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## **Optimal Monetary Policy in an Open Emerging Market Economy**

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#### Abstract

The majority of households across emerging market economies are excluded from the financial markets and cannot smooth consumption. I analyze the implications of this for optimal monetary policy and the corresponding choice of domestic versus external nominal anchor in a small open economy framework with nominal rigidities, aggregate uncertainty, and financial exclusion. I find that, if set optimally, monetary policy smooths the consumption of financially-excluded agents by stabilizing their income. Even though CPI inflation targeting approximates optimal monetary policy when financial inclusion is high, targeting the exchange rate is appropriate if financial inclusion is limited. Nominal exchange rate stability, upon shocks that create trade-offs for monetary policy, directly stabilizes the import component of financially-excluded agents' consumption baskets, which smooths their consumption and reduces macroeconomic volatility. This study provides a counterpoint to Milton Friedman's long-standing argument for a float.

**Keywords** Asymmetric Risk-Sharing, Fixed Exchange Rates, Financial Exclusion, Optimal Monetary Policy, Emerging Market Economies

**JEL classification** F21, F31, E24, E52, F43

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"In developing countries where financial markets are underdeveloped, the poor do not have access to credit and [are thus restricted to consuming disposable income]... The challenge is how to stabilize output more effectively and reduce the burden on the poor."

- The 2014 Global Economic Symposium (GES, 2014)

## 1 Introduction

The Global Financial Crisis of 2007-09 led to macroeconomic volatility in many emerging market economies (EMEs) and, in particular, adversely affected poor agents within these countries. One significant factor that contributed to the vulnerability of the poor was their inability to insure their consumption against the crisis-related contraction of incomes by drawing on savings held in the financial markets (World Bank, 2009). The macroeconomic consequences of this are not trivial, as on average, financially-excluded households constitute 60% of households in EMEs, compared to the corresponding average of under 5% in advanced economies (World Bank, 2015; see Figure 1).<sup>1</sup> This was highlighted by the crisis, which renewed policy interest in the following interlinked macroeconomic stabilization questions (Prasad, 2013; IMF, 2015a). How should monetary policy be designed in an open economy where the majority of agents cannot smooth consumption? Relatedly, should the appropriate monetary policy be implemented through a domestic or external nominal anchor? To complement the policy discussions, however, the corresponding theoretical analysis has been limited. I seek to fill in this gap by providing benchmark results on the implications of high financial exclusion for optimal monetary policy, and the corresponding choice of a nominal anchor, in an open economy.

I build an open economy model that incorporates asset market segmentation, with financially-included and financially-excluded agents. While the former can borrow and save, the latter fully consume their income each period, as they do not have the economic means to engage in financial transactions. The Euler Equation does not hold for financially-excluded agents, implying that a fraction of the economy cannot insure against shocks through international risk-sharing arrangements. The framework is that of a small open economy as EMEs are typically price takers for tradable goods (Frankel,

<sup>&</sup>lt;sup>1</sup>Regional abbreviations used in Figure 1 are as follows. MENA: Middle East and North Africa. SSA: Sub-Saharan Africa. ASIA: East, South, and Central Asia. LAC: Latin America and the Caribbean. EUR: Europe. CIS: Commonwealth of Independent States.

2010). Openness further permits a quantitative comparison of domestic versus external nominal anchors in approximating the optimal monetary policy. I characterize the optimal monetary in the presence of distortions caused by monopolistic competition and staggered price setting, and rank simple rules in terms of lowest welfare losses away from the optimum. The implications of different shocks are analyzed, with the focus primarily on adverse supply (or cost-push) shocks. This is of particular interest since over the past few years, cost-push shocks, due to domestic food price volatility (driven largely by weather conditions) and fuel input price volatility (driven largely by world market conditions), have been a significant source of concern in EMEs (Frankel, 2010).



Figure 1: Financial Inclusion Across the World Data: Percent Households with an Account at Any Financial Institution Sources: World Bank 2014 financial inclusion database (World Bank, 2015); IMF 2015 country classification scheme (IMF, 2015b)

There are two main results. The first is that with high financial exclusion, it is optimal to smooth the consumption of financially-excluded, or hand-to-mouth, agents by seeking to prevent deep recessions. These households cannot privately smooth consumption or insure themselves through risk-sharing arrangements with foreign agents. Their consumption thus fluctuates with shocks, leading to higher aggregate volatility. Optimal monetary policy, which is designed to reduce macroeconomic volatility, smooths the consumption of financially-excluded agents by stabilizing their income. In its loss function, the Central Bank implements this strategy by increasing the weight on stabilizing the output gap relative to inflation, as both financial exclusion and openness increase.<sup>2</sup> In a more open economy, domestic agents consume more imports relative to domestic goods. This increases the consequences of exchange rate volatility, which adversely affects hand-to-

<sup>&</sup>lt;sup>2</sup>A complementary reason is that as financial exclusion increases - the relative weight on stabilizing domestic inflation decreases, since inflation erodes the value of assets, and there are fewer asset holders left to consider. This is suggested in the closed economy model of **Bilbiie** (2008).

mouth agents as, unlike asset holders, they cannot smooth their consumption against the increasingly volatile price of imported goods. Optimal policy makes up for their inability to insure against exchange rate fluctuations by stabilizing their disposable income.

The second result is that the external anchor of a fixed nominal exchange rate approximates optimal monetary policy. I find that exchange rate stability is desirable in an economy with a large fraction of financially-excluded agents. Upon cost-push shocks, which simultaneously decrease output and increase inflation, thus creating a trade-off for monetary policy, targeting the exchange rate approximates the efficient dynamics through two channels. A fixed nominal exchange rate stabilizes the terms of trade, or the relative price of domestic goods, and hence domestic output and wage income. Mitigating exchange rate fluctuations also directly stabilizes the imported good component of the financiallyexcluded consumption basket, smoothing their consumption. These results provide a counterpoint to Milton Friedman's long-standing argument for flexible exchange rates, which provides the underlying intuition for the optimality of a float in the more recent complete financial inclusion reference model of Gali and Monacelli (2005).

Friedman advocated a float on the premise that it would allow for more efficient adjustment through international relative prices, given that nominal rigidities constrain real adjustment in practice. A peg would only exacerbate the lack of relative price adjustment and thus lead to greater macroeconomic volatility. However, this argument does not account for high financial exclusion. My results extend Friedman's classic arguments to an economic setting where the majority of agents cannot smooth consumption. Even though a float continues to allow for greater relative price adjustment with nominal rigidities, this actually leads to greater welfare loss in the presence of a large number of financially-excluded agents who cannot optimally hedge against exchange rate volatility. For emerging market economies, however, Friedman advocated a peg due to the gain in credibility by anchoring domestic monetary policy to a stable, advanced country's regime (Hanke, 2008). I corroborate these political economy reasons by providing a new theoretical rationale for the desirability of a peg in EMEs.

#### **Related Literature** This paper contributes to two main strands of research.

The first literature is on optimal monetary policy in open economies. There are a number of studies in this vein, reviewed in detail in Corsetti et al. (2010). Central Bank

intervention in these papers is required to correct different sources of distortions, for instance, local currency pricing and imperfect risk-sharing. Most papers incorporate nominal rigidities and monopolistic competition. The literature tends to approach optimal monetary policy design through a linear quadratic framework following the perturbation techniques discussed in Benigno and Woodford (2012). Gali and Monacelli (2005) develop a small open economy framework that has become well-cited in the literature, and find that flexible exchange rates allow for efficient adjustment in the presence of staggered price-setting. Papers including Devereux and Engel (2003) and Engel (2011) find that in the case of local currency pricing, or when producers set their prices in the currency of foreign buyers, unrestricted exchange rate movements are undesirable as they diminish efficient risk-sharing. Monacelli (2005) and Farhi and Werning (2012) are able to generate open economy optimal policy conclusions that are not isomorphic to the closed economy cases. A few studies have analyzed the appropriate choice of monetary policy with imperfect risk-sharing (Benigno, 2009; De Paoli, 2009; Corsetti et al., 2010), and find a case for exchange rate targeting as this can redress real exchange rate misalignments. The findings in this literature are based on representative agent models.

This paper contributes to the literature in two main ways. First, I derive the optimal monetary policy in an open economy with financial exclusion, which, to the best of my knowledge, has not been analyzed previously. This fundamental asymmetry in the ability of households to pool risk with foreign agents is new to the literature on the design of optimal policy in open economies, and has striking implications for the exchange rate. The inequality in access to finance of the heterogeneous agents in this study induces financial market incompleteness, complementing related research where a representative agent engages in imperfect risk-sharing. I also contribute in a technical capacity to the optimal policy literature, which has typically used a convenient subsidy to simplify the derivation of the micro-founded loss function (by eliminating the troublesome linear term in an ad-hoc manner). Benigno and Woodford (2012) argue that that approach is not fully correct, and outline perturbation techniques to address the issue. I implement their approach by analytically deriving second-order approximations of the structural equations to replace out for the linear term with the correct quadratic terms.

Related to the open economy model I use for optimal monetary policy are other papers with disposable-income driven agents. These studies, beginning with Mankiw (2000) who introduced the concept of hand-to-mouth households, pursue a variety of objectives. Galí et al. (2007), Bilbiie (2008), and Ascari et al. (2011) analyze optimal policy and the determinacy properties of ad-hoc monetary rules in a closed economy. Eser (2009) and Boerma (2014) analyze determinacy with hand-to-mouth agents, the former in a monetary union and the latter in an open economy. The literature suggests that active ad-hoc rules lead to indeterminacy for a high level of financial exclusion. Iyer (2014) develops an open economy model with hand-to-mouth agents to analyze commodity price shocks in low-income countries, a slightly simplified version of which is used here with the aim of analytically analyzing optimal policy. There are also a number of other studies that use hand-to-mouth agents in open economy models to analyze issues including the macroeconomic effects of scaling up aid and of public investment (eg. Berg et al., 2010, 2013), or the design of fiscal policy (eg. Kumhof and Laxton, 2013). The paper contributes by combining hand-to-mouth agents with open economy elements to describe the conduct of welfare-maximizing monetary policy.

This study also contributes to the burgeoning literature on monetary policy choices in emerging market economies. Prasad and Zhang (2015) build a multi-sector model with traded and non-traded sectors. They find that exchange rate targeting benefits households in the tradable sector as it fixes the domestic price of traded goods. Catão and Chang (2015) analyze monetary policy in an open economy model with imported food, to find that domestic inflation targeting is desirable in response to food price shocks with lower international risk-sharing. This paper contributes in two main ways. It shows that financial exclusion has significant implications for the optimal choice of an exchange rate regime. It further pursues an analytical approximation of monetary policy by employing rigorous perturbation techniques to micro-found the loss function. This affords a precise characterization of the sources of distortions in the economy.

Other papers, including Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010), have investigated business cycles in EMEs. Frankel (2010) and Holtemöller and Mallick (2016) find that cost-push shocks, due to food and fuel input price fluctuations, are a significant source of concern in driving business cycles in EMEs. Calvo and Reinhart (2002) find that most EMEs *de facto* target their exchange rates even though many are *de jure* classified as floats, in a phenomenon they term as "fear-of-floating". This paper add to the existing empirical literature on monetary policy and fluctuations in EMEs by

investigating the optimal Central Bank response to cost-push and productivity shocks, and as a result providing a rationale for the empirically observed fear-of-floating.

The rest of the paper proceeds as follows. Section 2 develops an open economy model with financial exclusion. Section 3 characterizes optimal monetary policy. Section 4 compares optimal policy with simple rules, and considers the choice of an implementable nominal anchor in the presence of financial exclusion. Section 5 concludes.

## 2 A Small Open Economy with Financial Exclusion

I develop an open economy general equilibrium framework with nominal rigidities, aggregate uncertainty, and two types of households. The model incorporates financiallyexcluded agents in addition to the financially-included agents typically considered in the open economy optimal policy literature, and builds upon the small open economy framework of Gali and Monacelli (2005). The model is a simplified version of my previous work, Iyer (2014), and also found in Boerma (2014). Section 2 is cast in non-linear terms for expositional ease, and the resulting equilibrium is linearized in Section 3 to solve for optimal monetary policy using perturbation techniques.

## 2.1 Households

There exists a continuum of households indexed by  $l \in [0, 1]$ . Fraction  $1 - \lambda$  of households participate in the financial markets, which are assumed to be complete both within and across countries. Through a complete set of state-contingent securities available internationally, they are able to share risk with foreign agents. However, a fraction  $\lambda$  of domestic agents do not engage in financial transactions as they do not have the economic means to do so, similar to the "hand-to-mouth" consumers in Mankiw (2000). Their sole source of income is wage income. Thus, although financially-excluded agents can optimize labour supply, they are unable to share risk internationally or smooth consumption. Throughout the paper, financially-included households are denoted with the symbol,  $\hat{}$ , while financially-excluded households are denoted with the symbol,  $\hat{}$ .

The domestic economy, with its heterogeneous consumers, is of measure zero compared to the world. This implies that domestic monetary policy decisions do not have any impact internationally, which consists of a continuum of economies,  $j \in [0, 1]$ , populated by identical, financially-included households. While different economies are subject to asymmetric shocks, initial net foreign asset positions are zero across countries, and preferences and market structures are identical. In each period  $t \ge 0$ , a stochastic event,  $s_t$ , is realized. Let  $s^t = (s_0, ..., s_{t-1}, s_t)$  be the history of events until period t, as in Chari et al. (2002). The unconditional probability, as of period 0, of observing any particular history  $s^t$  is  $\mu(s^t)$ . The probability of history  $s^{t+1}$ , conditional on  $s^t$ , is given by  $\mu(s^{t+1}|s^t)$ . The initial realization,  $s_0$ , is taken as given so that  $\mu(s_0) = 1$  for a particular  $s_0$ .

## 2.1.1 Financially-Excluded Agents

Financially-excluded agents,  $l \in [0, \lambda]$ , consume their income each period and cannot smooth consumption through the financial markets. These agents gain utility from consumption,  $\check{C}(s^t)$ , and disutility from hours worked,  $\check{N}(s^t)$ . The representative household chooses its period *t* allocation after the event,  $s_t$ , is realized to maximize its utility,  $\check{U}(x^t) = \sum_{s^t} \mu(s^t) U\{\check{C}(s^t), \check{N}(s^t)\}$ , in the following static optimization problem

$$\max_{\left\{\check{C}(s^{t}),\check{N}(s^{t})\right\}}\check{U}(s^{t}) = \sum_{s^{t}}\mu(s^{t})\left\{\frac{\check{C}\left(s^{t}\right)^{1-\sigma}}{1-\sigma} - \frac{\check{N}\left(s^{t}\right)^{1+\phi}}{1+\phi}\right\}$$
(1)

s.t. 
$$P(s^t) \check{C}(s^t) \le W(s^t) \check{N}(s^t)$$
 (2)

where  $\sigma$  is the inverse intertemporal elasticity of substitution (IES),  $\phi$  is the inverse Frisch elasticity of labour supply,  $P(s^t)$  is the consumer price index (CPI),  $W(s^t)$  denotes nominal wages,  $\check{N}(s^t)$  represents hours of labour supplied, and  $\check{C}(s^t)$  is consumption of domestically produced and imported goods from a constant elasticity of substitution (CES) consumption basket. These variables are denominated in domestic currency units. Preferences are locally non-satiated, so that the budget constraint binds. This implies that the consumption of excluded households derives directly from (2). As these agents do not hold bonds, they cannot smooth consumption across states or time.

Financially-excluded households trade off leisure for consumption goods, through their optimal labour supply condition, (3), where  $w(s^t)$  denotes the real wage rate,  $w(s^t) = \frac{W(s^t)}{P(s^t)}$ . While nominal wages,  $W(s^t)$ , are the same for both consumers and producers, *real* wages faced by households,  $w(s^t)$ , differ from the real wages faced by domestic firms,  $\frac{W(s^t)}{P_H(s^t)}$ , where  $P_H(s^t)$  is the domestic (monopolistically set) price index. This is because households take into account the CPI,  $P(s^t)$ , which consists of both domestic and imported good prices. The labour supply condition, derived by combining the first-order conditions on consumption and labour, is

$$\omega\left(s^{t}\right) = \check{N}\left(s^{t}\right){}^{\phi}\check{C}\left(s^{t}\right){}^{\sigma} \tag{3}$$

The consumption basket,  $\check{C}(s^t)$ , is comprised, as in Gali and Monacelli (2005), of domestic and foreign aggregates, respectively  $\check{C}_H(s^t)$  and  $\check{C}_F(s^t)$ .

$$\check{C}\left(s^{t}\right) = \left[\left(1-\alpha\right)^{\frac{1}{\varepsilon_{I}}}\check{C}_{H}\left(s^{t}\right)^{\frac{\varepsilon_{I}-1}{\varepsilon_{I}}} + \alpha^{\frac{1}{\varepsilon_{I}}}\check{C}_{F}\left(s^{t}\right)^{\frac{\varepsilon_{I}-1}{\varepsilon_{I}}}\right]^{\frac{\varepsilon_{I}}{\varepsilon_{I}-1}}$$
(4)

where  $\alpha \in [0, 1]$  is the share of imports in the CES basket and represents the degree of openness to international trade in goods (conversely,  $1 - \alpha$  is the degree of home bias), and  $\varepsilon_I$  is the elasticity of substitution between domestic and foreign goods. The associated price index is

$$P\left(s^{t}\right) = \left[(1-\alpha)P_{H}^{1-\varepsilon_{I}}\left(s^{t}\right) + \alpha P_{F}^{1-\varepsilon_{I}}\left(s^{t}\right)\right]^{\frac{1}{1-\varepsilon_{I}}}$$
(5)

where  $P(s^t)$  is the consumer price index (CPI),  $P_H(s^t)$  is the price index for domestic goods, and  $P_F(s^t)$  is the import price index. Minimizing expenditure on the CES consumption basket, (4), with respect to the CPI, (5), gives rise to the following downwardsloping demand functions for domestic and imported aggregates

$$\check{C}_{H}\left(s^{t}\right) = (1-\alpha) \left[\frac{P_{H}\left(s^{t}\right)}{P\left(s^{t}\right)}\right]^{-\varepsilon_{I}} \check{C}\left(s^{t}\right)$$
(6)

$$\check{C}_{F}(s^{t}) = \alpha \left[\frac{P_{F}(s^{t})}{P(s^{t})}\right]^{-\varepsilon_{I}}\check{C}(s^{t})$$
(7)

The imported good,  $\check{C}_F(s^t)$ , comprises of goods from each foreign country, *j*, and  $\varepsilon_F$  denotes the elasticity of substitution between imported goods

$$\check{C}_{F}\left(s^{t}\right) = \left[\int_{0}^{1}\check{C}_{j}\left(s^{t}\right)^{\frac{\varepsilon_{F}-1}{\varepsilon_{F}}}di\right]^{\frac{\varepsilon_{F}}{\varepsilon_{F}-1}}$$
(8)

e ...

The subcomponents of the domestic and imported good indices measure domestic consumption of individual varieties of goods, *i*. These varieties are produced at home as well as in the continuum of foreign small open economies,  $j \in [0, 1]$ , so that  $\check{C}_H(i, s^t)$  is the consumption of domestic variety *i* and  $\check{C}(i, s^t)$  is the consumption of variety *i* imported from country *j* 

$$\check{C}_{H}\left(s^{t}\right) = \left[\int_{0}^{1}\check{C}_{H}\left(i,s^{t}\right)^{\frac{\varepsilon_{p}-1}{\varepsilon_{p}}}di\right]^{\frac{\varepsilon_{p}}{\varepsilon_{p}-1}} \qquad \check{C}_{j}\left(s^{t}\right) = \left[\int_{0}^{1}\check{C}_{j}\left(i,s^{t}\right)^{\frac{\varepsilon_{p}-1}{\varepsilon_{p}}}di\right]^{\frac{\varepsilon_{p}}{\varepsilon_{p}-1}} \tag{9}$$

where, due to the assumption of identical market structures across countries,  $\varepsilon_p$  is the elasticity of substitution between individual varieties, *i*, produced in any country *j*, including home. The price indices associated with the demand for domestic and country *j*'s varieties are

$$P_{H}\left(s^{t}\right) = \left[\int_{0}^{1} \check{P}_{H}\left(i,s^{t}\right)^{1-\varepsilon_{p}} di\right]^{\frac{1}{1-\varepsilon_{p}}} \qquad P_{j}\left(s^{t}\right) = \left[\int_{0}^{1} \check{P}_{j}\left(i,s^{t}\right)^{1-\varepsilon_{p}} di\right]^{\frac{1}{1-\varepsilon_{p}}} \tag{10}$$

#### 2.1.2 Financially-Included Agents

Financially-included agents,  $l \in [\lambda, 1]$ , gain utility from consumption,  $\hat{C}(s^t)$  and disutility from hours worked,  $\hat{N}(s^t)$ . These agents own the domestic firms, and have access to a complete portfolio of state-contingent bonds, which are traded sequentially in spot markets with foreign households in each period,  $t \ge 0$ , before  $s_{t+1}$  occurs. The equilibrium that arises from this assumption of sequential trading is equivalent to that which would arise from a time 0 trading of a complete set of history-contingent bonds. Notably, although financial trades are executed before uncertainty realizes, the period *t* allocation is chosen after event  $s_t$  occurs.

I let  $B(s^t, s_{t+1})$  denote the representative financially-included agent's holdings of a complete vector of state-contingent claims at time t, given history  $s^t$ , on time t + 1domestic *currency*. One unit of this vector pays off one unit of domestic currency if the particular state  $s_{t+1}$  occurs and 0 otherwise (Chari et al., 2002).  $B(s^t, s_{t+1})$  is denominated in units of domestic currency and each of its units is priced at  $Z(s^{t+1}|s^t)$ , where  $s^{t+1} =$  $(s^t, s_{t+1})$ . Taking  $Z(s^{t+1}|s^t)$  as given, the representative household maximizes its utility,  $\hat{U}(s^t) = \sum_{s^t} \mu(s^t) U\{\hat{C}(s^t), \hat{N}(s^t)\}$ , subject to a sequence of budget constraints for  $t \ge 0$ , by solving the following dynamic optimization problem

$$\max_{\left\{\hat{C}(s^{t}),\hat{N}(s^{t}),B(s^{t},s_{t+1})\right\}} \sum_{t=0}^{\infty} \beta^{t} \hat{U}(s^{t}) = \sum_{t=0}^{\infty} \sum_{s^{t}} \mu(s^{t}) \beta^{t} \left\{ \frac{\hat{C}\left(s^{t}\right)^{1-\sigma}}{1-\sigma} - \frac{\hat{N}\left(s^{t}\right)^{1+\phi}}{1+\phi} \right\}$$
(11)

s.t. 
$$P(s^{t}) \hat{C}(s^{t}) + \sum_{s_{t+1}} Z(s^{t+1}|s^{t}) B(s^{t}, s_{t+1}) \leq B(s^{t}) + W(s^{t}) \hat{N}(s^{t}) + \Omega(s^{t}) - T(s^{t})$$
(12)

where  $\beta^t \in [0, 1]$  is the subjective discount factor,  $\hat{N}(s^t)$  represents hours of labour supplied,  $\hat{C}(s^t)$  is consumption of domestically produced and imported goods from a CES consumption basket,  $\Omega(s^t)$  denotes profits received from ownership of domestic firms, and  $T(s^t)$  is a nominal lump-sum tax. These variables are denominated in domestic currency units, and the initial condition,  $B(s_0)$ , is taken as given. The budget constraint binds, and as standard in the optimal policy literature, the lump-sum tax is used to finance a constant wage subsidy,  $\tau^e$ , to offset the steady state monopolistic distortion

$$T(s^{t}) = \tau^{e}W(s^{t})N(s^{t})$$
(13)

where  $N(s^t)$  is labour demanded by the monopolistic producers. Each period, financiallyincluded agents trade off leisure for consumption goods through their optimal labour supply condition, (14), derived by combining the first-order conditions on consumption and labour

$$\omega\left(s^{t}\right) = \hat{N}\left(s^{t}\right){}^{\phi}\hat{C}\left(s^{t}\right){}^{\sigma} \tag{14}$$

The first-order conditions on state-contingent bonds and consumption can be combined to yield the Euler Equation

$$Z\left(s^{t+1}|s^{t}\right) = \beta\mu\left(s^{t+1}|s^{t}\right) \left[\frac{\hat{C}\left(s^{t+1}\right)}{\hat{C}\left(s^{t}\right)}\right]^{-\sigma} \frac{1}{\Pi\left(s^{t+1}\right)}$$
(15)

where  $\Pi(s^{t+1}) = \frac{P(s^{t+1})}{P(s^t)}$  is CPI inflation. A similar equilibrium condition with respect to domestic currency state-contingent bonds, adjusted for the presence of the nominal bilateral exchange rate,  $e^j(s^t)$ , to ensure price equalization across the world in contingent claims, holds for the representative household in country *j* 

$$Z\left(s^{t+1}|s^{t}\right) = \beta\mu\left(s^{t+1}|s^{t}\right) \left[\frac{C^{j}\left(s^{t+1}\right)}{C^{j}\left(s^{t}\right)}\right]^{-\sigma} \frac{e^{j}\left(s^{t+1}\right)}{e^{j}\left(s^{t}\right)} \frac{1}{\Pi^{j}\left(s^{t+1}\right)}$$
(16)

where  $C^{j}(s^{t})$ ,  $\Pi^{j}(s^{t})$ , and  $\varepsilon^{j}(s^{t})$  denote the consumption basket, CPI inflation, and bilateral nominal exchange rate of country *j*.<sup>3</sup> Combining the Euler Equation of the domes-

<sup>&</sup>lt;sup>3</sup>The representative household in country *j* faces the following budget constraint in each period  $t \ge$ 

tic household, (15), with that of each foreign household  $j \in [0, 1]$ , (16), and integrating across foreign households,  $C^*(s^t) = \int_0^1 C_j(s^t) dj$ , yields the Backus-Smith (Backus and Smith, 1993) international risk-sharing condition from the perspective of the domestic small open economy

$$\hat{C}(s^t) = vC^*(s^t)Q(s^t)^{\frac{1}{\sigma}}$$
(17)

which assumes that agents across the world make appropriate ex-ante international insurance transfers through complete financial markets, to ensure that risk is pooled internationally, ie. v = 1, and that world consumption is exogenous. The value of the insurance transfers to the domestic economy is given in equation (38). International risksharing, (17), means that the marginal utility of consumption, weighted by the real effective exchange rate,  $Q(s^t) = e(s^t) \frac{P^*(s^t)}{P(s^t)}$ , is equalized across countries, where  $P^*(s^t)$  is the world price index and  $e(s^t)$  is the nominal effective exchange rate ie. an index of the prices of foreign currencies  $j \in [0, 1]$  in terms of domestic currency.

Efficient risk-sharing thus implies that demand, by financially-included agents worldwide, is directed at countries where it is cheaper to consume. Risk-sharing is possible because of the timing of financial trades. Financial markets open before monetary policy decisions are made, implying that financially-included agents are able to smooth consumption in the face of uncertainty implied by choice of monetary regime. Further, this pooling of risk with foreign agents insures financially-included agents in an open economy, compared to the closed economy case (without capital) where it is not possible to insure against aggregate uncertainty.

Asset market arbitrage opportunities do not exist, as these would lead to indeterminacy in the international portfolio allocation problem. This implies that the equilibrium prices - in domestic currency - of risk-free one-period uncontingent nominal bonds at home and in foreign country j, are related to their gross returns as follows

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^{t}\right) = \frac{1}{1+i(s^{t})}$$
(18)

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) e_j\left(s^{t+1}\right) = \frac{e\left(s^t\right)}{1+i_j\left(s^t\right)}$$
(19)

<sup>0:</sup>  $P^{j}(s^{t}) C^{j}(s^{t}) + e^{j}(s^{t}) \sum_{s_{t+1}} Z(s^{t+1}|s^{t}) B^{j}(s^{t}, s_{t+1}) \leq e^{j}(s^{t}) B^{j}(s^{t}) + W^{j}(s^{t}) N^{j}(s^{t}) + \Omega^{j}(s^{t}) - T^{j}(s^{t}),$ where  $B^{j}(s^{t})$  represents the foreign household's holdings of the state-contingent bond denominated in units of domestic currency and  $e^{j}(s^{t})$  is the bilateral nominal exchange rate with respect to the domestic economy ie. the price of domestic currency in units of foreign currency.  $e^{j}(s^{t})$  serves to convert the domestic currency payoffs into foreign currency (Chari et al., 2002).

where the domestic currency price of a domestic bond,  $\sum_{s_{t+1}} Z(s^{t+1}|s^t)$ , is inversely related to the gross domestic nominal interest rate,  $1 + i(s^t)$ , and the domestic currency price of a foreign bond,  $\sum_{s_{t+1}} Z(s^{t+1}|s^t) e_j(s^{t+1})$ , is inversely related to the gross foreign nominal interest rate,  $1 + i_j(s^t)$ , adjusted by the bilateral nominal exchange rate. Domestic monetary policy has direct leverage over  $i(s^t)$ . Combining the domestic and foreign bond pricing equations, and aggregating across countries  $j \in [0, 1]$ , yields the uncovered interest rate parity (UIP) condition

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^{t}\right) \left\{ (1+i\left(s^{t}\right)) - (1+i^{*}\left(s^{t}\right))\frac{e\left(s^{t+1}\right)}{e\left(s^{t}\right)} \right\} = 0$$
(20)

where  $i^*(s^t) = \int_0^1 i_j(s^t) dj$  is the world nominal interest rate, taken as given by the domestic economy. UIP implies that the nominal exchange rate is expected to adjust to equalize the domestic currency returns on domestic and foreign contingent bonds.

The consumption basket of the financially-included agents,  $\hat{C}(s^t)$ , is similar to (4), and is comprised of domestic and foreign aggregates, respectively  $\hat{C}_H(s^t)$  and  $\hat{C}_F(s^t)$ 

$$\hat{C}(s^{t}) = \left[ (1-\alpha)^{\frac{1}{\varepsilon_{I}}} \hat{C}_{H}(s^{t})^{\frac{\varepsilon_{I}-1}{\varepsilon_{I}}} + \alpha^{\frac{1}{\varepsilon_{I}}} \hat{C}_{F}(s^{t})^{\frac{\varepsilon_{I}-1}{\varepsilon_{I}}} \right]^{\frac{\varepsilon_{I}}{\varepsilon_{I}-1}}$$
(21)

Minimizing expenditure on the CES consumption basket, (21), with respect to the CPI, (5), gives rise to the following downward-sloping demand functions for domestic and imported aggregates for financially-included agents

$$\hat{C}_{H}(s^{t}) = (1-\alpha) \left[ \frac{P_{H}(s^{t})}{P(s^{t})} \right]^{-\epsilon_{I}} \hat{C}(s^{t})$$
(22)

$$\hat{C}_F(s^t) = \alpha \left[ \frac{P_F(s^t)}{P(s^t)} \right]^{-\epsilon_I} \hat{C}(s^t)$$
(23)

## 2.2 Relative Prices and Exchange Rates

The model equilibrium, following Ferrero and Seneca (2015), is defined in terms of the effective terms of trade,  $X(s^t) = \frac{P_F(s^t)}{P_H(s^t)}$ , which is an index of the bilateral terms of trade between the domestic economy and all foreign economies  $j \in [0, 1]$ . That is,  $X(s^t) = \left(\int_0^\infty X_{jH}(s^t)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$ , where  $X_{jH}(s^t) = P_j(s^t) / P_H(s^t)$  denotes the price of country j's goods in terms of domestic goods.

To put prices in terms of  $X(s^t)$ , I first normalize all prices by the CPI, (5), to define them in relative terms. These are the relative domestic price index,  $p_H(s^t) = P_H(s^t) / P(s^t)$ , and the relative import price index,  $p_F(s^t) = P_F(s^t) / P(s^t)$ . As functions of the effective terms of trade,  $X(s^t)$ , these are

$$p_{H}\left(s^{t}\right) = \left[1 - \alpha + \alpha X\left(s^{t}\right)^{1 - \varepsilon_{I}}\right]^{-\frac{1}{1 - \varepsilon_{I}}} \qquad p_{F}\left(s^{t}\right) = X\left(s^{t}\right)\left[1 - \alpha + \alpha X\left(s^{t}\right)^{1 - \varepsilon_{I}}\right]^{-\frac{1}{1 - \varepsilon_{I}}}$$
(24)

The effective terms of trade,  $X(s^t)$ , is thus a ratio of relative prices

$$X(s^{t}) = \frac{p_{F}(s^{t})}{p_{H}(s^{t})}$$
(25)

The nominal effective exchange rate,  $e(s^t) = \left(\int_0^1 e_j(s^t)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$ , is an index of the nominal bilateral exchange rates among foreign countries,  $j \in [0,1]$ , and the domestic economy. The nominal exchange rate between home and any other country j,  $\varepsilon_j(s^t)$ , is the price of foreign currency, j, in units of domestic currency.  $P^*(s^t) = \int_0^1 P^j(s^t) dj$  is the world price index, where  $P^j(s^t)$  is the CPI in country j. The real effective exchange rate

$$Q\left(s^{t}\right) = \frac{e\left(s^{t}\right)P^{*}\left(s^{t}\right)}{P\left(s^{t}\right)}$$

is defined as the domestic currency price of a foreign basket of consumption,  $e(s^t) P^*(s^t)$ , relative to the domestic currency price of a domestic basket of consumption,  $P(s^t) \cdot Q(s^t) = (\int_0^1 Q_j(s^t)^{1-\varepsilon_F} dj)^{\frac{1}{1-\varepsilon_F}}$ , where  $Q_j(s^t)$  is the real bilateral exchange rate between home and country *j*.  $Q(s^t)$  can be expressed in terms of  $X(s^t)$ . Assuming that law of one price holds in the imported goods market, so that  $P_F(s^t) = e(s^t) P_F^*(s^t)$ , and that from the perspective of the domestic economy, which is small, the world as a whole behaves like a closed economy, so that  $P^*(s^t) = P_F^*(s^t)$ , I derive

$$Q(s^{t}) = \left[ (1-\alpha)X(s^{t})^{\varepsilon_{I}-1} + \alpha \right]^{-\frac{1}{1-\varepsilon_{I}}}$$
(26)

Note that the real exchange rate is a function of the effective terms of trade in this framework, so that these two relative prices co-move upon shocks.

#### 2.3 Firms

The firm's problem is standard in the literature. Firms,  $i \in [0, 1]$ , are monopolistic and set prices in a staggered fashion. Every period, a fraction  $(1 - \theta)$  of (randomly selected) firms can re-optimize prices, as in Calvo (1983). Fraction  $\theta$  of firms cannot re-optimize and instead adjust labour demand to meet changes in output demand upon shocks. Firms that do reset prices upon shocks take into account that the probability of keeping this period's price *k* periods ahead is given by  $\theta^k$ .

With production function  $Y(i, s^t) = A(s^t) N(i, s^t)$ , each reoptimizing firm *i* sets its optimal reset price as a markup over current and expected marginal costs, where  $MC(i, s^t) = W(s^t) / A(s^t)$ , giving rise to domestic inflation. Noting that a firm that reoptimizes in period *t* will choose the price  $P_H^*(i, s^t)$  that maximizes current and future expected discounted profits until period t + k while this price remains effective, so that  $P_H^*(i, s^{t+k}) = P_H^*(i, s^t)$  for  $k = 0, ..., \infty$ , the optimal reset price at time *t* solves the following problem

$$\begin{aligned} \max_{P_{H}^{*}(i,s^{t})} \sum_{k=0}^{\infty} \sum_{s^{t+k}} Z\left(s^{t+k}|s^{t}\right) \theta^{k} \left\{ (1+\tau^{e}) P_{H}^{*}\left(i,s^{t}\right) Y\left(i,s^{t+k}\right) - W\left(s^{t+k}\right) N\left(i,s^{t+k}\right) \right\} \\ s.t. \ Y\left(i,s^{t+k}\right) = \left(\frac{P_{H}^{*}\left(i,s^{t}\right)}{P_{H}\left(s^{t}\right)}\right)^{-\varepsilon_{p}} \left(C_{H}\left(i,s^{t+k}\right) + \int_{0}^{1} C_{H}^{j}\left(i,s^{t+k}\right) dj\right) \end{aligned}$$
(27)

where  $Z(s^{t+k}|s^t)$  is the stochastic discount factor (as households own the firms) in period t + k given history  $s^t$  (recall that  $s^{t+k} = (s^t, s_{t+k})$ ),  $\tau^e$  is a steady state wage subsidy,  $Y(i, s^{t+k})$  and  $W(s^{t+k}) N(i, s^{t+k})$  are respectively the output and total cost in period t + k for a firm that last reset its price in period t, and  $C_H(i, s^{t+k})$  and  $\int_0^1 C_H^j(i, s^{t+k}) dj$  represent demand for good i in period t + k respectively by domestic consumers and foreign consumers in countries j. In a symmetric equilibrium, as derived in Chari et al. (2002) and Galí (2007), the same price is chosen by all firms that can re-optimize so that  $P_H^*(i, s^t) = P_H^*(s^t) \forall i$ . The first-order condition is

$$\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^{t}\right) \theta^{k} Y\left(i, s^{t+k}\right) \left[P_{H}^{*}\left(s^{t}\right) - \frac{1}{1+\tau^{e}} \frac{\varepsilon_{P}}{\varepsilon_{P}-1} \frac{W\left(s^{t+k}\right)}{A\left(s^{t+k}\right)}\right] = 0$$
(28)

where productivity shocks,  $A(s^t)$ , are determined relative to their steady state value A

and follow the following stationary autoregressive process

$$Ln(1 + A(s^{t})) - Ln(1 + A) = \rho_{A}Ln(1 + A(s^{t-1})) - Ln(1 + A) + s_{t}^{A}$$

with  $\rho_A \in (0,1)$  and  $s_t^A \sim N(0, \sigma_A^2)$ . The optimal reset price relates to the domestic inflation rate as follows

$$\frac{P_{H}^{*}\left(s^{t}\right)}{P_{H}\left(s^{t}\right)} = \left(\frac{1 - \theta \Pi_{H}\left(s^{t}\right)^{\varepsilon_{p}-1}}{1 - \theta}\right)^{\frac{1}{1 - \varepsilon_{p}}}$$
(29)

where  $\Pi_H(s^t) = \frac{P_H(s^t)}{P_H(s^{t-1})}$ .<sup>4</sup> Combining the constraint (27), the labor-market clearing condition  $N_t = \int_0^1 N(i, s^t) di$ , the price index associated with monopolistic goods  $P_H(s^t) = \left[\int_0^1 P_H(i, s^t)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}}$ , and the definition of price dispersion,  $\Delta(s^t) \equiv \int_0^1 \left(\frac{P_H(i, s^t)}{P_H(s^t)}\right)^{-\varepsilon_p} di$ , which follows law of motion

$$\Delta\left(s^{t}\right)^{\frac{\varepsilon_{p}-1}{\varepsilon_{p}}} = \theta \pi_{H}\left(s^{t}\right)^{\varepsilon_{p}-1} \Delta\left(s^{t-1}\right) + (1-\theta) \frac{1-\theta \Pi_{H}\left(s^{t}\right)^{\varepsilon_{p}-1}}{1-\theta}$$
(30)

the aggregate production function, (31), is

$$Y(s^{t}) \Delta(s^{t}) = A(s^{t}) N(s^{t})$$
(31)

The monopolistic sector faces exogenous cost-push shocks which directly increase domestic inflation without excess aggregate demand pressures. These disturbances represent unanticipated food price or fuel input price shocks that arise in markets outside the authority's control. As standard in the literature, and discussed further in Sutherland (2005),  $V(s^t)$  is the net monopolistic markup,  $\frac{\varepsilon_P}{\varepsilon_P-1}\frac{1}{1+\tau^{\epsilon}}$ . Markup shocks,  $s_t^V$ , are assumed to arise from random changes in the production subsidy or the degree of monopoly power. Cost-push/ markup shocks are determined relative to their steady state value V and follow the following stationary autoregressive process

$$Ln(1+V(s^{t})) - Ln(1+V) = \rho_{V}Ln(1+V(s^{t-1})) - Ln(1+V) + s_{t}^{V}$$

<sup>&</sup>lt;sup>4</sup>The linearized version of the optimal price-setting rule, (28), is a standard result and can be found in chapter 3 of Galí (2007):  $\pi_{H,t} = \beta \pi_{H,t+1} + \xi mc_t + v_t$ , where  $mc_t$  is marginal cost,  $\xi = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ , and  $v_t$  is a cost-push shock that is appended on the Phillips Curve, as standard in the literature.

where  $\rho_V \in (0, 1)$  and  $s_t^V \sim N(0, \sigma_V^2)$ .

## 2.4 Central Bank

Monetary policy is set according to either simple rules or a model-specific optimized rule. The strict simple rules (in linearized terms) are domestic inflation targeting (DIT),  $\pi_H(s^t) = 0$ , CPI inflation targeting (DIT),  $\pi(s^t) = 0$ , and a fixed exchange rate,  $\Delta e(s^t) = 0$ , and the flexible ones are Taylor-type rules where the nominal interest rate responds to a measure of inflation and the output gap.

## 2.5 Market-Clearing and Accounting

The demand for each monopolistic good  $i \in [0, 1]$ , is

$$Y_t(i,s^t) = C_H(i,s^t) + \int_0^1 C_H^j(i,s^t) \, dj$$

where  $C_H(i, s^t)$  is consumption of home good *i* by domestic consumers and  $C_H^j(i, s^t)$  denotes consumption of home good *i* by country *j*. Replacing in the expressions for the consumption of individual varieties, *i*, international risk-sharing, and the definitions of the bilateral and effective terms of trade

$$Y\left(s^{t}\right) = \left(\frac{P_{H}\left(s^{t}\right)}{P\left(s^{t}\right)}\right)^{-\varepsilon_{I}} \left[\left(1-\alpha\right)C\left(s^{t}\right) + \alpha\hat{C}\left(s^{t}\right)\int_{0}^{1}Q_{j}\left(s^{t}\right)^{\varepsilon_{I}-\frac{1}{\sigma}}\left(X_{j}\left(s^{t}\right)X^{j}\left(s^{t}\right)\right)^{\varepsilon_{F}-\varepsilon_{I}}dj\right]$$
(32)

where  $X_j(s^t)$  and  $X_j(s^t)$  denote bilateral variables for the domestic economy, and  $X^j(s^t)$  denotes the effective terms of trade for country *j*. The labour market is Walrasian, with the real wage,  $w(s^t)$ , moving instantly to clear demand and supply imbalances

$$\lambda \check{N}\left(s^{t}\right) + (1-\lambda)\hat{N}\left(s^{t}\right) = N\left(s^{t}\right) = \int_{0}^{1} N\left(i, s^{t}\right) di$$
(33)

where  $\int_0^1 N_t(i, s^t)) di$  is the demand for labour by each firm, *i*. Aggregate consumption is a weighted average, with weights given by  $\lambda$ , of consumption by financially-included and financially-excluded agents

$$C(s^{t}) = \lambda \check{C}(s^{t}) + (1 - \lambda)\hat{C}(s^{t})$$
(34)

Aggregate consumption of domestic and imported foreign aggregates is likewise

$$C_H(s^t) = \lambda \check{C}_H(s^t) + (1-\lambda)\hat{C}_H(s^t)$$
(35)

$$C_F(s^t) = \lambda \check{C}_F(s^t) + (1-\lambda)\hat{C}_F(s^t)$$
(36)

The real trade balance, in terms of the CPI,  $P(s^t)$ , is defined as the imbalance between domestic production and consumption, and given by

$$NX(s^{t}) = p_{H}(s^{t})Y(s^{t}) - (\lambda \check{C}(s^{t}) + (1-\lambda)\hat{C}(s^{t}))$$
(37)

The trade balance is non-zero, unlike Gali and Monacelli (2005), which is nested in (37) when  $\lambda = 0$  in this framework. This point is independent of calibration, but is seen more easily for  $\sigma = \varepsilon_I = \varepsilon_F = 1$ , as in the welfare analysis, so that  $NX(s^t)$  boils down to  $NX(s^t) = \alpha\lambda (\hat{C}(s^t) - \check{C}(s^t))$  upon replacing out for (32) in (37). The fact that  $NX(s^t)$  is away from zero is driven by  $\lambda > 0$ , ie. a positive degree of financial exclusion, and has implications for the domestic net foreign asset position.

Net foreign assets, *NFA* ( $s^t$ ) are non-zero upon shocks (they are zero in the initial steady state), and quantify the net present value of insurance transfers to the domestic financially-included households when uncertainty realizes. In each period, the stock of *NFA* ( $s^t$ ), normalized by the CPI, *P* ( $s^t$ ), derived by iterating the financially included household's budget constraint, is given by

$$NFA(s^{t}) = -\sum_{k=0}^{\infty} \sum_{s^{t+k}} Z(s^{t+k}|s^{t}) \left[ NX(s^{t+k}) + \left( C(s^{t+k}) - \hat{C}(s^{t+k}) \right) - w(s^{t+k}) \left( N(s^{t+k}) - \hat{N}(s^{t+k}) \right) \right]$$
(38)

Equation (38) is a useful summary of the difference between closed economies with financially-excluded agents, for example, Galí et al. (2007) and Bilbiie (2008), and an open economy. It arises because of (i) international risk-sharing and the associated insurance transfers, possible only in an open economy and (ii) the fact that the trade balance is not zero with  $\lambda > 0$ . If  $\lambda = 0$ , then *NFA* ( $s^t$ ) = 0 in each period, as in Gali and Monacelli (2005), and the open economy model is isomorphic to the closed case.

In the current framework, however, as only a fraction of domestic agents can share risk with foreign agents, open economy elements do not affect the two types of agents in a symmetric fashion. In particular, financially-included agents can share risk and smooth consumption upon exchange rate fluctuations, whereas financially-excluded agents cannot. This fundamental asymmetry, connected with international risk-sharing, is why  $NFA(s^t) \neq 0$  and optimal policy in an open economy model with financially-excluded agents is not isomorphic to the closed economy case.

## 2.6 Equilibrium

For a particular specification of monetary policy (which determines the nominal interest rate,  $i(s^t)$ ), an equilibrium for the model is a state-contingent sequence of prices

$$\{X(s^t), Z(s^{t+1}|s^t), \Pi_H(s^t), \Pi(s^t), MC(s^t), \Delta(s^t), e(s^t)\}_{t=0}^{\infty}$$

and quantities

$$\{\check{C}(s^{t}),\check{C}_{H}(s^{t}),\check{C}_{F}(s^{t}),\hat{C}(s^{t}),\hat{C}_{H}(s^{t}),\hat{C}_{F}(s^{t}),\\ \check{N}(s^{t}),\hat{N}(s^{t}),N(s^{t}),C(s^{t}),C_{H}(s^{t}),C_{F}(s^{t}),Y(s^{t})\}_{t=0}^{\infty}$$

such that

- Financially-excluded agents optimize: (3), (4), (6), and (7)
- Financially-included agents optimize: (14), (15), (21), (22), and (23)
- International-risk sharing and no-arbitrage conditions hold: (17), (18), and (20)
- Aggregate consumption is given by (34), (35), and (36)
- Goods, (32), and labour, (33), markets clear
- Firms optimize: (29), (30), and (31)

taking as given initial conditions,  $B(s^0)$ ,  $\Delta(s^0)$ ,  $X(s^0)$ , and exogenous processes for shocks and foreign variables  $\{A(s^t), V(s^t), i^*(s^t), P^*(s^t)\}_{t=0}^{\infty}$ . The effective terms of trade,  $X(s^t)$ , given by (25), is the only relative price that matters for the characterization of equilibrium.

## **3** Optimal Monetary Policy

I proceed to characterize the optimal monetary policy. I provide a generalized analysis of the optimum, before specifically analyzing cost-push shocks and ranking simple rules. Some salient results emerge. These include the insurance role played by optimal monetary policy in the presence of nominal rigidities and financial exclusion, and the desirability of stabilizing the nominal exchange rate in these circumstances. I focus on the Cole-Obstfeld parameterization (Cole and Obstfeld, 1991),  $\sigma = \varepsilon_I = \varepsilon_F = 1$ , to keep the analysis tractable. Two flexible price allocations are characterized, to serve as references, before turning to the constrained efficient case where monetary policy plays a role.

## 3.1 Flexible Prices

I begin by describing the efficient allocation of economic resources in the absence of market imperfections. This is the allocation away from which optimal monetary policy, in the presence of real and nominal rigidities, seeks to minimize deviations (Gali and Monacelli, 2005). The efficient allocation corresponds to the solution of a Planner's problem, that in this study maximizes a weighted sum of household utilities and faces the flexible price and perfectly competitive versions of the constraints in the optimal monetary policy problem. The efficient allocation can be decentralized through a wage subsidy, funded through lump-sum taxes on the financially-included household, (13).

I will show that the efficient allocation does not coincide with the flexible price version of the model, sometimes called the natural allocation, implying that the flexible price business cycle and steady state of the model are inefficient. I choose an appropriate wage subsidy,  $\tau^e$ , to implement the efficient allocation at the steady state. To derive the efficient allocation, the Planner maximizes a weighted sum of household utilities,  $U(s^t) = \lambda \check{U}(s^t) + (1 - \lambda)\hat{U}(s^t)$ , with weights given by  $\lambda$ , and where  $\chi_1(s^t) - \chi_8(s^t)$ are Lagrange multipliers attached to the constraints.

The optimization problem is below. The first two constraints in the optimization problem are aggregate consumption and labour, the third is the production function, the fourth and fifth are the optimal labour supply conditions of the financially-excluded and financially-included agents respectively, the sixth is the financially-excluded agents' consumption function through their budget constraint, the seventh is the international risksharing condition of financially-included agents where exogenous world consumption is assumed to be constant, and the eighth is the goods-market clearing condition.

$$\begin{split} \underset{\Sigma(s^{t})}{\text{Max}} U\left(s^{t}\right) &= \lambda \left( Ln\check{C}^{e}\left(s^{t}\right) - \frac{\check{N}^{e}\left(s^{t}\right)^{1+\phi}}{1+\phi} \right) + (1-\lambda) \left( Ln\hat{C}^{e}\left(s^{t}\right) - \frac{\hat{N}^{e}\left(s^{t}\right)^{1+\phi}}{1+\phi} \right) \\ \text{s.t.} & \left[ \chi_{1}\left(s^{t}\right) \right] \quad C^{e}\left(s^{t}\right) &= \lambda\check{C}^{e}\left(s^{t}\right) + (1-\lambda)\hat{C}^{e}\left(s^{t}\right) \\ & \left[ \chi_{2}\left(s^{t}\right) \right] \quad N^{e}\left(s^{t}\right) &= \lambda\check{N}^{e}\left(s^{t}\right) + (1-\lambda)\hat{N}^{e}\left(s^{t}\right) \\ & \left[ \chi_{3}\left(s^{t}\right) \right] \quad Y^{e}\left(s^{t}\right) &= A\left(s^{t}\right)N^{e}\left(s^{t}\right) \\ & \left[ \chi_{4}\left(s^{t}\right) \right] \quad w^{e}\left(s^{t}\right) &= \check{C}^{e}\left(s^{t}\right)\check{N}^{e}\left(s^{t}\right)^{\phi} \\ & \left[ \chi_{5}\left(s^{t}\right) \right] \quad \check{C}^{e}\left(s^{t}\right) &= \tilde{C}^{e}\left(s^{t}\right)\check{N}^{e}\left(s^{t}\right) \\ & \left[ \chi_{7}\left(s^{t}\right) \right] \quad \check{C}^{e}\left(s^{t}\right) &= X^{e}\left(s^{t}\right)^{1-\alpha} \\ & \left[ \chi_{8}\left(s^{t}\right) \right] \quad Y^{e}\left(s^{t}\right) &= \alpha X^{e}\left(s^{t}\right)^{\alpha}\check{C}^{e}\left(s^{t}\right) + (1-\alpha)X^{e}\left(s^{t}\right)^{\alpha}C^{e}\left(s^{t}\right) \end{split}$$

where  $\Sigma\left(s^{t}\right) = \left\{\hat{C}^{e}\left(s^{t}\right), \check{C}^{e}\left(s^{t}\right), C^{e}\left(s^{t}\right), \hat{N}^{e}\left(s^{t}\right), \check{N}^{e}\left(s^{t}\right), N^{e}\left(s^{t}\right), \omega^{e}\left(s^{t}\right), \Upsilon^{e}\left(s^{t}\right), X^{e}\left(s^{t}\right)\right\}.$ 

The optimum derives as a non-linear system of seventeen endogenous variables, which solve an equilibrium with nine first-order conditions and eight constraints. Further to some manipulations, it is possible to summarize the equilibrium in a more compact system of three equations with three unknowns:  $X^e(s^t)$ ,  $\hat{N}^e(s^t)$ , and the shadow value of the resource constraint,  $\chi_8(s^t)$ .

$$\begin{split} X^{e}\left(s^{t}\right) - A\left(s^{t}\right)\hat{N}^{e-\phi}\left(s^{t}\right) &= 0\\ \frac{1}{X^{e}\left(s^{t}\right)} - \chi_{8}\left(s^{t}\right)\left(\lambda\hat{N}^{e}\left(s^{t}\right)^{\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= 0\\ \frac{\lambda\phi\hat{N}^{e}\left(s^{t}\right)^{-1} - (1-\lambda)\hat{N}^{e}\left(s^{t}\right)^{\phi} - \chi_{8}\left(s^{t}\right)A\left(s^{t}\right)\left(1-\lambda\right)}{X^{e}\left(s^{t}\right)\left(1-\alpha\right)\phi\lambda} - \chi_{8}\left(s^{t}\right) &= 0 \end{split}$$

Proposition 1 gives the closed-form expressions for the efficient labour allocation,  $N^e(s^t)$ , the financially-included agent's consumption,  $\hat{C}^e(s^t)$ , and the financially-excluded agent's consumption,  $\check{C}^e(s^t)$ .

**Proposition 1.** (Efficient Allocation) *In the first-best allocation for the economy when prices are flexible and firms operate in perfect competition, employment is given by* 

$$\lambda \phi \left( \frac{N^{e}(s^{t})}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{-1} - (1-\lambda) \left( \frac{N^{e}(s^{t})}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{\phi}$$
$$= \frac{(1-\alpha)\phi\lambda - (1-\lambda) \left( \frac{N^{e}(s^{t})}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{\phi}}{\lambda \left( \frac{N^{e}(s^{t})}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}}$$
(39)

and financially-included and financially-excluded consumption are functions of aggregate employment

$$\begin{split} \check{C}^{e}\left(s^{t}\right) &= A\left(s^{t}\right)\left(\frac{N^{e}\left(s^{t}\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi\alpha} \\ \hat{C}^{e}\left(s^{t}\right) &= A\left(s^{t}\right)\left(\frac{N^{e}\left(s^{t}\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{-\phi\left(1-\alpha\right)} \end{split}$$

*Closed-form solutions for the other endogenous variables, as well as the Lagrange multipliers, are backed out from the remaining constraints and optimality conditions.* 

The solution to the Planner's problem with heterogeneous agents, when  $\lambda = 0$ , nests Gali and Monacelli (2005) where  $N^e(s^t) = (1 - \alpha)^{\frac{1}{1+\phi}}$ . Furthermore, note that the consumption of financially-included and financially-excluded agents differ as only  $1 - \lambda$  fraction of the economy shares risk internationally. This is in contrast to the closed-economy case of Bilbiie (2008), where consumption of both agents are limited to be identical in both flexible price allocations. The competitive allocation for the flexible price economy derives from the definition of equilibrium provided in the previous section, and is given in closed-form in Proposition 2.

**Proposition 2.** (Natural Allocation) *In the second-best allocation with flexible prices and monopolistic competition, employment is* 

$$N^{n}\left(s^{t}\right)^{\phi}\left(N^{n}\left(s^{t}\right)-(1-\alpha)\lambda\frac{\varepsilon_{P}-1}{\varepsilon_{P}}\right)=\frac{\varepsilon_{P}-1}{\varepsilon_{P}}\left(1-\lambda\right)^{\phi}\left((1-\alpha)(1-\lambda)+\alpha\right)$$
(40)

Other variables are backed out in closed-form from the decentralized equilibrium conditions. The subsidy  $\tau^e$  that restores steady state efficiency is of size  $\tau^e = 1 - \frac{\varepsilon_P - 1}{\varepsilon_P} / (1 - \alpha)$ .

Monetary policy does not play a stabilization role in either of the flexible price allocations since prices can instantly jump. When prices are sticky, however, the relative price distortions that arise due to sluggish real adjustment justify policy intervention. Upon trade-off creating shocks, welfare with sticky prices is strictly lower than with flexible prices, and for the purposes of comparison, it will be useful to express the allocation with nominal rigidities as log-deviations,  $\tilde{x}_t$ , from the flexible price reference allocations.

## 3.2 Sticky Prices

It is convenient to work with a linearized version of the model hereafter, as the non-linear equilibrium is a complicated system of stochastic difference equations. Computing analytical solutions to the non-linear problem with nominal rigidities becomes intractable. This approach is also followed by much of the analytical literature on optimal monetary policy, for eg. Gali and Monacelli (2005) and Farhi and Werning (2012). I characterize the constrained efficient allocation away from a symmetric deterministic steady state. This is done by taking a linear approximation of the constraints and a quadratic approximation of the welfare loss function. The optimal targeting rule that results is internally consistent and a locally linear approximation of the non-linear optimal policy. These perturbation techniques are described further in Benigno and Woodford (2012).

#### 3.2.1 Constraints

The constraints in the Central Bank optimization problem are the first-order linear approximations, in log-deviation or "gap" terms, of the equilibrium conditions defined in section 2.6. A linearized variable,  $f(s^t)$ , is related to its non-linear value and steady state,  $F(s^t)$  and F, approximately as  $f(s^t) \approx \frac{F(s^t) - F}{F}$ .<sup>5</sup> A variable in log-deviation terms,  $\tilde{f}(s^t)$  is defined as the sticky price linearized variable,  $f(s^t)$ , in deviation from the efficient linearized variable,  $f(s^t)^e$ , so that  $\tilde{f}(s^t) = f(s^t) - f(s^t)^e$ . Note that each linearized variable is measured in deviations from its *efficient* version, since the first-best case is the relevant welfare benchmark. Henceforth, I refer to  $\tilde{f}(s^t)$  as an "efficient gap". I also use

<sup>&</sup>lt;sup>5</sup>At the symmetric and efficient steady state with zero inflation, shocks  $\{A, V, C^*, i^*\}$  are normalized to 1. Quantities and relative prices are endogenous. As financial assets and profits are zero in the steady state, the budget constraints of both households coincide. Then due to the same functional form for preferences,  $C = \hat{C} = \check{C}$  and  $N = \hat{N} = \check{N}$ . It is possible to derive that X = 1 by combining the resource constraint,  $Y = CX^{1-\alpha}$ , risk-sharing condition,  $C = X^{1-\alpha}$ , production function, Y = N, marginal cost condition,  $1 = wX^{\alpha}$ , and labour supply condition,  $w = CN^{\phi}$ . The remaining variables are directly backed out.

 $\tilde{f}(s^t) \equiv \tilde{f}_t$ , and the expectations operator,  $E_t \{.\}$ , to save on notation.

It is possible to summarize the constraints in demand (dynamic IS Equation) and supply blocks (New Keynesian Phillips Curve, or NKPC), similar to Boerma (2014). Notably, in the presence of financially-excluded agents, it is still possible to derive a condition that resembles the canonical IS Equation for the small open economy. This is done by substituting the resource constraint, (32), into the aggregate Euler Equation. The latter combines the consumption of financially-excluded agents, (2), and the intertemporal optimality condition of financially-included agents, (15), so thus summarizes the joint consumption evolution of the heterogeneous households in the model

$$\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\tilde{y}_t - \alpha\tilde{x}_t = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}E_t\tilde{y}_{t+1} - \alpha E_t\tilde{x}_{t+1} - (i_t - E_t\pi_{t+1})$$

The Phillips Curve is derived by combining the evolution of domestic inflation resulting from firm optimization,  $\pi_{Ht} = \beta \pi_{Ht+1} + \xi m \tilde{c}_t$ , where  $\xi = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ , and  $m \tilde{c}_t = \frac{\phi}{1-\lambda} \tilde{y}_t + \tilde{x}_t$ , which is a version of the marginal cost condition derived by replacing the resource constraint, (32), risk-sharing condition (17), financially-excluded consumption, (2), and the optimal labour supply conditions, (3) and (14), into real marginal costs given by  $m \tilde{c}_t = \tilde{\omega}_t + \alpha \tilde{x}_t$ . In the NKPC, I also make use of the relation between the terms of trade and output,  $\tilde{x}_t = Y \tilde{y}_t$ , which is derived by combining the resource constraint with risk-sharing. Proposition 3 formalizes the constraints in the optimal policy problem.

**Proposition 3.** (Canonical Equilibrium) *For a particular specification of monetary policy, the equilibrium with inflexible prices is* 

$$\tilde{y}_{t} = E_{t} \tilde{y}_{t+1} - \frac{1}{\gamma} \left( i_{t} - E_{t} \pi_{Ht+1} - r_{t}^{e} \right)$$
(41)

$$r_t^e = -\Upsilon \left(1 - \rho_a\right) a_t \tag{42}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{\xi}{\Lambda} \tilde{y}_t + v_t \tag{43}$$

where  $\Upsilon = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}$ ,  $\xi = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ ,  $\Lambda = \frac{1-\lambda}{1-\lambda-\lambda\phi(1-\alpha)+\phi}$ ,  $v_t$  is an exogenous cost-push shock in the NKPC, and  $r_t^e$  is the equilibrium efficient rate of interest.

These constraints nest Gali and Monacelli (2005) when  $\lambda = 0$ . Note, however, that  $\alpha$  enters the canonical equilibrium when  $\lambda > 0$ . In the presence of financial exclusion,

openness,  $\alpha$  (or conversely home bias,  $1 - \alpha$ ), plays an explicit role in linking inflation and output. As financially-excluded agents cannot smooth consumption, they are increasingly adversely affected by exchange rate volatility as the economy opens up.

The property of Divine Coincidence (Blanchard and Galí, 2007) is present in the model, implying that the planner can simultaneously stabilize domestic inflation and the *efficient* output gap. In the absence of cost-push shocks, setting  $\pi_{H,t} = 0$  closes the *natural* output gap,  $y_t - y_t^n$ , when  $\tilde{f}_t = f_t - f_t^n$  in Proposition 3. But natural and efficient output are scalar functions of each other, so that strict domestic inflation targeting (DIT),  $\pi_{H,t} = 0$ , is able to achieve the first-best allocation of  $\pi_{H,t} = y_t - y_t^e = 0$  in the absence of cost-push shocks. This has implications for the optimal monetary policy response to productivity shocks, as discussed further in Section 2.4.2. Productivity shocks in these types of models do not create any meaningful trade-offs, unlike cost-push shocks that are typically considered for the optimal design of monetary policy (Clarida et al., 2001).

The equilibrium is determinate as long as the optimal targeting rule, (49), is implemented. This is the focus of this study. For ad-hoc rules, the equilibrium is indeterminate for high values of  $\lambda$ , as is typical of DSGE models with hand-to-mouth agents, for instance in Galí et al. (2007).<sup>6</sup> This threshold is given by  $\lambda < \lambda^* = \frac{1}{1+\phi(1-\alpha)}$ , a result shown in Bilbiie (2008) and Boerma (2014). However, even with ad-hoc rules, it is possible to have  $\lambda^*$  as high as 0.8 with a standard range of calibrations for  $\phi$  and  $\alpha$ , so that the threshold value does not affect any of the results. The percentage of financially excluded agents,  $\lambda$ , in the vast majority of emerging market economies, is less than 80%.

#### 3.2.2 Central Bank Loss Function

The objective of optimal monetary policy is to maximize the expected utility of households. The Central Bank loss function is a second-order approximation of expected utility, as described in Benigno and Woodford (2012) who show how to derive an approximation to household welfare that takes the form of a discounted quadratic loss function with terms including those in inflation and the output gap. The advantage of using this method to micro-found the monetary policy objective is that is affords an internally con-

<sup>&</sup>lt;sup>6</sup>When  $\lambda < \lambda^*$ , the elasticity of output with respect to the nominal interest rate,  $-\frac{1}{Y}$ , from the IS Equation, (41), is negative so that, ceteris paribus, contractionary monetary policy would lead to a real contraction as one would expect. However, when  $\lambda \ge \lambda^*$ , the interest elasticity of output is positive, ie.  $-\frac{1}{Y} > 0$ , which leads to indeterminacy (discussed further in Bilbiie, 2008).

sistent and precise characterization of which terms appear in the loss function, with relative weights that are contingent on the specific distortions and monetary transmission mechanism in the model considered.

In this paper, I extend the closed-economy, representative agent approach of Benigno and Woodford (2012) to an open economy, heterogeneous agent setting. The Central Bank maximizes aggregate welfare,  $W_t$ , through the *weighted sum* of household expected utilities, as in Bilbiie (2008), with the weight,  $\lambda$ , depending on the degree of financial exclusion

$$W_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ \lambda \check{U}_{t+k} + (1-\lambda) \hat{U}_{t+k} \right\}$$
(44)

A second-order approximation of  $\lambda \check{U}_{t+k} + (1-\lambda) \hat{U}_{t+k}$  initially yields some non-zero linear terms, where *t.i.p.* stand for "terms independent of policy" (constants and functions of disturbances), and o(3) contains terms of third-order and higher (to place a bound on the amplitude of the perturbations)

$$\mathscr{L}_{t} = -E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ \frac{1}{2} \frac{\varepsilon}{\xi} \pi_{H,t+k}^{2} + \frac{U_{N}}{U_{C}} \frac{N}{C} \tilde{y}_{t+k} + \tilde{c}_{t+k} \right\} + t.i.p. + o(3)$$

To be able to evaluate optimal policy upto second-order, the linear terms in  $\tilde{y}_{t+k}$  and  $\tilde{c}_{t+k}$  need to be eliminated. I follow the method of Benigno and Woodford (2005) to eliminate  $\tilde{y}_{t+k}$  and  $\tilde{c}_{t+k}$  through the analytical approach of replacing for these terms through second-order approximations of the following non-linear equilibrium conditions: the consumption of financially-excluded agents, (2), labour supply by both agents, (3) and (14), risk-sharing, (17), evolution of price dispersion, (30), the aggregate production function, (31), the optimal price-setting equation, (28), the resource constraint, (32), and finally aggregate labour and consumption, (33) and (34). The resulting loss function is an expression with *purely quadratic* terms, as described in Proposition 4.

**Proposition 4.** (Welfare Loss Function) Central Bank preferences in a small open economy with financially-excluded agents, with  $\tilde{x}_t = x_t - x_t^e$  and  $\tilde{y}_t = y_t - y_t^e$ , are represented as

$$\mathscr{L}_{t} = -\frac{1}{2} E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ \Psi_{\pi} \pi_{H,t+k}^{2} + \Psi_{y} \tilde{y}_{t+k}^{2} + \Psi_{x} \tilde{x}_{t+k}^{2} \right\}$$
(45)

where the weights on domestic inflation, output gap, and terms of trade gap, are

$$\begin{split} \Psi_{\pi} &= \frac{\varepsilon_{P}}{\xi} \left( \Xi \Lambda \left( 1 + \nu \right) - \left( 1 + (1 - \alpha) \Phi \right) \nu \lambda \right) - \frac{\varepsilon_{P}}{\xi} \frac{U_{N}}{U_{C}} \frac{N}{C} \\ \Psi_{y} &= \lambda (1 - \alpha)^{2} \nu^{2} - (1 - \alpha) - \lambda (1 - \lambda) \nu^{2} - \Xi \Lambda \lambda (1 - \alpha) \nu^{2} \\ &+ \Xi \left( \Lambda^{-1} + \Lambda \right) - \frac{1 + \phi}{1 - \lambda} \frac{U_{N}}{U_{C}} \frac{N}{C} \\ \Psi_{x} &= (1 - \alpha) \left( 1 - 2\alpha \right) + 2(1 - \alpha) \omega \Upsilon - \Xi \Lambda \left( 1 - 2\alpha \right) - 2\Xi \Lambda \omega \Upsilon \end{split}$$

and the composite parameters are

$$Y = \frac{1 - \lambda - \lambda \phi (1 - \alpha)}{1 - \lambda}$$

$$\Lambda = \frac{1 - \lambda}{1 - \lambda - \lambda \phi (1 - \alpha) + \phi}$$

$$\omega = \frac{\lambda \phi (1 - \alpha)^2 + \alpha (1 - \lambda)}{1 - \lambda}$$

$$\nu = \frac{\phi}{1 - \lambda}$$

$$\Xi = (1 - \alpha)Y + \lambda \nu + \frac{U_N}{U_C} \frac{N}{C}$$

$$\Phi = \Xi \Lambda - (1 - \alpha)$$

with the efficient variables  $x_t^e$  and  $y_t^e$  as functions of structural parameters and shocks.

The loss function in an open economy model with financial exclusion features a quadratic term in the terms of trade beyond what is usually found in models where all households are financially-included. When  $\lambda = 0$ , the criterion collapses to that in Gali and Monacelli (2005) where  $\tilde{x}_t^2 = \tilde{y}_t^2$ :  $\Psi_{\pi}|_{\lambda=0} = \Psi_{\pi}^{GM} = \frac{\varepsilon_p}{\zeta} \frac{U_N}{U_C} \frac{N}{C}$  and  $\Psi_y|_{\lambda=0} + \Psi_x|_{\lambda=0} = \Psi_y^{GM} = (1 + \phi) \frac{U_N}{U_C} \frac{N}{C}$ . I give the loss function below with  $\lambda = 0$ , ie. the nested Gali and Monacelli (2005) case, as this serves as a useful point of comparison for some of the results. (46) requires that  $\frac{U_N}{U_C} \frac{N}{C} = -(1 - \alpha)$ , ie. the steady state is rendered efficient.

$$\mathscr{L}_t|_{\lambda=0} = \mathscr{L}_t^{GM} = -\frac{1-\alpha}{2} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\varepsilon_p}{\xi} \pi_{H,t+s}^2 + (1+\phi) \tilde{y}_{t+s}^2 \right\}$$
(46)

The key driver for the differences in Central Bank preferences with financial exclusion is the asymmetry in the ability of domestic households to share risk. Recall that financiallyincluded households can pool risk with foreign agents, (17), by receiving (or making) appropriate international insurance transfers, (38). This smooths their consumption upon fluctuations in their income. In sharp contrast, financially-excluded agents must fully consume their income, (2). This implies that, unlike the  $\lambda = 0$  case, (46), the stabilization of the terms of trade gap,  $\tilde{x}_t^2$ , is required in (45) to minimize welfare loss.

# **Result 1.** *Terms of trade fluctuations have first-order distortionary effects in the presence of financial exclusion.*

In an economy where all agents can smooth consumption, shocks to purchasing power due to exchange rate fluctuations are offset through international risk-sharing arrangements. For instance, import price volatility, caused by exchange rate movements, does not affect financially-included agents by much as they are able to adjust their financial assets in a manner that consumption is smoothed. However, financially-excluded agents cannot similarly hedge against exchange rate volatility. Their inability to smooth their consumption against the volatility in international relative prices increases macroeconomic volatility, leading to aggregate welfare losses. This incentivizes the Central Bank to stabilize the terms of trade.

**Policy Trade-offs** Result 1 is one dimension of the loss-minimizing objective, but what do Central Bank preferences look like overall? It is useful to provide more intuition on the micro-founded objective, (45), as this drives most of the optimal monetary policy results. However, the weights in (45) are complicated functions of structural parameters, so that their implications are not transparent. This can be addressed by re-writing the loss function in terms of only domestic inflation,  $\pi_{H,t}$ , and the output gap,  $\tilde{y}_t$ , as (47), by using the proportionality between the terms of trade gap and the output gap,  $\tilde{x}_t = Y \tilde{y}_t$ . The optimal policy trade-offs that arise are then captured by the relative weight, (48).<sup>7</sup>

$$\mathscr{L}_t = -\frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^s \left\{ \pi_{H,t+k}^2 + \Phi^\lambda \tilde{y}_{t+k}^2 \right\}$$
(47)

where the relative weight on  $\tilde{y}_t$ ,  $\Phi^{\lambda}$ , is given by

$$\Phi^{\lambda} = \frac{\Xi \left(\Lambda^{-1} + \Lambda\right) - \left(1 - 2\alpha\right)\Phi Y^{2} - \left(\left(1 - \lambda\right) + \left(1 - \alpha\right)\Phi\right)\lambda\nu^{2} - 2\Phi\omega Y - \left(1 - \alpha\right) - \frac{1 + \phi}{1 - \lambda}\frac{U_{N}}{U_{C}}\frac{N}{C}}{\frac{\varepsilon}{\zeta}\left(\Xi\Lambda\left(1 + \nu\right) - \left(1 + \left(1 - \alpha\right)\Phi\right)\nu\lambda\right) - \frac{\varepsilon_{P}}{\zeta}\frac{U_{N}}{U_{C}}\frac{N}{C}}{(48)}$$

**Result 2.** The relative weight on output gap stabilization,  $\Phi^{\lambda}$ , is increasing in  $\alpha$  and  $\lambda$ .

It is instructive to compare the relative weight with limited financial market participation,  $\Phi^{\lambda}$ , with the Gali and Monacelli (2005) complete financial inclusion case, where the relative weight,  $\Phi^{GM} = \frac{1+\phi}{\frac{\xi}{\xi}}$ , is invariant with respect to openness,  $\alpha$ . In contrast,  $\Phi^{\lambda}$  in this model increases with both  $\alpha$  and  $\lambda$ , as shown in Figure 2.



Figure 2: Optimal Relative Weight on Output Gap

Greater openness implies that domestic agents consume more imports relative to domestic goods. This increases the consequences of exchange rate fluctuations, which adversely affects financially-excluded agents as unlike asset holders, they cannot smooth

<sup>&</sup>lt;sup>7</sup>A numerical check yields that the relative weight,  $\Phi^{\lambda}$ , is strictly positive when the structural parameters in the loss function,  $\alpha$ ,  $\lambda$ ,  $\phi$ ,  $\varepsilon$ ,  $\beta$ ,  $\theta$ , jointly satisfy the following system of inequalities:  $0 < \alpha < 0.95$ ,  $0 < \lambda < 0.95$ ,  $0 < \theta < 1$ ,  $0 < \beta < 1$ ,  $\varepsilon_P > 0$ ,  $\phi > 0$ . The positive weight implies that the quadratic loss function is convex so that the second-order sufficient conditions are satisfied. Note that convexity ensues for a wide range of parameter values in this model due to the logarithm and isoelastic functional forms in utility, as also in Benigno and Woodford (2005).

their consumption against the increasingly volatile price of imported goods. The resulting consumption volatility leads to higher aggregate welfare loss, incentivizing the Central Bank to smooth the consumption of hand-to-mouth agents by stabilizing their income through the output gap.  $\frac{\partial \Phi^{\lambda}}{\partial \alpha} \neq 0$  in the model is thus contingent on  $\lambda > 0$ , i.e. a fraction of agents unable to insure against risk. Similarly,  $\Phi^{\lambda}$  increases with  $\lambda$  to minimize the greater loss as progressively more agents cannot smooth consumption.

## **Result 3.** *The closed economy case is not isomorphic for* $\lambda > 0$ *.*

This point is of independent interest to the question at hand, but merits a mention. Optimal policy in the open economy model with financially-excluded agents does not necessarily converge to the closed economy case when  $\alpha \rightarrow 0$ . Usually considered desirable from a modeling perspective, other instances can be found in Monacelli (2005) and Farhi and Werning (2012). There is an irreversible open economy asymmetry in this model compared to the closed economy version of Bilbiie (2008), as only a fraction of domestic consumers can share risk with foreign agents as the economy opens up. Exchange rate movements affect the heterogeneous agents in completely different ways, implying that consumption patterns fundamentally diverge over the business cycle, overriding the type of closed economy isomorphism that characterizes Gali and Monacelli (2005).

The argument for this is similar to Farhi and Werning (2012). In the closed economy limit ie. as  $\alpha \rightarrow 0$ , a fixed nominal exchange rate,  $e_t - e_{t-1} = 0$  implies a fixed nominal interest rate (equal to the exogenous foreign interest rate,  $i_t^*$ ) through the UIP condition,  $i_t = i_t^* + E_t e_{t+1} - e_t$ . However, it is known that a fixed nominal interest rate could imply equilibrium indeterminacy in a closed economy (Galí, 2009). I numerically find that a fixed nominal interest rate,  $i_t = 0$ , also results in indeterminacy in my model when  $\alpha = 0$ , and indeed for all  $\alpha$ . When  $\alpha > 0$ , however, an exchange rate peg,  $e_t - e_{t-1} = 0$ , ensures a unique equilibrium and moreover approximates the optimal policy, as will be discussed in Section 4. This result is also found in Monacelli (2005) and Farhi and Werning (2012), and would suggest caution in using closed economy models to approximate the open.

**Targeting Rule** I derive the flexible inflation targeting rule for the model under dynamically consistent timeless commitment, where the optimal plan set in later periods is the same as the one that would have been set initially (Woodford, 2011). This requires minimizing the quadratic loss function (45) with respect to the linear aggregate supply relation, (43). The IS Equation does not bind since the nominal interest rate is unconstrained. I attach the Lagrange multiplier,  $\chi_{\pi,t}$ , on the NKPC and take first-order conditions with respect to domestic inflation,  $\pi_{H,t}$ , and the output gap,  $\tilde{y}_t$ 

$$\begin{aligned} 2\Psi_{\pi}\pi_{H,t} + \chi_{\pi,t} - \chi_{\pi,t-1} &= 0 \\ 2\Psi_{y}\tilde{y_{t}} - \xi\Lambda^{-1}\chi_{\pi,t} &= 0 \end{aligned}$$

These can be combined to yield the flexible targeting rule, which is a locally linear approximation to optimal policy, assuming that shocks are sufficiently small in amplitude.

**Proposition 5.** (Optimal Policy) *The optimal plan under timeless commitment is implemented as follows* 

$$\pi_{H,t} = -\frac{\Lambda}{\xi} \Phi^{\lambda} \left( \tilde{y}_t - \tilde{y}_{t-1} \right)$$
(49)

The optimal plan, (49), implies the classic "leaning against the wind" analogy of contracting the output gap to bring down domestic inflation, whenever the latter is inefficiently high. The targeting rule holds only if prices are slow to adjust,  $\xi \neq \infty$ , as it would be redundant in a flexible price environment where inflation creates no real distortions.

The equilibrium is unique with optimal policy for all  $\lambda \in [0, 1)$ , corroborating the closed economy analog in Bilbiie (2008). The targeting rule, (49), is combined with the NKPC, (43), to get the following second-order difference equation

$$E_t \tilde{y_{t+1}} = \left[1 + \frac{1}{\beta} \left(1 + \frac{\xi^2}{\Lambda^2} \frac{1}{\Phi^\lambda}\right)\right] \tilde{y_t} - \frac{1}{\beta} \tilde{y_{t-1}} + \frac{1}{\beta} \frac{\xi}{\Lambda} \frac{1}{\Phi^\lambda} v_t$$

whose roots are numerically found to be on the opposite sides of the unit circle for all  $\lambda$  (Blanchard and Kahn, 1980).

## 3.2.3 Optimal Policy Dynamics

I proceed to analyze how the optimal targeting rule, (49), is employed to mitigate welfare loss in response to unexpected disturbances. The focus is primarily on cost-push shocks as these have been a significant concern in emerging market economies, and create a meaningful trade-off between stabilizing inflation and output (Frankel, 2010). I also analyze productivity shocks. Dynamics are analyzed based on a calibrated version of the framework. Calibration is a challenging task, as the required micro-level data is scarce for EMEs. I thus select parameters from the existing open economy literature, and pair this with extensive sensitivity analysis. The fraction of randomly chosen monopolistic producers that can reset prices,  $\theta$ , is set at 0.75, which implies an average period of one year between price adjustments, as in Gali and Monacelli (2005).

The household discount factor  $\beta$  equals 0.99, which implies a steady state real interest rate of around four percent. The elasticity of substitution between differentiated monopolistic goods is set at  $\varepsilon_P = 6$ , which implies a steady state markup of around 20%. The degree of openness,  $\alpha$ , is set at 0.5 and the inverse Frisch elasticity of labour supply,  $\phi$ , at 1. The persistence of shocks, ie.  $\rho_j$  in the stationary autoregressive process  $j_t = \rho_j j_{t-1} + s_{j,t}$ , where  $j_t = \{v_t, a_t, i_t^*\}_{t=0}^{\infty}$ , is set at 0.9, consistent with the evidence for emerging market economies provided in Aguiar and Gopinath (2007). In most cases, optimal policy with complete financial market participation,  $\lambda = 0$ , is contrasted with  $\lambda = 0.6$ , which is around the EME financial exclusion average (World Bank, 2015).

Figure 3 provides dynamics for varying degrees of financial exclusion upon a unit positive cost-push shock. There is an immediate rise in domestic inflation that arises independently from variations in domestic demand. The Central Bank contracts output to control inflation, and a recession ensues. The rise in domestic prices leads to a fall in aggregate consumption, while also causing a terms of trade appreciation. The variations in dynamics, depending on  $\lambda$ , are based on the trade-off embodied in the optimal targeting rule, (49). With higher financial exclusion, the Central Bank is incentivized to stabilize the output gap by more to smooth the disposable income of hand-to-mouth agents. As these households are unable to insure their consumption against the rise in prices, their response would otherwise be very volatile and lead to aggregate welfare losses. As per the optimal plan, domestic inflation is allowed to rise by a bit more so that a deep recession can be prevented. To support this optimal trade-off, the nominal exchange rate appreciates to put downward pressure on inflation when financial inclusion is high. The exchange rate appreciates by less as exclusion increases to stabilize the terms of trade, and hence the demand for tradable output, as per the optimal plan, (49).



Figure 3: 1% Cost-Push Shock, Optimal Policy and Financial Exclusion

**Result 4.** Monetary policy plays an additional insurance role in the presence of financial exclusion.

For  $\lambda > 0$ , optimal monetary policy takes into account the inability to share risk and hence the more volatile consumption of financially-excluded agents. Implementation of the optimal plan, (49), results in disposable income being stabilized by more as financial exclusion increases. This can be interpreted as provision of insurance by monetary policy to agents who cannot privately smooth consumption, and is a required transmission channel in the efficient plan. Thus, besides macroeconomic stabilization, monetary policy plays an additional insurance role when  $\lambda > 0$ . The extent of insurance varies depending on the degrees of openness and price stickiness, as shown next.

## 3.2.4 Sensitivity Analysis

I analyze the implications of the optimal plan, (49), for  $\alpha = \{0.1, 0.4, 0.7\}$  and  $\theta = \{0.7, 0.75, 0.8\}$ . The financial exclusion case ( $\lambda = 0.6$ ) while varying  $\alpha$  is depicted below in Figure 4 (corresponding  $\lambda = 0$  case is in the appendix). The dynamics suggest that the insurance provided by monetary policy is increasing in openness and price flexibility.

## Openness

When financial inclusion is complete,  $\lambda = 0$ , the trade-off is independent of openness as changes to real income from exchange rate fluctuations are offset through international risk-sharing. This independence is broken in the presence of financial exclusion,  $\lambda > 0$ . Here, exchange rate volatility causes equivalent fluctuations in the consumption of financially-excluded agents (and by a greater amount as  $\alpha \rightarrow 1$  due to the increased consumption of imports), unlike that of financial asset holders. To mitigate the resulting greater macroeconomic volatility, optimal monetary policy smooths the consumption of hand-to-mouth agents by more. The optimal plan thus places greater relative weight on stabilizing output in response to cost-push shocks. To support this, the nominal exchange rate depreciates by more to mitigate the terms of trade appreciation, hence relatively stabilizing the demand for output as per the optimal trade-off, (48). This helps smoothen the disposable income of hand-to-mouth households as the economy opens up.



Figure 4: 1% Cost-Push Shock, Optimal Policy with  $\lambda = 0.6$ , Varying Openness

## **Nominal Rigidities**

Stickier prices,  $\theta \rightarrow 1$ , imply that fewer randomly chosen price-setters re-optimize each period. Thus, upon a cost-push shock, the required downward re-optimization of prices (in anticipation of future high domestic inflation) is hindered. This leads to more domestic inflation volatility, requiring a greater contraction of the output gap as per the optimal

plan. The resulting steeper recession dampens labour demand by more. Consequently, lower real wages clear the market, implying that hand-to-mouth consumption decreases by more. Nominal rigidities thus hamper the provision of insurance by optimal monetary policy. These dynamics are found in the appendix.

## 4 Nominal Anchor: Peg or Float?

I analyze the appropriate choice of a nominal anchor in an economy with financial exclusion. To do so, simple and implementable rules are ranked according to the loss function, (47), in terms of the lowest welfare losses away from the optimum. Simple rules as approximations of optimal monetary policy are in particular useful, since the optimum can sometimes be cast as a complicated and unintuitive function of model parameters. Optimal policy is thus often not as transparent and implementable in practice as a simple rule. I show that targeting the exchange rate is an implementable way to internalize the insurance properties of the optimal plan characterized in the previous section.

Simple Rule	Financial Inclusion	Financial Exclusion
Strict CPI IT (CIT)		
$\pi_t = 0$	0.019	0.024
Strict domestic IT (DIT)		
$\pi_{H,t} = 0$	0.032	0.143
Fixed exchange rate (PEG)		
$\Delta e_t = 0$	0.139	0.001

Table 1: All Entries are in % Units of Steady State Consumption

Table 1 provides the relative welfare loss numbers of some standard simple rules compared to the benchmark optimal policy, upon trade-off creating cost-push shocks. There are two columns: the first reports welfare losses in the case of complete financial inclusion,  $\lambda = 0$ , whereas the second does likewise with high financial exclusion,  $\lambda = 0.6.^{8}$  All entries in Table 1 are, as in Gali and Monacelli (2005), percentage units of steady-state consumption and in deviation from the first-best case of optimal monetary policy. It can be seen that while strict CPI Inflation Targeting,  $\pi_t = 0$ , best approximates the

<sup>&</sup>lt;sup>8</sup>Many variations of simple rules were compared, including flexible Taylor-type rules with inflation and the output gap, but the ranking remains the same ie. CIT leads to lowest welfare losses with high financial inclusion and PEG leads to lowest welfare losses with high financial exclusion. Also, while the table here discusses the cases of  $\lambda = 0$  versus  $\lambda = 0.6$ , CIT is appropriate till around a threshold of 40% financial inclusion ( $\lambda = 0.4$ ), after which PEG is preferred.
optimal policy when all agents can smooth consumption, a fixed exchange rate,  $\Delta e_t = 0$ , is least suboptimal when financial exclusion is high.

Figure 5 connects the optimal monetary policy, described in (45) and (49), with the implementation of simple monetary rules. Complementing Table 1 that reports whether a peg or float is preferred for  $\lambda = 0, 0.6$  with cost-push shocks, Figure 5 displays the appropriate choice of a nominal anchor for  $\lambda, \alpha \in \{0, 0.0.8\}$ . I reproduce the optimal relative weight on output gap stabilization, Figure 2, to show that while flexible exchange rates (CIT) are preferred for lower values of  $\lambda$  and  $\alpha$ , exchange rate targeting (PEG) approximates the optimal plan with higher financial exclusion and openness.



Figure 5: Optimal Relative Weight on Output Gap: Implemented by Fixed (PEG) versus Flexible (CIT) Exchange Rates

#### **Result 5.** A nominal exchange rate peg is least suboptimal with high financial exclusion.

Table 1 indicates that fixed exchange rates are preferred to flexible exchange rates when the majority of households in the economy are financially-excluded. Targeting the nominal exchange rate mitigates fluctuations in the relative price of domestic goods, which stabilizes output and disposable income similar to the optimal plan, (49). This policy is desirable in the presence of high asset market inequality, when a large fraction of agents cannot smooth consumption. A fixed exchange rate also stabilizes the fluctuations in imported goods prices, smoothening hand-to-mouth consumption with cost-push shocks. Notably, dynamics under a fixed exchange rate almost completely mirror those under optimal policy, as shown in Figure 6, which compares PEG with the optimal targeting rule, (49), CIT (least suboptimal policy when  $\lambda = 0$  with cost-push shocks), and DIT (optimal policy in this model with productivity shocks).



Figure 6: 1% Cost-Push Shock, Optimal vs Simple Rules, Financial Exclusion ( $\lambda = 0.6$ )

**Result 6.** CPI Inflation Targeting is appropriate with high financial inclusion.

When financial inclusion is close to complete, domestic inflation volatility leads to greater aggregate welfare loss compared to when financial market participation is limited. Thus, optimal policy places greater weight on stabilizing domestic inflation. With fixed exchange rates, the muted real appreciation implies that marginal costs, and hence domestic inflation, increase by too much. Thus, PEG is highly suboptimal. DIT, on the other hand, leads to extreme output gap volatility (which is still penalized, albeit by less) in fully stabilizing domestic inflation. CIT strikes the appropriate in-between as it results in less output gap volatility than DIT (it stabilizes *both* domestic and imported good inflation, and the output gap responds to *domestic* inflation only as per the optimal targeting rule, (49), and less domestic inflation volatility than PEG due to greater real appreciation. These dynamics are found in the appendix.

### 4.1 Role of Openness

I vary the degrees of openness and nominal rigidities, to analyze whether CIT and PEG remain robust. I find that they do. Price stickiness matters to the extent that the simple

rules approximate optimal policy better as nominal rigidities lessen ie.  $\theta \rightarrow 0$  (because the role of optimal policy diminishes with increased price flexibility); however, varying  $\alpha$  yields an interesting result.

### **Result 7.** When $\alpha > 0$ , CIT and PEG work better with lower degrees of openness.

CIT,  $\pi_t = 0$ , approximates the  $\lambda = 0$  optimal monetary policy better, and PEG,  $\Delta e_t = 0$ , does likewise for the  $\lambda = 0.6$  optimal policy, as  $\alpha \to 0$ . Note, however, that this does *not* imply that  $\pi_{H,t} = 0$  and  $i_t = 0$  (the closed economy analogs of CIT and PEG) perform well when  $\alpha = 0$ . In the former case,  $\pi_{H,t} = 0$  is achieved at the expense of high output gap volatility, which is suboptimal - CIT does not require this. Furthermore,  $i_t = 0$  results in an indeterminate equilibrium for  $\alpha = 0$ . Intuition for the better CIT and PEG approximations with low  $\alpha > 0$  is as follows.

Consider first the case when  $\lambda = 0$ . It is useful to note first the following: (i) lower  $\alpha$  implies that more domestic goods are consumed relative to imported goods, (ii) CIT targets both domestic and import price inflation, and (iii) upon a cost-push shock, while relative domestic prices increase, relative import prices *decrease* due to an expenditure-switching effect. Now, as  $\alpha \rightarrow 0$ , CIT implies the stabilization of progressively more *domestic* inflation (which has increased), which requires higher real appreciation (preventing import deflation would, in contrast, require real *depreciation*). This higher real appreciation as  $\alpha$  decreases matches that under optimal policy for the complete financial inclusion case. PEG does not provide the required high real appreciation for  $\lambda = 0$ .

Now, consider the  $\lambda = 0.6$  case, where the weight on output gap stabilization increases with  $\alpha$  to smooth the consumption of financially-excluded agents. This requires progressively less real appreciation, as discussed in section 3.2.4. With very high  $\alpha$ , the optimal real appreciation is required to be so low that although a nominal peg comes closest to engendering this, even it cannot completely mirror the optimal policy (a different simple rule, perhaps targeting the *real* exchange rate itself, might be appropriate here). However, for low ( $\alpha = 0.1$ ) and moderate ( $\alpha = 0.4$ ) degrees of openness, the real appreciation under the peg is appropriate. In contrast, CIT is inefficient for all  $\alpha$  with financial exclusion, since it requires suboptimally high real appreciation. The corresponding dynamics for  $\lambda = 0$  and  $\lambda = 0.6$  are found in the appendix.

### 4.2 Productivity Shocks

**Result 8.** A fixed nominal exchange rate in a high financial exclusion economy is appropriate only if cost-push shocks predominate.

It is useful to understand the circumstances under which a fixed exchange rate provides the most efficient stabilization. I find that the type of shock matters for the choice of domestic versus external nominal anchor. As shown in Proposition 3, optimal policy with productivity shocks leads to perfect stabilization and is strict domestic inflation targeting (DIT) with any degree of financial exclusion, as in Gali and Monacelli (2005). This can be seen by setting  $\pi_{H,t} = 0$  in the system of equations, (41), (42), and (43), in the absence of cost-push shocks,  $v_t$ , so that the output gap is also perfectly stabilized,  $\tilde{y}_t = 0$ . This DIT policy requires considerable nominal exchange rate volatility. In the presence of financial exclusion, exchange rate stability is appropriate only in the presence of shocks that create a meaningful trade-off for monetary policy.

### 5 Conclusion

This study analyzed optimal monetary policy and the corresponding choice of an appropriate nominal anchor in a small open economy with financial exclusion. There are two main findings. The first result is that that optimal policy seeks to smooth the consumption of financially-excluded agents who cannot privately do so themselves, by increasing the relative weight on the output gap in the micro-founded welfare loss function as financial exclusion and openness increase. The second result is that a fixed nominal exchange rate internalizes the insurance provision properties of the optimal plan when the majority of households do not have financial market access. The desirability of exchange rate stability provides a counterpoint to Milton Friedman's long-standing argument for a float. This paper sought to establish benchmark analytical results, and lends itself to some extensions. It might be interesting to interact imperfect risk-sharing with the asset market segmentation considered in this study, and I leave this for future work.

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## Appendix



Figure 7: 1% Cost-Push Shock, Optimal Monetary Policy with Complete Financial Inclusion: Varying Openness



Figure 8: 1% Cost-Push Shock, Optimal Monetary Policy with Financial Exclusion: Varying Stickiness



Figure 9: 1% Cost-Push Shock, Optimal Policy Versus Simple Rules: Complete Financial Inclusion



Figure 10: 1% Cost-Push Shock, Optimal vs Simple Rules: Complete Financial Inclusion, Low Openness



Figure 11: Optimal Policy Versus Simple Rules: Financial Exclusion, Low Openness

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