpha^h}{1-\alpha}\frac{q^w}{q^h}\right)^{\sigma-1}+(1-\alpha)\right]^{\frac{\sigma}{\sigma-1}}}, \\ k &= l\left(\frac{\alpha^k}{1-\alpha}\frac{q^w}{q^k}\right)^{\sigma}, m = l\left(\frac{\alpha^m}{1-\alpha}\frac{q^w}{q^m}\right)^{\sigma}, h = l\left(\frac{\alpha^h}{1-\alpha}\frac{q^w}{q^h}\right)^{\sigma}. \end{aligned} \tag{7}$$

6.4 Exit Age

A restaurant exits when the marginal cost of producing exceeds the marginal benefit. The marginal costs of continued operation is $r\eta p^k k+hw^h+mp^m+lw$, where the first term reflects the shadow cost of staying open and thus delaying the sale of k units of capital at a price ηp^k. Because factor prices are assumed constant and input choices are fixed for the establishment's life, these costs are constant over the life of the restaurant. The flow of marginal benefits at age j are the revenue that the restaurant produces, $e^{-\delta j}y_0P$. However, unlike marginal costs, marginal benefits decline over the life of a restaurant because TFP falls as the restaurant ages.

Because marginal benefit declines while marginal cost remains constant over the life of the restaurant, eventually the restaurant will exit. The exit age J equates the marginal cost and marginal benefit of operating:

$$e^{-\delta J}Py_0 = r\eta p^k k + hw^h + mp^m + lw. \tag{8}$$

Substituting the restaurant's factor demands (equation 7) into the exit age equation (equation 8) results in one equation with two unknowns (P and J). The thin lines in Figure 1 show the determination of exit age of a restaurant for a given product price P, given parameter values we found to be consistent with the results presented earlier. In this particular case, the restaurant exits during year 18 as the marginal cost of operating exceeds the marginal benefit thereafter.

6.5 Market Price Determination

In steady state, free entry pins down the market price. Let f denote the steady state mass of entrants each period. The free entry condition indicates that any profit opportunities will be bid away by new entrants, implying either expected profits or entry is zero:

$$\pi \leq 0, f \geq 0, \text{ and } \pi f = 0. \tag{9}$$

This free entry condition is written in complementary slackness form since there is no entry when expected profits are negative. In steady state, however, there is entry and profits are zero. Plugging the conditional factor demands from equations (7) and setting $\pi = 0$ in equation (5), and also using equation (8), yields two equations with two unknowns (P and J). Although the analytic expressions for P and J are complicated, their solution is straightforward.

6.6 Market-Level Equilibrium

Having determined the restaurant's problem, we can solve for the total number of restaurants in a market. The industry faces an isoelastic product demand curve with elasticity γ:

$$Q = \theta P^{-\gamma}. \tag{10}$$

Product market clearing implies that quantity demanded equals quantity supplied, where the quantity supplied is:

$$Q = \int_0^J e^{-j\delta} y_0 f \, dj. \tag{11}$$

Market supply comes from restaurants of vintage j supplying quantity $e^{-\delta j} y_0$, the density of each vintage of restaurant (and the mass of entrants each period) f, and the mass of different vintages of incumbent restaurants J. Integrating (11) and rearranging provides an explicit solution to the steady state mass of restaurants that enter in every time period:

$$f = \frac{\delta Q}{y_0(1 - e^{-J\delta})}, \tag{12}$$

where Q is a function of P as in equation (10), and P and J are solved as in section 6.5.

6.7 Steady State Equilibrium

A steady state equilibrium is given by endogenous objects $\{k, h, m, l, Q, P, J, f\}$ taking factor prices $\{p^k, p^m, w^h, w, r\}$ and the environment $\{\delta, \eta, \theta, \gamma, \sigma, \alpha^k, \alpha^m, \alpha^h, y_0\}$ as given such that:

- Restaurants maximize profits, where profits are defined in equation (5)
- Free entry holds (equation 9)
- The product market clears (equation 11).

7 A Minimum Wage Hike

In this section, we consider a permanent but unexpected minimum wage increase from w_o to w_n at time t_n.[20] Such a hike affects employment through both a scale and substitution effect, sometimes referred to as the Hicks-Marshall channels.

When there is free entry and expected profits are zero, restaurants pass the higher labor costs to consumers in the form of higher prices. As a result, consumers purchase fewer meals and restaurants require fewer inputs. This reduction in sales causes net exit of restaurants immediately following a minimum wage hike and consequently an immediate fall in the employment of minimum wage workers (Aaronson and French (2007)). This channel is sometimes known as the "scale effect."

A hike in the minimum wage also makes low-skilled workers more expensive, causing restaurants to substitute to cheaper factors of production. However, in a putty-clay model, all substitution occurs through entry and exit. Because remaining incumbents maintain their input mix, the substitution effect occurs gradually as the incumbents exit and are replaced by new restaurants that are free to choose the optimal input mix given the higher price of minimum wage labor.

7.1 Exit Dynamics

Since incumbent restaurants are committed to their input mix, the only margin on which they can respond to higher labor costs is by exiting earlier (or later). In this section, we endogenize exit.

[20] Note t denotes calendar time whereas j denotes the age of a restaurant.

Let $J(w_o, w_n)$ be the exit age of a restaurant that entered when the minimum wage was w_o but is deciding to exit when the minimum wage is w_n. We rewrite the restaurant exit decision equation (8) as

$$e^{-\delta J(w_o,w_n)} P_n y_0 = r\eta p^k k_o + h_o w^h + m_o p^m + l_o w_n, \tag{13}$$

where the left hand side is the marginal benefit of continuing to operate at exit age $J(w_o, w_n)$, and the right hand side is the marginal cost of continuing to operate. Note that we assume the product price jumps to its new steady state P_n immediately. In the next section, we show when this assumption is satisfied.

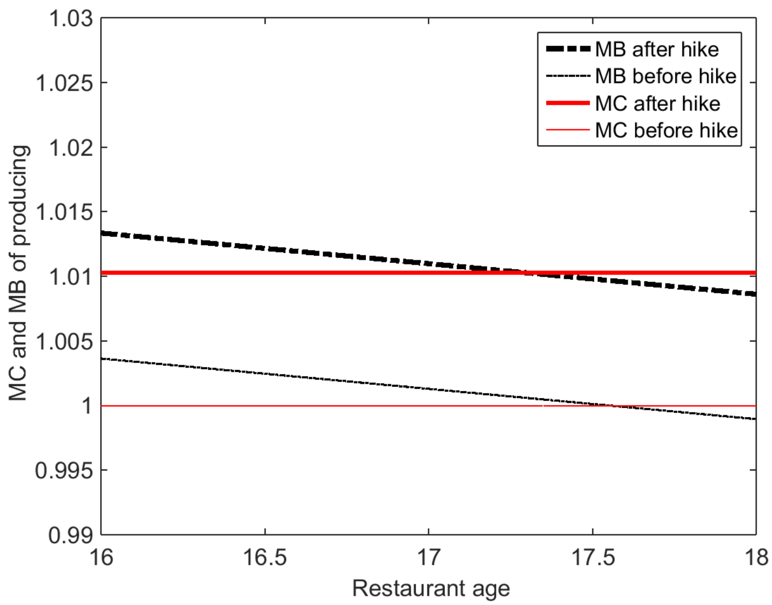
If $\sigma \leq 1$ and restaurants become more capital-intensive after a minimum wage hike (i.e., $\frac{dk}{dw} \geq 0$), which is the empirically relevant case, then incumbent restaurants respond to the hike by exiting early ($J(w_o, w_n) < J(w_o, w_o)$). Exit spikes as all incumbents between the ages of $J(w_o, w_n)$ and $J(w_o, w_o)$ simultaneously leave the market. This finding is proven in result 2 of Appendix D.

Figure 1 illustrates the exit decision of an incumbent restaurant, both before and after the minimum wage hike. The intersection of the marginal benefit and marginal cost curves determine the age at which the restaurant exits. The marginal benefit of operating at every age rises after the hike because the market price rises. This rise in the market price, however, is not enough to compensate the restaurant for an increase in the wage. Indeed, after the minimum wage hike, the marginal cost curve rises by enough that the restaurant exits earlier than it would have otherwise, i.e., $J(w_o, w_n) < J(w_o, w_o)$. In particular, there is a mass of restaurants caught between the old and the new exit age who exit early. This mass of restaurants produces the spike in exit.

Following the spike in exit, the density of restaurants exiting in a given period remains the same as before the minimum wage hike until all of the incumbent restaurants have exited. Appendix E provides a detailed discussion of exit dynamics.

As a result of the higher minimum wage, marginal cost curves rise for both incumbents and new entrants. However, because new entrants can substitute away from minimum wage labor, their marginal cost rises by less than for incumbents (entrants' marginal costs rise by 0.97 percent, whereas incumbents marginal costs rise by 1.03 percent). This relative cost disadvantage causes incumbents to exit.

Figure 1: Exit decision of incumbents after a 10 percent hike in the minimum wage



Note: Figure shows marginal benefit and marginal cost of restuarants both before and after a 10 percent minimum wage hike. The intersection of the marginal benefit and marginal cost curves determines the exit age. Marginal cost before the hike is normalized to 1. Parameter values used in the calibration are shown in Table 4.

7.2 Entry and product price response

This subsection discusses when and why product prices jump immediately to their new steady state level, as in equation (13) and consistent with the empirical findings of Aaronson (2001) and Aaronson, French, and MacDonald (2008) among others.

Product prices jump instantaneously if there is entry. The free entry condition (equation (9)) implies that the profits of new entrants are zero. Because profits depend only on product and factor prices, for there to be zero profits, a change in factor prices is instantaneously transmitted to the product price. Importantly, because the restaurant's decision depends only on product and factor prices, the distribution of incumbent restaurants does not affect the product price. This feature of the model greatly simplifies the solution and equilibrium computation, and means that the equilibrium conditions defined in section 6.7 hold in and out of steady state. The directed search literature (e.g., Menzio and Moen (2010) and Menzio and Shi (2011)) uses the free entry condition in a similar way. Furthermore, the model implies a spike in entry and exit $J(w_n, w_n)$ days after the shock, as well as $2 \times J(w_n, w_n)$,

$3 \times J(w_n, w_n), etc.$ days after the shock. Indeed, the model never returns to steady state. However, this is not a problem because our equilibrium conditions hold in and out of steady state.

The product demand curve (equation 10) then determines how the change in product prices map into changes in product quantity and entry. The distribution of incumbent restaurants matters for entry because it determines how many restaurants exit given the minimum wage hike. The condition for there to be entry following the minimum wage hike is that the extent of quantity exiting the market exceeds the drop in the market clearing quantity coming from the jump in the product price. Letting P_n be the new steady state product price and $Q_n = \theta P_n^{-\gamma}$ be the new steady state market output, Appendix D shows that, to a first-order approximation, market output drops instantly from Q_o to Q_n when:

$$\frac{J(w_o, w_o) - J(w_o, w_n)}{J(w_o, w_o)} \geq \frac{Q_o - Q_n}{Q_o}.$$

The left hand side is the percent of incumbent restaurants that exit, and the right hand side is the percent change in market quantity. If the percent of restaurants exiting is greater than the percent change in market output, new restaurants must enter to fill the gap. More details on entry behavior can be found in Appendix E.

7.3 Chains versus non-chains in the context of the model

The results in Table 2 suggest that entry and exit are more responsive among chains. This empirical result is consistent with the putty-clay model for two reasons. First, chains appear to be more capital intensive than non-chains. According to a survey conducted by the National Restaurant Association (NRA),[21] the compensation to sales ratio per full-time equivalent employee is 0.26 for chains and 0.30 for non-chains (Table D-6). Adjusted for seating capacity, the difference is more dramatic: 0.21 for chains versus 0.33 for non-chains (Table D-12). The NRA survey responses among chains are similar to financial 10-K reports for 22 large limited service restaurant chains, which on average report a payroll to revenues ratio of 24 percent.[22] This gives additional confidence in the quality of the survey

[21] See the 2013-2014 NRA Restaurant Operations Report at http://www.restaurant.org/News-Research/Research/Operations-Report

[22] We found these companies, which do not correspond to the chains we identify in the QCEW, by using a list of the largest restaurant companies published by Nation's Restaurant News (www.nrn.com). We excluded full service restaurant companies, as well as those that did not file a 10-K in 2014.

responses. The entry of chains is consistent with the hypothesis that the new entrants are more capital-intensive.

Second, chains are likely to be less flexible in response to minimum wage hikes than non-chains. Chains typically have national operating manuals. For example, "[t]he McDonald's operations manual dictates every move made inside one of its restaurants."[23] Because of these formal procedures, chains are likely to be less flexible than non-chains in responding to location-specific cost shocks like a minimum wage hike.

We conjecture that an extension of the model that allows chains to be less flexible than non-chains, in combination with greater heterogeneity in capital intensity in chains, would result in a larger spike in exit and entry for chains than non-chains. The additional exit would be concentrated among the inflexible labor intensive chains, whereas the additional entry would be concentrated among the more capital intensive restaurants, which are more likely to be chains.

8 Calibration

Because the only input that we observe at the restaurant-level is labor—and we do not have any measure of output— estimating the production function is infeasible. Instead, we calibrate the model.

Our calibration proceeds in two steps. First, we select parameter values $\{\sigma, p^m, p^k, w, w^h, r\}$ for which there are well-agreed-upon values in the literature. Panel A in Table 4 report those values. Second, we choose 6 parameters $\{\alpha^k, \alpha^m, \alpha^h, \eta, \delta, \gamma\}$ to match 6 moments – the unique factor shares $\{s^m, s^k, s^h\}$,[24] the average lifespan of a restaurant J, and the elasticities of entry and exit with respect to the minimum wage hike using a minimum distance estimator.

While we draw upon Aaronson and French (2007), we augment their calibration targets to accommodate the more sophisticated dynamics in this model. The calibration targets are listed in Table 5, the parameters chosen to match those targets are in panel B of Table 4, and more detail is provided in Appendix A. As is standard with CES technology, the moments that identify the α^k, α^m, and α^h parameters are the factor shares s^m, s^k, and s^h. Aaronson and French (2007) use financial reporting data to obtain s^m and s^k and use Current

[23] See see http://www.forbes.com/forbes/2004/0329/058.html.

[24] $s^l = 1 - (s^m + s^k + s^h)$ and thus does not contribute any useful information.

Table 4: Calibration

Parameter	Value	Description	Source
	A. Exogenously set parameters		
w	1	Minimum wage	Normalization
p^k	1	Capital price	Normalization
p^m	1	Materials price	Normalization
w^h	2.76	High skill wage	Aaronson and French (2007)
σ	0.80	Elasticity of substitution	Aaronson and French (2007)
r	0.05	Interest rate	Standard
	B. Parameters chosen to match targets*		
δ	0.002	Depreciation rate	
η	0.95	Resale price	
γ	0.57	Elasticity of product demand	
α^k	0.49	Productivity of capital	Match s_k
α^h	0.11	Productivity of h labor	Match s_h
α^m	0.34	Productivity of materials	Match s_m

Note: Targets shown in table 5.

Table 5: Calibration targets

Moment	Target	Result	Description	Source
s^k	0.30	0.30	Capital share	Aaronson and French (2007)
s^h	0.20	0.20	High-skill labor share	Aaronson and French (2007)
s^m	0.40	0.40	Materials share	Aaronson and French (2007)
Exit Spike	2.40	2.40	Elasticity of exit with respect to w	This paper
Entry Spike	1.37	1.37	Elasticity of entry with respect to w	This paper
J	17.54	17.54	Average life of a restaurant	This paper

Population Survey (CPS) to obtain the share of labor cost that is paid to workers making above the minimum wage. Since the calibration of δ, η, and γ is less standard, we outline our reasoning in more detail.

The moment that identifies the depreciation rate, δ, is the *exit* elasticity with respect to the minimum wage. All else equal, δ determines the slope of the marginal benefit curve (see Figure 1). If δ is small, then the restaurants' productivity, and consequently marginal benefit of producing, declines slowly over time. In that case, productivity levels are similar for many incumbents, including those that are close to exiting. Thus a small hike causes many restaurants to exit. In contrast, a steep marginal benefit curve (high δ) means that few restaurants are close to exiting and the exit elasticity is small.

The moment that identifies the resale price of capital (η) is the steady state exit age J. All else equal, η determines the level of the marginal cost curve shown in Figure 1. When the resale price is low, the opportunity cost of re-selling capital is low and thus the marginal cost of operating is low as well. When marginal cost is low, the restaurant remains open longer. In contrast, a high marginal cost curve (high η) signifies that the opportunity cost of operating is high and restaurants exit at a younger age.

Finally, the moment that identifies the elasticity of demand for restaurant output (γ) is the entry elasticity. Because price pass-through immediately follows a minimum wage hike, all else equal, γ determines the change in market quantity and hence output. A low γ indicates that output is unresponsive to a minimum wage hike and most exiting output is replaced by entry. A high γ means that output is very responsive to a minimum wage hike and therefore a spike in entry is unlikely.

8.1 Three questions of the calibrated model

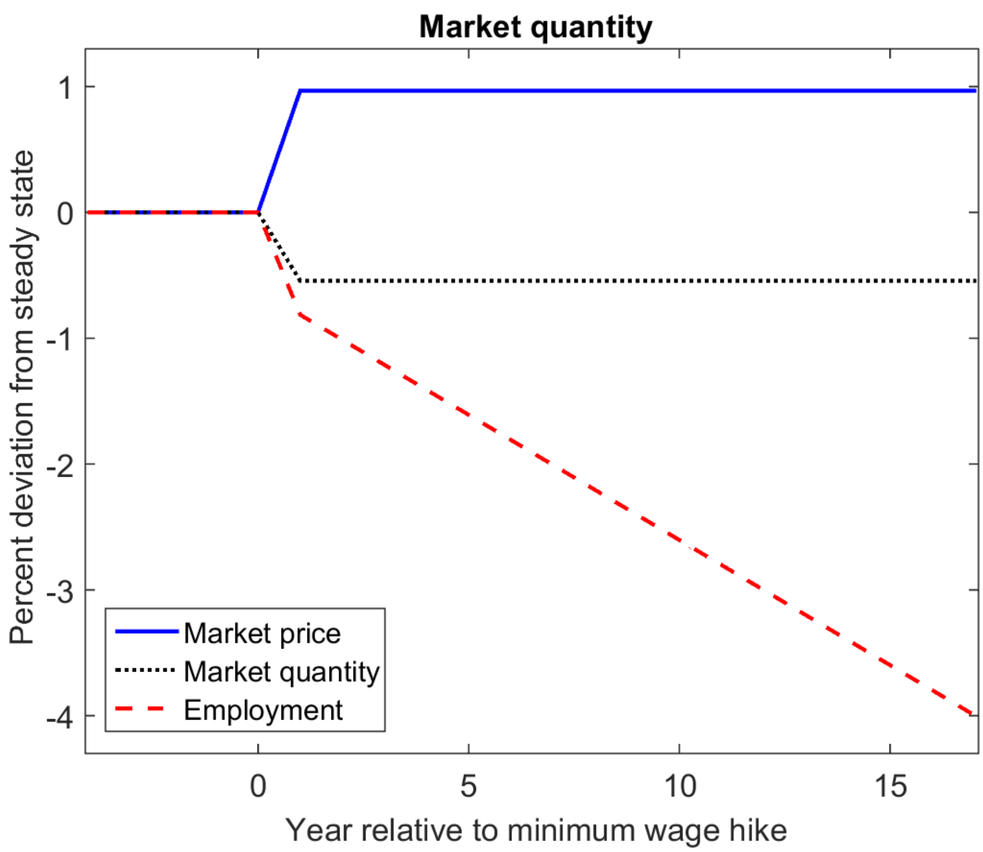
We ask three questions of our calibrated model.

First, are the implied parameter values plausible? We find that the answer is yes. The model interprets the estimated spike in exit and entry rates after the minimum wage as a small δ and a high η (Table 4, panel B). Although our calibrated value of δ is lower and our calibrated value of η is higher than many estimates in the literature, this disparity is to be expected since much of the productivity of a restaurant is derived from its land and location, which may not decline much over time. Because both the entry and exit elasticities are of similar size, the total disemployment effect is small. Our product demand elasticity, γ, is similar to Aaronson and French's (2007) preferred value of 0.5, albeit higher than the 0.2 estimated in Harasztosi and Lindner (2015).

Second, is our model quantitatively consistent with other findings in the minimum wage literature? Again, we find the answer is yes. Figure 2 shows industry price, quantity and employment in the 17 years following a 10 percent one-time, unanticipated and permanent minimum wage increase from steady state.[25] Because of entrants, the product price jumps to the new steady state immediately, about 0.97 percent higher than before the hike. That price increase implies an elasticity of 0.097, in line with the evidence discussed in Aaronson

[25] The hike occurs at time 0. We aggregate the model's predicted response to an annual frequency to be consistent with the data.

Figure 2: Market-level variables after a 10 percent minimum wage hike



Note: The minimum wage rises 10 percent immediately after time 0. We aggregate the data to an annual frequency. Panels depict the percent change in market prices for the output good, market quantity, and employment, relative to their levels before the hike. Employment is employment of high and low skill workers.

and French (2007). Because the price jumps immediately, the industry quantity drops to its new steady state level as well. After one year, total employment (including both high- and low-skill workers) falls by 0.8 percent, implying an elasticity of -0.08. This short-run employment response is in line with both our estimates in this paper, as well as recent work studying the restaurant industry, such as Dube, Lester, and Reich (2010), Neumark, Salas, and Wascher (2014), and Allegretto et al. (2016).[26]

Although the free entry condition means there are zero profits for entrants, both before and after the hike, incumbent profits and market value drop in response to a minimum wage hike, consistent with the empirical evidence in Draca, Machin, and Reenen (2011) and Bell and Machin (2016) respectively. In particular, we find that the elasticity of incumbent restaurant value with respect to a 10 percent minimum wage hike is -0.04. The elasticity of accounting profit among the incumbent restaurants (calculated as revenue less payments to labor and materials) with respect to the same minimum wage hike is -0.01. See appendix F for calculation details.

Third, what is the *timing* of the effect of minimum wage hikes? We find that the short-run employment effect captures only a small share of the long-run employment effect generated in the model. Because restaurants turn over slowly following a minimum wage hike, the full employment effect of the minimum wage also unfolds slowly. The employment response grows over time such that in the steady state determined by the new minimum wage, the long-term elasticity is -0.40, or five times the short-run employment elasticity of -0.08 (table 6, row 1). Allegretto et al. (2016) also find some evidence that the disemployment effect among restaurants grows over time, although their estimates up to 5 years after the hike are smaller than our predicted long-run effects and often not statistically different from their own short-run estimates.

To understand the timing of the disemployment effects, recall the two Hicks-Marshall channels by which employment falls. The first is the scale effect. Free entry implies that expected profits are 0, so restaurants will pass the higher labor costs to consumers in the form of higher prices. As a result, consumers will purchase fewer meals and restaurants will require fewer inputs. Because the price increase is instantaneous–for reasons discussed in

[26] It is worth emphasizing that we report the elasticity of total restaurant employment with respect to the minimum wage. Thus, this elasticity combines the decrease in low-skill labor with a smaller increase in high-skill labor. We do this is to be comparable to most studies which, like ours, measure the disemployment effect of restaurant industry employment with respect to the minimum wage.

Table 6: Short and long run disemployment effects

		SR employment	LR employment
1.	Baseline	-0.08	-0.40
2.	Alternate elasticity of substitution: $\sigma = 0.4$	-0.07	-0.24
3.	Alternate factor shares: $s^l = .051$	-0.07	-0.24
4.	Both: $\sigma = 0.4$ and $s^l = 0.051$	-0.06	-0.15

Note: This table reports the elasticity of employment with respect to the minimum wage. SR is the employment elasticity one year after the minimum wage hike; LR is the employment elasticity after the market has fully adjusted to the minimum wage hike (approximately 18 years). In the alternate factor share calibration, minimum wage workers are 30 percent of all workers.

section 7.2— the scale effect occurs instantaneously also. Quantitatively, the size of the scale effect is pinned down by the share of costs from low-skill labor and the product demand elasticity γ. In the calibration, we use external information to calibrate the share of low-skill labor costs, and γ is pinned down by our estimates of the entry elasticity. Specifically, the scale effect implies an employment elasticity of -0.054, or over half of the total employment effect in the first year.

The second Hicks-Marshall channel is the substitution effect. A hike in the minimum wage makes low-skilled workers more expensive, causing restaurants to substitute to cheaper factors of production. Quantitatively, the size of the substitution effect is pinned down by the factor shares and the elasticity of substitution. This channel only occurs through entry and exit of restaurants since there is no scope for continuing restaurants to substitute away from minimum wage labor. It is for this reason that the short run and long run disemployment effects are different. Some of the substitution to more capital intensive restaurants occurs immediately because of the jump in entry and exit. However, as it turns out, this effect is small. Specifically, in the first year following the hike, the employment elasticity through the substitution effect is -0.027. Endogenous churn contributes only a small amount to shifting the timing of the employment effects. More importantly, the rest of the disemployment unfolds slowly over the next 16 years as the remaining incumbent restaurants exit and are replaced.

Table 6 provides robustness checks concerning two key parameters in our model, σ and s^l, that are not easily measured. Since much of the disemployment response comes from the substitution channel, our results depend fundamentally on the assumed elasticity of substitution σ. We use a value of 0.8, taken from Aaronson and French (2007), but lower values of σ would lead to smaller disemployment effects in the long-run. In row 2, we report one such exercise, where $\sigma = 0.4$, as in Harasztosi and Lindner (2015). The disemployment effect in the first year is -0.07 and in the long-run is -0.24. While the long-run effect is smaller than our baseline of -0.40, it is still over three times larger than the short-run effect.

The calibrations are also somewhat sensitive to assumptions about the share of costs attributable to minimum wage labor, s^l (and thus the share of all workers who are paid the minimum wage). As we describe in appendix A, there are reasons to believe that our baseline assumption of $s^l = 0.10$ may be too high or too low. Row 3 of table 6 provides one assessment of how the employment elasticities could vary by setting $s^l = 0.051$, our estimate of its likely lower bound value.[27] In this case, the short- and long- run employment elasticities are -0.07 and -0.24.[28] If we set $\sigma = 0.4$ and $s^l = 0.051$, the long-run employment elasticity falls to -0.15, or about $2\frac{1}{2}$ times larger than the short-run employment elasticity. These estimates should be interpreted with caution, however, as the elasticity of product demand is pushed above 1, which is likely too high based on Harasztosi and Lindner (2015) and Aaronson and French (2007).

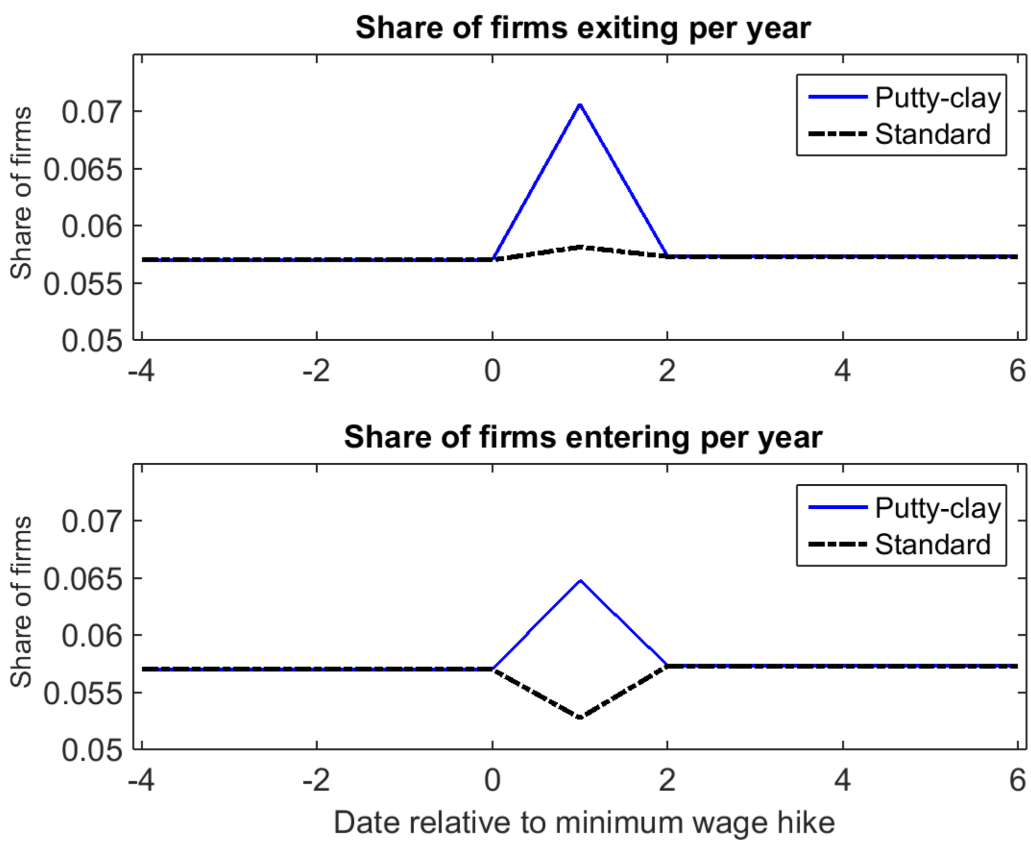
Regardless of the precise parameter values chosen, perhaps as little as twenty to thirty percent of the employment response generated in our model occurs in the first year. We believe this has important implications for assessing the consequences of minimum wage hikes. Using short-run employment responses to evaluate the implications of minimum wage hikes, as is standard in the literature and among policymakers, may understate the negative employment effects and overstate the effectiveness of minimum wage hikes as a redistributive tool.[29] As Sorkin (2015) emphasizes, it is not easy to read the long-run effects of minimum wage hikes off of simple regressions, so a model based exercise is informative.

[27]See Appendix A. Of particular note, Aaronson, French, and MacDonald (2008) estimate a price elasticity for all restaurants of 0.071 but 0.155 among limited service restaurants only. Aaronson (2001) also finds a sizable difference between the price responses in limited and full service establishments, consistent with economically important disparities in the usage of minimum wage labor across subsector.

[28]Aaronson and French (2007) use only data for the factor shares from public restaurants which must file 10k reports, whereas our estimates in tables 2 and 3 of this paper are from all firms. As we pointed out in section 7.3, there are some differences in capital shares between chains (which are more likely to be publicly traded companies) versus non-chains. If anything, this leads us to use too small of a value for labor's share in the calibrations. Using a larger labor's share would yield a larger short run effect but a (slightly) smaller long run effect.

[29]In the model, the elasticity of the earnings of workers as a whole with respect to the minimum wage hike is one minus the employment elasticity. So long as the employment elasticity is less than 1, the minimum wage hike increases the income of workers.

Figure 3: Share of restaurants entering and exiting after a 10 percent minimum wage hike, putty-clay model versus standard model



Note: The solid blue line (labeled "putty-clay") depicts the entry and exit behavior using our calibrated putty-clay model. The black dotted line ("standard") depicts entry and exit behavior when restaurants can adjust their factor demands after a minimum wage hike. The exit share is the share of restaurants in operation a year ago that are not currently in operation. The entry share is the share of restaurants not in operation a year prior that are in operation. In these calibrations, the minimum wage is boosted by 10 percent immediately after time 0.

8.2 The contribution of putty-clay to entry and exit behavior

Finally, to highlight the role of the ex-post inflexibility built into the putty-clay model in generating the spike in exit and entry, we consider an alternative model where incumbent restaurants can re-optimize their factor mix after the hike. The models are otherwise identical. Figure 3 contrasts entry and exit behavior of the putty-clay model with this more standard alternative. In the absence of putty-clay technology (where restaurants can re-optimize their factor mix after the hike), there is still an increase in exit after the minimum wage hike if $\sigma < 1$. However, the increase is barely perceptible. Since the exit response generated by the model without putty-clay technology is smaller, the entry response will be smaller as well. In fact, entry drops. This decline is a robust qualitative feature of a model without putty-clay technology, highlighting that putty-clay is central to understanding a rise in entry.

9 Conclusion

We present new evidence on the effect of minimum wage hikes on establishment entry, exit, and employment among employers of low-wage labor. We show that small net employment changes in the restaurant industry may hide a significant amount of establishment churning that arises in response to a minimum wage hike. To capture these dynamics, we develop a putty-clay model with endogenous entry and exit. The key feature of the putty-clay model is that, after entry, technology and input mix is fixed for the life of the restaurant. After minimum wage hikes, inflexible incumbents are replaced by potential entrants who can optimize on input mix. Thus, the model is capable of predicting both restaurant entry and exit in response to a minimum wage hike.

Furthermore, we show that the putty-clay model generates employment and output price responses to minimum wage hikes that are consistent with those reported in the literature. In particular, the model predicts that restaurant prices are immediately and fully passed onto consumers in the form of higher prices, again consistent with the literature. Similarly, putty-clay yields sluggish employment responses to minimum wage hikes, with a short-run disemployment effect of just under -0.1 that likely grows by three to five times in the long-run. This finding has important implications for evaluating the implications of minimum wage hikes, especially since most empirical studies concentrate solely on short-run responses.

Other models, such as those that incorporate adjustment costs, can reconcile some of these facts but not others, especially the simultaneous rise of exit and entry. As such, we believe putty-clay models could be potentially useful for understanding the response to other labor market policies, including taxes, hiring subsidies, and firing costs and we view our paper as a novel contribution in that we provide micro level evidence on the empirical relevance of putty-clay in an important policy setting.

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Appendix A: Calibration

This appendix details the parameter values we use in the calibration exercise. It borrows heavily from Aaronson and French (2007).

Factor Shares, s^l, s^h, s^m, s^k There are a number of sources for labor share, all of which tend to report similar numbers for the food away from home industry. First, 10-K company reports contain payroll to total expense ratios. Of the 17 restaurant companies that appear in a search of 1995 reports using the SEC's Edgar database, the unconditional mean and median of this measure of labor share is 30 percent and it ranges from 21 to 41 percent.[30] These numbers are in-line with a sampling of 1995 corporate income tax forms from the Internal Revenue Service's Statistics on Income Bulletin. Because operating costs are broken down by category, it is possible to estimate labor's share.[31] According to these tax filings, labor cost as a share of operating costs for eating place partnerships is roughly 33 percent. Consequently, we set $s^l + s^h$ to 30 percent.

We are particularly interested in labor share in low wage restaurants. We use the 1997 Economic Census for Accommodations and Food Services, which reports payroll for full service (FS) and limited service (LS) restaurants. LS includes fast-food stores and any restaurant without sit-down service and where customers pay at the counter prior to receiving their meals. They tend to be the primary employer of minimum wage labor. According to this 1997 census, labor share, as a fraction of sales, is slightly higher at FS (31 percent) than LS (25 percent) stores.[32] Therefore, there is little evidence of a significant difference in labor share across establishment type.

Aaronson and French (2007) use Current Population Survey data to show that $\frac{1}{3}$ of restaurant industry workers are paid less than 150 percent of the minimum wage, and are thus likely to be affected by the minimum wage. This group accounts for 17 percent of the wage bill in the restaurant industry. Given these shares, Aaronson and French (2007) argue that the share of costs from minimum wage labor in the total restaurant industry is likely between 0.05 and 0.10. To derive the higher value suppose all restaurants either pay none or all

[30] The search uses five keywords: restaurant, steak, seafood, hamburger, and chicken.

[31] The IRS claims that labor cost is notoriously difficult to decompose for corporations and therefore we restrict our analysis to partnerships, where there is less concern about reporting.

[32] Several 10-K reports of individual restaurant companies show that wages account for 85 percent of compensation. Therefore, labor's share based on compensation is roughly 36 and 29 percent at full and limited service restaurants.

their workers the minimum wage and also that all restaurants have the same sized workforce. Combining these assumptions and that 33 percent of all workers are paid the minimum wage, then 33 percent of all restaurants pay the minimum wage and 67 percent do not. Thus, the average minimum wage labor's share at all restaurants is $33\% \times \ 30\% + 67\% \times 0\% = 0.099$. As an alternative assumption, suppose there are multiple labor types at each restaurant, and that all restaurants have identical factor shares, including for above minimum wage labor and for minimum wage labor. Then, each restaurant must have 17 percent of its labor costs going to minimum wage labor and therefore minimum wage labor share is $30\% \times \ 17\% = 0.051$ at every restaurant. We believe the correct estimate of minimum wage labor share in the restaurant industry is somewhere in between 0.05 and 0.10. Indeed Aaronson, French, and MacDonald (2008), find that prices rise by 7.1 (standard error of 1.4) percent in response to a 10 percent minimum wage hike, which should approximately equal the average minimum wage labor's share in the putty-clay model. As a baseline, we set $s^l = 0.1$ and $s^h = 0.2$. We chose to use a higher value of s^l to acknowledge that limited service establishments will have a higher minimum wage labor share than the average of all restaurants.[33] However, we also show results where we set $s^l = 0.051$ and $s^h = 0.249$.

We should note that these values are for the restaurant industry. Both minimum wage labor's share, and also the the share of all workers paid the minimum wage are likely higher in the fast food industry.

Based on the same sample of company financial reports used to compute $s^l + s^h$, we assume that capital's share is 30 percent and material's share is 40 percent.

The Elasticity Parameter σ Aaronson and French (2007) could not find estimates of the elasticity of substitution σ for restaurants specifically so instead uses 0.8, which a review of the literature suggests is an average estimate across all industries.

Targets: age, exit and entry elasticity The exit age of a restaurant, J, is picked to match the average exit probability of 0.057 (see appendix Table A5): $J = \frac{1}{0.057} = 17.54$ years. The entry and exit elasticities are from table 2.

[33]While Aaronson, French, and MacDonald (2008) estimate a price elasticity for all restaurants of 0.071, they find a price elasticity of 0.155 among limited service restaurants, roughly five times larger than the 0.032 price elasticity for full service restaurants. Aaronson (2001) also finds a sizable difference between the price responses in limited and full service establishments, consistent with economically important disparities in the usage of minimum wage labor across subsector.

Appendix B: Comparative Static Result: Product Price

This appendix first derives the explicit expression for the market price almost in terms of model fundamentals (the exit age J is left implicit). The appendix then solves for the elasticity of product price with respect to the minimum wage.

The (effective) product price

Free entry implies that the maximand in (6) is equal to zero. Substituting in the equilibrium factor demands from equation (7) and the definition of y_0 from equation (5) to the maximand in equation (6) set equal to zero:

$$q^p y_0 = q^k k + q^m m + q^h h + q^w l \tag{A1}$$

$$q^p = \frac{\left(q^k\left(\frac{q^w}{q^k}\frac{\alpha^k}{1-\alpha}\right)^{\sigma} + q^m\left(\frac{q^w}{q^m}\frac{\alpha^m}{1-\alpha}\right)^{\sigma} + q^h\left(\frac{q^w}{q^h}\frac{\alpha^h}{1-\alpha}\right)^{\sigma} + q^w\right)}{\left(\alpha^k\left(\frac{q^w}{q^k}\frac{\alpha^k}{1-\alpha}\right)^{\sigma-1} + \alpha^m\left(\frac{q^w}{q^m}\frac{\alpha^m}{1-\alpha}\right)^{\sigma-1} + \alpha^h\left(\frac{q^w}{q^h}\frac{\alpha^h}{1-\alpha}\right)^{\sigma-1} + (1-\alpha)\right)^{\frac{\sigma}{\sigma-1}}}. \tag{A2}$$

We now want to simplify each term. For example, the term involving low-skill wages simplifies as follows:

$$\frac{q^w}{1-\alpha}\alpha^k\left(\frac{\alpha^k}{1-\alpha}\frac{q^w}{q^k}\right)^{\sigma-1} = q^k\left(\frac{q^w}{q^k}\frac{\alpha^k}{1-\alpha}\right)^{\sigma}. \tag{A3}$$

Exploiting analogous simplifications on each term in (A1) gives the effective product price:

$$q^p = \frac{q^w}{1-\alpha}\left(\alpha^k\left(\frac{\alpha^k}{1-\alpha}\frac{q^w}{q^k}\right)^{\sigma-1} + \alpha^m\left(\frac{\alpha^m}{1-\alpha}\frac{q^w}{q^m}\right)^{\sigma-1} + \alpha^h\left(\frac{\alpha^h}{1-\alpha}\frac{q^w}{q^h}\right)^{\sigma-1} + (1-\alpha)\right)^{\frac{-1}{\sigma-1}}. \tag{A4}$$

To convert the effective product price to the product price, explicitly solve the expression relating these two prices given in the paragraph above equation (5):

$$q^p = \int_0^J e^{-(r+\delta)j} P dj \tag{A5}$$

$$q^p \frac{r+\delta}{1-e^{-(r+\delta)J}} = P. \tag{A6}$$

Combining equations (A4) and (A5) gives an explicit expression for the product price:

$$P = \frac{r+\delta}{1-e^{-(r+\delta)J}} \frac{q^w}{1-\alpha} \left[\alpha^k \left(\frac{q^w}{q^k} \frac{\alpha^k}{1-\alpha} \right)^{\sigma-1} + \alpha^m \left(\frac{q^w}{q^m} \frac{\alpha^m}{1-\alpha} \right)^{\sigma-1} + \alpha^h \left(\frac{q^w}{q^h} \frac{\alpha^h}{1-\alpha} \right)^{\sigma-1} + (1-\alpha) \right]^{\frac{-1}{\sigma-1}}. \tag{A7}$$

Response of product price to a minimum wage hike

We are interested in the effect of a change in the low-skill wage on the price level. The effective low-skill wage, $q^w = \frac{1-e^{-rJ}}{r} w$, depends on w directly and indirectly through J, the exit age, because it depends on w. We study the effect of w on J in appendix C. To see where J enters the expression, substitute in the definitions of the effective prices in the paragraph above equation (5) into (A7). To keep the expression somewhat more compact, define:

$$\hat{k}(J,w) = \left(\frac{w}{p^k} \frac{\frac{1-e^{-rJ}}{r}}{1-e^{-rJ}\eta} \frac{\alpha^k}{1-\alpha} \right)^{\sigma}, \hat{m}(w) = \left(\frac{w}{p^m} \frac{\alpha^m}{1-\alpha} \right)^{\sigma}, \text{ and } \hat{h}(w) = \left(\frac{w}{p^h} \frac{\alpha^h}{1-\alpha} \right)^{\sigma}.$$

Then the product price depends on the flow prices and exit age (J) as follows, where in this expression only we write $J(w)$ to emphasize the dependence of J on w:

$$P = \frac{r+\delta}{r} \frac{1-e^{-rJ(w)}}{1-e^{-(r+\delta)J(w)}} \frac{w}{1-\alpha} \left[\alpha^k \hat{k}(J(w),w)^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}(w)^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}(w)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \right]^{\frac{-1}{\sigma-1}}.$$

Take the derivative of the product price with respect to the low-skill wage:

$$\begin{aligned}
\frac{\partial P}{\partial w} = & \left\{ \frac{r+\delta}{r} \frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}} \frac{1}{1-\alpha} + \frac{r+\delta}{r} \frac{w}{1-\alpha} \frac{\partial \frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}}}{\partial J} \frac{\partial J}{\partial w} \right\} \\
& \times \left[\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \right]^{\frac{-1}{\sigma-1}} \\
& - \frac{r+\delta}{r} \frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}} \frac{w}{1-\alpha} \left[\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \right]^{\frac{-1}{\sigma-1}-1} \\
& \times \left[\alpha^k \frac{1}{w} \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \frac{1}{w} \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \frac{1}{w} \hat{h}^{\frac{\sigma-1}{\sigma}} + \frac{1}{\sigma} \alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}-1} \left(\frac{\partial \hat{k}}{\partial J} \frac{\partial J}{\partial w} \right) \right].
\end{aligned} \tag{A8}$$

Convert to an elasticity (the expression for $\frac{w}{P}$ comes from rearranging (A7)):

$$\begin{aligned}\frac{\partial P}{\partial w}\frac{w}{P} &= \frac{1-\alpha}{\left[\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right]} \\ &\quad - \frac{\frac{1}{\sigma}\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}}\left(\frac{\partial \hat{k}}{\partial J}\frac{J}{\hat{k}}\frac{\partial J}{\partial w}\frac{w}{J}\right)}{\left[\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right]} + \frac{\partial \frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}}}{\partial J}\frac{J}{\frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}}}\frac{\partial J}{\partial w}\frac{w}{J}. \qquad \text{(A9)}\end{aligned}$$

To simplify this expression further, derive expressions for some steady state factor shares. For low-skill labor:

$$\begin{aligned} s_L &= \frac{q^w}{q^k\hat{k} + q^m\hat{m} + q^h\hat{h} + q^w} \qquad \text{(A10)} \\ &= \frac{1-\alpha}{\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)} \qquad \text{(A11)}\end{aligned}$$

For capital:

$$s_K = \frac{\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}}}{\alpha^k \hat{k}^{\frac{\sigma-1}{\sigma}} + \alpha^m \hat{m}^{\frac{\sigma-1}{\sigma}} + \alpha^h \hat{h}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)}. \qquad \text{(A12)}$$

Hence, substituting the factor shares ((A10) and (A12)) into (A9) gives the following expression for the elasticity of the product price with respect to the low-skill wage:

$$\frac{\partial P}{\partial w}\frac{w}{P} = s_L - \frac{1}{\sigma}s_K\left(\frac{\partial \hat{k}}{\partial J}\frac{J}{\hat{k}}\frac{\partial J}{\partial w}\frac{w}{J}\right) + \frac{\partial \frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}}}{\partial J}\frac{J}{\frac{1-e^{-rJ}}{1-e^{-(r+\delta)J}}}\frac{\partial J}{\partial w}\frac{w}{J}.$$

The dependence of the exit age on the low-skill wage introduces two additional terms relative to the standard result that the product price elasticity is s_L.

Appendix C: Response of Steady State Exit Age J to Minimum Wage Hike

Start with the exit condition for a restaurant, equation (8), which equates the marginal cost and the marginal benefit of operating in the period. Everything in this expression except for J can be written in terms of model primitives. Thus, the expressions give an implicit equation for J:

$$\underbrace{\frac{r+\delta}{r}\frac{e^{-\delta J}-e^{-(r+\delta)J}}{1-e^{-(r+\delta)J}}}_{a}$$

$$\times\frac{1}{1-\alpha}\left(\alpha^k\underbrace{\left(\frac{\alpha^k}{1-\alpha}\frac{1}{rp^k}\frac{1-e^{-(r)J}}{1-\eta e^{-rJ}}\right)^{\sigma-1}}_{b}+\alpha^m\left(\frac{\alpha^m}{1-\alpha}\frac{1}{p^m}\right)^{\sigma-1}+\alpha^h\left(\frac{\alpha^h}{1-\alpha}\frac{1}{w^h}\right)^{\sigma-1}\right)=$$

$$r\eta p^k\underbrace{\left(\frac{\alpha^k}{1-\alpha}\frac{1}{rp^k}\frac{1-e^{-(r)J}}{1-\eta e^{-rJ}}\right)^{\sigma}}_{c}+\left(\frac{\alpha^h}{1-\alpha}\frac{1}{w^h}\right)^{\sigma}w^h+\left(\frac{\alpha^m}{1-\alpha}\frac{1}{p^m}\right)^{\sigma}p^m+\underbrace{\left(1-\frac{e^{-\delta J}-e^{-(r+\delta)J}}{1-e^{-(r+\delta)J}}\frac{r+\delta}{r}\right)}_{d}\underbrace{w^{1-\sigma}}_{e}. \tag{A13}$$

We use this expression for J to characterize the response of J to a change in w.

Result 1. *When $\sigma < 1$, $\frac{\partial J}{\partial w} < 0$. When $\sigma = 1$, $\frac{\partial J}{\partial w} = 0$.*

Proof. The proof is by contradiction. It relies on facts collected in Table A1. The table shows what happens to the terms in equation (A13) that depend on J and w following an increase in w when $\sigma < 1$ under two different assumptions on J: first if J increases and second if J stays constant. Straightforward (though tedious) calculations sign the derivatives in column (4).

Suppose that $\frac{\partial J}{\partial w} > 0$ so that both w and J increase simultaneously. Column (2) of Table A1 shows that the equality no longer holds since the left hand side of equation (A13) decreases while the right hand side of equation (A13) increases. Hence, $\frac{\partial J}{\partial w} \leq 0$ when $\sigma < 1$.

Suppose that $\frac{\partial J}{\partial w} = 0$ so that w increases and J remains constant. Column (3) of Table A1 shows that the equality no longer holds since the left hand side of equation (A13) remains constant while the right hand side of equation (A13) increases. Hence, $\frac{\partial J}{\partial w} \neq 0$ when $\sigma < 1$.

Combining, when $\sigma < 1$ then $\frac{\partial J}{\partial w} < 0$.

The equality in equation (A13) must still hold following an increase in w. When $\sigma = 1$, the term involving w drops out, and so $\frac{\partial J}{\partial w} = 0$. ∎

Appendix D: Exit behavior and market price response

This appendix proceeds in two steps:

Table A1: Effect of an increase in w on equation (A13) for $\sigma < 1$.

	$J \uparrow$	J constant	
Term	Movement	Movement	Reason
(1)	(2)	(3)	(4)
a	$\downarrow$	*constant*	$\frac{\partial}{\partial J}\frac{e^{-\delta J}-e^{-(r+\delta)J}}{1-e^{-(r+\delta)J}} < 0$
b	$\downarrow$	*constant*	$\frac{\partial}{\partial J}\left(\frac{\alpha^k}{1-\alpha}\frac{1}{r}\frac{w}{p^k}\frac{1-e^{-(r)J}}{1-\eta e^{-rJ}}\right) > 0$
c	$\uparrow$	*constant*	$\frac{\partial}{\partial J}\left(\frac{\alpha^k}{1-\alpha}\frac{1}{r}\frac{w}{p^k}\frac{1-e^{-(r)J}}{1-\eta e^{-rJ}}\right) > 0$
d	$\uparrow$	*constant*	$\frac{\partial}{\partial J}\left(-\frac{e^{-\delta J}-e^{-(r+\delta)J}}{1-e^{-(r+\delta)J}}\right) > 0$
e	$\uparrow$	$\uparrow$	$\frac{\partial w^{1-\sigma}}{\partial w} = (1-\sigma)w^{-\sigma} > 0$, $\left(1 - \frac{e^{-\delta J}-e^{-(r+\delta)J}}{1-e^{-(r+\delta)J}}\frac{r+\delta}{r}\right) > 0$
$LHS(A13)$	$\downarrow$	*constant*	
$RHS(A13)$	$\uparrow$	$\uparrow$	

- Solve for the exit behavior of the incumbent assuming the product price jumps to the new steady state level immediately.

- Derive the condition for the product price to jump immediately to its new steady state level.

Exit behavior assuming market price jumps to its new steady state

Result 2. *If* $\sigma < 1$ *and* $\frac{\partial k}{\partial w} \geq 0$ *then for a minimum wage increase* $J(w_o, w_n) < J(w_n, w_n) < J(w_o, w_o)$. *If* $\sigma = 1$ *and* $\frac{\partial k}{\partial w} \geq 0$ *then for a minimum wage increase* $J(w_o, w_n) < J(w_n, w_n) = J(w_o, w_o)$.

Proof. For $\sigma < 1$, Result 1 gives that $J(w_n, w_n) < J(w_o, w_o)$ and for $\sigma = 1$ $J(w_n, w_n) = J(w_o, w_o)$.

The proof strategy is to analyze the exit condition. The difficulty arises because in steady state the relative prices that restaurants face when they enter differs from the relative prices they face when they exit because some of the cost of capital is sunk. A restaurant exits when MC=MB. So consider the exit condition, equation (8), for both the new entrants:

$$e^{-\delta J(w_n, w_n)} y_0 P_n = r\eta p^k k_n + w^h h_n + p^m m_n + w_n l_n, \tag{A14}$$

and the incumbents:

$$e^{-\delta J(w_o,w_n)}y_0P_n = r\eta p^k k_o + w^h h_o + p^m m_o + w_n l_o. \tag{A15}$$

Rearrange these expressions so that the left hand sides are equal:

$$y_0P_n = \left(r\eta p^k k_n + w^h h_n + p^m m_n + w_n l_n\right) e^{\delta J(w_n,w_n)} \tag{A16}$$

$$y_0P_n = \left(r\eta p^k k_o + w^h h_o + p^m m_o + w_n l_o\right) e^{\delta J(w_o,w_n)}. \tag{A17}$$

Set them equal and rearrange:

$$\frac{(r\eta p^k k_n + w^h h_n + p^m m_n + w_n l_n)}{(r\eta p^k k_o + w^h h_o + p^m m_o + w_n l_o)} = e^{\delta J(w_o,w_n)}e^{-\delta J(w_n,w_n)}. \tag{A18}$$

Note that $J(w_n, w_n) \leq J(w_o, w_o)$ for $\sigma \leq 1$ so that showing that the left hand side is less than 1 proves what we want. Hence, we would like to show:

$$\left(r\eta p^k k_o + w^h h_o + p^m m_o + w_n l_o\right) > \left(r\eta p^k k_n + w^h h_n + p^m m_n + w_n l_n\right). \tag{A19}$$

To do so, note that input bundles (k_o, h_o, m_o, l_o) and (k_n, h_n, m_n, l_n) are both on the y_0-isoquant (both produce y_0 in a brand new restaurant). Cost minimization implies that:

$$\left(q_n^k k_o + q_n^h h_o + q_n^m m_o + q_n^w l_o\right) > \left(q_n^k k_n + q_n^h h_n + q_n^m m_n + q_n^w l_n\right). \tag{A20}$$

Converting to flow prices by multiplying by $\frac{r}{1-e^{-rJ(w_n,w_n)}}$:

$$\left(\frac{1-\eta e^{-rJ(w_n,w_n)}}{\eta(1-e^{-rJ(w_n,w_n)})} rp^k\eta k_o + w^h h_o + p^m m_o + w_n l_o\right) > \left(\frac{1-\eta e^{-rJ(w_n,w_n)}}{\eta(1-e^{-rJ(w_n,w_n)})} rp^k\eta k_n + w^h h_n + p^m m_n + w_n l_n\right) \tag{A21}$$

$$\begin{aligned}&\left(\left(\frac{1-\eta}{\eta(1-e^{-rJ(w_n,w_n)})}+1\right)rp^k\eta k_o + w^h h_o + p^m m_o + w_n l_o\right)\\ &> \left(\left(\frac{1-\eta}{\eta(1-e^{-rJ(w_n,w_n)})}+1\right)rp^k\eta k_n + w^h h_n + p^m m_n + w_n l_n\right)\end{aligned} \tag{A22}$$

$$\left(\left(\frac{(1-\eta)r\eta p^k}{\eta(1-e^{-rJ(w_n,w_n)})}\right)(k_o-k_n) + rp^k\eta k_o + w^h h_o + p^m m_o + w_n l_o\right) > \left(rp^k\eta k_n + w^h h_n + p^m m_n + w_n l_n\right). \tag{A23}$$

Note that if $\left(\frac{(1-\eta)r\eta p^k}{\eta(1-e^{-rJ(w_n,w_n)})}\right)(k_o-k_n) \leq 0$, then Equation (A19) holds. Since $\frac{(1-\eta)r\eta p^k}{\eta(1-e^{-rJ(w_n,w_n)})} >$

0, we need that $k_o \leq k_n$: following a minimum wage hike, the usage of capital increases. This is true by assumption. This completes the proof. ∎

When would a minimum wage *increase* lead to a *decrease* in the use of capital and our high-level sufficient condition to fail? This cannot happen when $\sigma = 1$, because in this case $\frac{k_n}{k_o} = \left(\frac{w_n}{w_o}\right)^{1-\alpha}$ and the sufficient condition is always satisfied. This might happen if the exit age is incredibly responsive to the minimum wage (i.e. if $\frac{\partial 1-e^{-rJ}}{\partial w}\frac{w}{1-e^{-rJ}} > 1$). Then it is possible that capital use declines. The central difficulty in ruling out this case is that we cannot solve for J in closed form so it is hard to bound its responsiveness to w.

Condition for the product price to jump immediately to its new steady state level

Under the assumption that the product price immediately jumps to its new steady state level, the output of the exiting restaurants is:

$$\int_{J(w_o,w_n)}^{J(w_o,w_o)} e^{-\delta j} y_0 f_o dj = \frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0. \tag{A24}$$

Under this assumption, the change in market quantity is $Q_o - Q_n$, where the market quantity is a function of the product price.

What has to happen for the exit spike to be large enough to accommodate the hypothesized decline in market quantity? The relevant inequality is:

$$\frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0 \geq Q_o - Q_n. \tag{A25}$$

That is, the exit spike has to be (weakly) larger than the change in market quantity. This leaves room for there to be an entry spike as well (if the inequality is strict).

Now we manipulate (A25) to ask what has to be true for the inequality to be satisfied. Divide both sides by Q_o, where $Q_o = \frac{f_o y_0}{\delta}(1 - e^{-\delta J(w_o,w_o)})$:

$$\frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{1 - e^{-\delta J(w_o,w_o)}} \geq \frac{Q_o - Q_n}{Q_o}.$$

Multiply through by $\frac{e^{\delta J(w_o,w_o)}}{e^{\delta J(w_o,w_o)}}$ on the left hand side and simplify:

$$\frac{e^{\delta J(w_o,w_o)-\delta J(w_o,w_n)} - e^{\delta J(w_o,w_o)-\delta J(w_o,w_o)}}{e^{\delta J(w_o,w_o)} - e^{\delta J(w_o,w_o)-\delta J(w_o,w_o)}} \geq \frac{Q_o - Q_n}{Q_o} \tag{A26}$$

$$\frac{e^{\delta J(w_o,w_o)-\delta J(w_o,w_n)} - 1}{e^{\delta J(w_o,w_o)} - 1} \geq \frac{Q_o - Q_n}{Q_o}. \tag{A27}$$

Result 2 shows that $J(w_o, w_o) > J(w_o, w_n)$ so that the numerator is positive.

The condition for the product price to jump immediately to the new steady state level is:

$$\frac{e^{\delta J(w_o,w_o)-\delta J(w_o,w_n)} - 1}{e^{\delta J(w_o,w_o)} - 1} \geq \frac{Q_o - Q_n}{Q_o}. \tag{A28}$$

Or, dividing by the right hand side, multiplying through by the denominator on the left hand side, adding one to both sides, and taking logs, the condition can be rewritten as:

$$\ln\left\{\frac{e^{\delta J(w_o,w_o)-\delta J(w_o,w_n)} - 1}{\frac{Q_o-Q_n}{Q_o}} + 1\right\} \geq \delta J(w_o, w_o). \tag{A29}$$

Appendix E: Entry and Exit Dynamics Following a Minimum Wage Hike

Exit Dynamics

In the old steady state the number of restaurants that exit in a period interval Δ is:

$$\Delta f_o \tag{A30}$$

and the implied output of these exiting restaurants is:

$$\Delta e^{-\delta J(w_o,w_o)} f_o y_0. \tag{A31}$$

The minimum wage increase results in an exit of restaurants with ages between $J(w_o, w_n)$

Table A2: **Exit dynamics, with a permanent minimum wage increase at t_n**

Time	Number	Quantity	Eqn. #
$(-\infty, t_n)$	Δf_o	$\Delta e^{-\delta J(w_o,w_o)} f_o y_0$	A30, A31
t_n	$f_o(J(w_o,w_o) - J(w_o,w_n))$	$\frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0$	A32, A33
$(t_n, t_n + J(w_o,w_n))$	Δf_o	$\Delta e^{-\delta J(w_o,w_n)} f_o y_0$	A34, A35
$[t_n + J(w^o,w_n), t_n + J(w_n,w_n)]$	0	0	

Note: This table summarizes results in equations (A30)-(A35). A Δ indicates that the pdf is bounded so that instantaneously there is no entry/exit. The Δ is a time interval.

and $J(w_o, w_o)$. So the number of restaurants exiting is:

$$\int_{J(w_o,w_n)}^{J(w_o,w_o)} f_o dj = f_o(J(w_o,w_o) - J(w_o,w_n)). \tag{A32}$$

The total output of exiting restaurants is:

$$\int_{J(w_o,w_n)}^{J(w_o,w_o)} e^{-\delta j} y_0 f_o dj = \frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0. \tag{A33}$$

Appendix D showed that when $\sigma \leq 1$ then $J(w_o,w_n) < J(w_n,w_n) < J(w_o,w_o)$. In the interval $(t_n, t_n + J(w_o,w_n)]$ only the old restaurants exit. Hence, the number of restaurants exiting is:

$$\Delta f_o \tag{A34}$$

and the output that exits is:

$$\Delta f_o e^{-\delta J(w_o,w_n)} y_0. \tag{A35}$$

After time $t_n + J(w_o,w_n)$, all of the old restaurants have exited. In the interval $(t_n + J(w_o,w_n), t_n + J(w_n,w_n))$, the old restaurants do not exit, nor do the new restaurants. Table A2 summarizes this discussion.

Entry Dynamics

In the old steady state the number of entrants is:

$$\Delta f_o \tag{A36}$$

and the output of entrants is

$$\Delta f_o y_0. \tag{A37}$$

At implementation of the minimum wage hike, the market quantity declines from Q_o to Q_n and remains constant thereafter. Hence, the entry at implementation must accommodate this decline. Using this fact and equation (A33), the output that is replaced is:

$$\frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0 + (Q_n - Q_o). \tag{A38}$$

The fact that the output of new restaurants is y_0 along with equation (A38) implies that the number of entrants is:

$$\frac{\frac{e^{-\delta J(w_o,w_n)} - e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0 + (Q_n - Q_o)}{y_0}. \tag{A39}$$

Over the time interval $(t_n, t_n + J(w_o, w_n))$, Q_n remains constant, and the amount of exiting and depreciating output is given by

$$\Delta \left\{ e^{-\delta J(w_o,w_n)} f_o y_0 + \delta Q_n \right\} \tag{A40}$$

so that the entering number is

$$\frac{\Delta \left\{ e^{-\delta J(w_o,w_n)} f_o y_0 + \delta Q_n \right\}}{y_0}. \tag{A41}$$

Finally, in $(t_n + J(w_o, w_n), t_n + J(w_n, w_n))$, there is no exit and thus entry just replaces the depreciation. Output is then:

$$\Delta \delta Q_n \tag{A42}$$

Table A3: **Entry dynamics, with a permanent minimum wage increase at t_n**

Time	Number	Quantity	Eqn. #
$(-\infty, t_n)$	Δf_o	$\Delta f_o y_0$	A36, A37
t_n	$\frac{\frac{e^{-\delta J(w_o,w_n)}-e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0+(Q_n-Q_o)}{y_0}$	$\frac{\frac{e^{-\delta J(w_o,w_n)}-e^{-\delta J(w_o,w_o)}}{\delta} f_o y_0+(Q_n-Q_o)}{y_0}$	A39, A38
$(t_n, t_n + J(w_o, w_n))$	$\Delta \frac{\left\{e^{-\delta J(w_o,w_n)} f_o y_0+\delta Q_n\right\}}{y_0}$	$\Delta \left\{e^{-\delta J(w_o,w_n)} f_o y_0 + \delta Q_n\right\}$	A41, A40
$[t_n + J(w_o, w_n),$ $t_n + J(w_n, w_n)]$	$\Delta \frac{\delta Q_n}{y_0}$	$\Delta \delta Q_n$	A43, A42

Note: This table summarizes results in equations (A36)-(A43). A Δ indicates that the pdf is bounded so that instantaneously there is no entry/exit. The Δ is a time interval.

and the resulting number of new entrants is

$$\Delta \frac{\delta Q_n}{y_0}. \tag{A43}$$

Table A3 summarizes this discussion.

Appendix F: Firm Values and Profits

Firm values

The steady state value of a firm at age $a > 0$ with exit age J is given by:

$$\pi(a, J) = \int_a^J e^{-(r+\delta)j} P dj y_0 - \int_a^J e^{-rj} \left(wl + p^m m + w^h h \right) dj + e^{-r(J-a)} p^k \eta k.$$

The first term is the discounted revenues, which reflects discounting for both depreciation and the discount rate. The second term is the discounted flow factor payments. The third term is the resale value of the capital. The reason that this term is typically positive is that the cost of capital is sunk.

Evaluating this expression, we get:

$$\pi(a,J) = (e^{-(r+\delta)a} - e^{-(r+\delta)J})\frac{Py_0}{r+\delta} - (e^{-ra} - e^{-rJ})\frac{(wl + p^m m + w^h h)}{r} + e^{-r(J-a)}p^k \eta k.$$

To get the aggregate firm value in the economy, note that firm age a is uniformly distributed on $[0, J]$. Hence, total firm value is:

$$\begin{aligned}
&\int_0^J \frac{1}{J}\pi(a,J)da \\
&= \int_0^J \frac{1}{J}\left[(e^{-(r+\delta)a} - e^{-(r+\delta)J})\frac{Py_0}{r+\delta} - (e^{-ra} - e^{-rJ})\frac{(wl + p^m m + w^h h)}{r} + e^{-r(J-a)}p^k \eta k\right] da \\
&= \frac{1-(1+J(r+\delta))e^{-(r+\delta)J}}{J(r+\delta)^2}Py_0 - \frac{1-J(1+r)e^{-rJ}}{Jr^2}\left(wl + p^m m + w^h h\right) + \frac{e^{-rJ}(1-e^{-rJ})}{Jr}p^k \eta k.
\end{aligned} \tag{A44}$$

Note that at a minimum wage change, three terms change: P, w and J. To think about what happens to the value of incumbents who exist before and after the hike, note that before the hike the value of the restaurants that do not exit is :

$$\begin{aligned}
&\int_0^{J'} \frac{1}{J'}\left[(e^{-(r+\delta)a} - e^{-(r+\delta)J})\frac{P_o y_0}{r+\delta} - (e^{-ra} - e^{-rJ})\frac{(w_o l_o + p^m m_o + w^h h_o)}{r} + e^{-r(J-a)}p^k \eta k_o\right] da \\
&= \frac{1-e^{-(r+\delta)J'} - J'(r+\delta)e^{-(r+\delta)J}}{J'(r+\delta)^2}P_o y_0 - \frac{1-e^{-rJ'} - J're^{-rJ}}{J'r^2}\left(w_o l_o + p^m m_o + w^h h_o\right) \\
&+ \frac{e^{-rJ}(1-e^{-rJ'})}{J'r}p^k \eta k_o.
\end{aligned} \tag{A45}$$

where $J' = J(w_o, w_n)$ is the exit age of restaurants that entered at the old minimum wage and are deciding to exit at the new minimum wage, and w_o and p_o are wages and prices at the "old" minimum wage, and o subscripts on factor demands denotes that they were chosen at the old minimum wage. After the minimum wage hike the value of firms is:

$$\frac{1-(1+J'(r+\delta))e^{-(r+\delta)J'}}{J'(r+\delta)^2}P_n y_0 - \frac{1-J'(1+r)e^{-rJ'}}{J'r^2}\left(w_n l_o + p^m m_o + w^h h_o\right) + \frac{e^{-rJ'}(1-e^{-rJ'})}{J'r}p^k \eta k_o. \tag{A46}$$

Note that this is the steady state formula, but with J' plugged in everywhere (since the minimum wage hike shortens the time horizon), and with the new w and p (but no change in the factor demands). To calculate the change in firm values, we compare the percent difference between equation (A45) and equation (A46).

Firm profits

We measure the steady state flow profit of a firm of age a as:

$$e^{-\delta a} P y_0 - \left(wl + p^m m + w^h h \right).$$

We measure profits as accounting profits, and do not include payments to capital. To get the aggregate flow profits in the economy, note that firm age a is uniformly distributed on $[0, J]$. Hence, total flow profits are:

$$\int_0^J \frac{1}{J} \left[e^{-\delta a} P y_0 - \left(wl + p^m m + w^h h \right) \right] da = \frac{1 - e^{-\delta J}}{J\delta} P y_0 - wl - p^m m - w^h h.$$

To solve for profits for continuing incumbents before the hike, we use the notation above; this involves just resolving the integral, while holding the factors demands constant:

$$\begin{aligned} &\int_0^{J'} \frac{1}{J'} \left[e^{-\delta a} P_o y_0 - \left(w_o l_o + p^m m_o + w^h h_o \right) \right] da \\ =&\frac{1 - e^{-\delta J'}}{J'\delta} P_o y_0 - w_o l_o - p^m m_o - w^h h_o. \end{aligned} \tag{A47}$$

For profits of incumbents after the hike we get

$$\frac{1 - e^{-\delta J'}}{J'\delta} P_n y_0 - w_n l_o - p^m m_o - w^h h_o. \tag{A48}$$

We calculate the percent change in profits by evaluating the percent difference between equation (A48) and equation (A47).

The elasticity of value with respect to the minimum wage hike (estimated using a 10% minimum wage hike) turns out to be -0.0373. The elasticity of profit with respect to the minimum wage hike (estimated using a 10% minimum wage hike) is -0.0123.

Appendix G: Additional Tables

Table A4: QCEW Sample Construction

	Exit sample	Entry sample
A. Limited service, minimum employee size is 1		
All establishments in counties with > 10 establishments in final sample	61,595	56,225
delete establishments not passing size threshold	51,297	50,253
and delete breakouts/consolidations (Final sample)	40,739	39,769
B. Limited service, minimum employee size is 15		
All establishments in counties with > 10 establishments in final sample	60,375	55,226
delete establishments not passing size threshold	23,158	23,473
and delete breakouts/consolidations (Final sample)	16,191	16,513
C. Full service, minimum employee size is 15		
All establishments in counties with > 10 establishments in final sample	54,925	50,122
delete establishments not passing size threshold	21,484	22,081
and delete breakouts/consolidations (Final sample)	18,184	18,529

Note: This table reports how three of our key samples were constructed. In the first row of each panel (labeled "all establishments in counties..."), we report the total number of establishments in our exit and entry samples. To appear in our sample, BLS confidentiality requires that counties ultimately have a minimum number of establishments. The difference in samples between panels A and B reflect the counties that meet this minimum number of establishments with at least 1 employee but not with at least 15 employees. Recall to be in the exit sample, an establishment must meet minimum employment requirements at time $t-1$ but may or may not remain open at time t. Analogously, to be included in the entry sample, an establishment must meet minimum employment requirements at time t but may or may not be open at time $t-1$. The next row in each panel deletes establishments that do not meet the minimum size threshold (of 1 in panel A and 15 in panels B and C). Finally, the third row (labeled "and delete breakouts/consolidations") is our final sample after additionally removing establishments that are part of a QCEW breakout or consolidation.

Table A5: Descriptive Statistics, QCEW

			Average size of establishment	
	Exit rate	Entry rate	Exit sample	Entry sample
Limited service restaurants	0.057	0.087	31.7	31.7
Chains	0.033	0.071	31.6	31.2
Non-chains	0.075	0.099	31.8	32.0
Full service restaurants	0.068	0.095	42.6	43.3

Note: This table reports the exit and entry rates, as well as the average employment size, for limited and full service restaurants with a minimum employment threshold of 15. The average size of limited service restaurants in the exit sample with at least 1 employee is 16.7 (all), 25.7 (chains), and 13.9 (non-chains).