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Open Mouth Operations

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Abstract

We examine the standard New Keynesian economy's Ramsey problem written in terms of instrument settings instead of allocations. Its standard formulation makes two instruments available: the path of current and future interest rates, and an "open mouth operation" which selects one of the many equilibria consistent with the chosen interest rates. Removing the open mouth operation by imposing a finite commitment horizon yields pathological policy advice that relies on the model's forward guidance puzzle.

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1 Introduction

Consider a standard New Keynesian economy with an intertemporal-substitution (IS) curve and a forward-looking Phillips curve. Its central banker seeks to minimize deviations of output and inflation from their steady state values following a transitory markup shock. The standard solution to this policy problem first calculates the best allocation of inflation and output consistent with the sequence of Phillips curves and then calculates the required interest rates from this allocation and the sequence of IS curves. This sequence has been interpreted as the central banker's optimal Odyssean forward guidance (Giannoni and Woodford, 2005; Campbell, Evans, Fisher, and Justiniano, 2012).

We demonstrate that this standard exercise does not yield useful policy advice. Writing the Ramsey problem in terms of optimal *instrument settings* instead of allocations clarifies that the Ramsey planner has two instruments: the path of current and future interest rates, and control over a variable which indexes which path of inflation, out of the multitude consistent with those interest rates, private agents implement. The model gives no operational guidance on how the central banker coordinates private-sector expectations on her desired equilibrium, so we label the communications tool she uses for this task an "open mouth operation" or OMO.

Policy advice so calculated tempts policy makers to interpret outcomes driven by OMOs as consequences of interest rate choices. In this way, it can diminish rather than enhance understanding. Considering the outcome of the policy problem as an optimal active interest rate rule fails to add clarity. Following King (2000), we demonstrate that all such rules have a time-varying intercept and an "active" response to deviations of inflation from its time-varying target. The adoption of such a rule commits the central banker to participate in an economic disaster with exploding inflation and output gaps if households and firms fail to coordinate on the chosen equilibrium. If such a commitment successfully implements the central banker's chosen allocation, then the threatened disaster never occurs. Therefore, "adoption" of an optimal active interest rate rule requires persuasion rather than action. This is less credible than a central banker directly coordinating private agents' beliefs, because it requires a declaration of time-inconsistent promises (Cochrane, 2011)

In spite of their empirical implausibility, *OMOs are mathematically essential for implementing the policy prescriptions of the standard New Keynesian economy's Ramsey planning problem.* We provide two complementary demonstrations of this essentiality. First, we note the existence of a knife edge case in which open mouth operations are the *only* instrument which the policy maker changes in response to a cost-push shock. That is, the central banker chooses not to fight inflation with contractionary interest rate policy. Second, we recast the

Ramsey problem from one of choosing allocations subject to the constraints of private-sector optimality to one of choosing policy instruments directly. Here, the OMO appears as an option to choose the initial inflation rate *given all current and future interest rates.*

Since the equilibrium multiplicity which underlies OMOs requires an infinite horizon economy, imposing a finite commitment horizon on the central banker (after which which the output gap and inflation both return to their steady state values) removes it. For a very long commitment horizon, this constraint imposes almost no welfare cost. However, the policy relies on small changes to the final interest rate under the central banker's control to substitute for the OMO. That is, a central banker with a finite horizon leans heavily upon the forward guidance puzzle of Del Negro, Giannoni, and Patterson (2012) and Carlstrom, Fuerst, and Paustian (2015) to implement her desired outcomes.

The next section presents our results characterizing the optimal monetary policy problem in the face of a one-time cost-push shock, as in Giannoni and Woodford (2005). Section 3 contains our characterization of the monetary policy problem with a finite horizon, and Section 4 concludes with a brief discussion of a modification to the standard New Keynesian model which can both remove open mouth operations from the central banker's toolkit and resolve the forward guidance puzzle: stochastically arriving opportunities for central bank reoptimization (Schaumburg and Tambalotti, 2007; Bodenstein, Hebden, and Nunes, 2012; Debortoli and Lakdawala, 2016).

2 Ramsey Planning

Here we demonstrate how the well-known indeterminacy present in the standard model makes an extra instrument available to the central banker in her corresponding Ramsey problem. We begin in a standard New Keynesian economy.[1] The Phillips curve (PC) is

$$\pi_t = \kappa y_t + \beta \pi_{t+1} + m_t \tag{1}$$

with cost-push shock $m_0 \neq 0$ and $m_t = 0$ for all $t > 0$, and the Intertemporal Substitution (IS) curve is

$$y_t = -\frac{1}{\sigma}(i_t - \pi_{t+1} - i^\natural) + y_{t+1}. \tag{2}$$

Here i_t, $i^\natural$, π_t and y_t denote the nominal interest rate, the natural rate of interest, inflation and the output gap. The parameters satisfy $\sigma, \kappa \in (0, \infty)$ and $\beta \in (0, 1)$. We ignore the possibility of an effective lower bound on nominal interest rates, so i_t can take any value.

[1] See Galí (2008) for a derivation of these now-standard equations.

The central banker seeks to minimize a loss function which is quadratic in current and future output gaps and inflation rates.

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}\pi_t^2 + \frac{\lambda}{2} y_t^2 \right) \tag{3}$$

This can be derived from the representative household's utility function (Woodford, 2003) or from the central banker's legislative environment (Evans, 2011).

A central banker solving the Ramsey problem chooses paths for i_t, π_t, and y_t to minimize her loss in (3) while satisfying the sequences of constraints given by (1) and (2). Since i_t only appears in the IS curve, it can be selected to satisfy that equation given arbitrary values for π_{t+1}, y_t and y_{t+1}. Therefore, the only binding constraints on the central banker's choices of inflation and output gap sequences come from the sequence of PCs. For this reason, the standard approach to solving this Ramsey problem first minimizes the loss function constrained only by that sequence and then backs out the necessary values for i_t using the IS curve.

Figure 1 presents an example solution with $m_0 = 1$, or a one percent shock to the Phillips curve in the initial time period. (All of our numerical examples use the same value of $\beta = 0.99$.) If the central banker did nothing, so that $i_t = i^\natural$ for all t, and agents' inflation expectations for $\pi_1, \pi_2, ... = 0$ remained unchanged, then inflation today would have to increase by one percent. Even while taking future outcomes as given, the central banker can improve on this outcome by raising i_t above $i^\natural$ and thereby reducing both inflation and output. Because the loss function is quadratic, the central banker receives a first order gain from reducing inflation, at the cost of a second order loss from reducing output. The optimal setting of i_0 equates the marginal benefit of reducing inflation with the marginal cost of diminishing output. Given the parameter values used in Figure 1 and no control over expectations of the future, this policy would yield $\pi_0 = .8$ and $y_0 = -.8$.

Since the central banker controls π_1, she can further improve outcomes by promising a small deflationary recession. Mechanically, the promised deflation in period one partially offsets the cost-push shock, allowing the central banker to achieve both lower inflation today and a smaller output gap (e.g. Campbell, 2013). This reduces inflation and increases output today, yielding a first order gain, at the expense of a second order loss from the deflationary recession in the future. Intuitively, such a policy improves outcomes by "spreading the pain" of a transitory shock across multiple future periods. The dark blue line plots the first few periods of the optimal plan for inflation, while the red line plots the same for output. Although it is feasible to close these gaps at any time after $t = 0$, they close only asymptotically. This is because the central banker who closes both gaps by some finite time

T can always lower her loss by spreading the pain into $T+1$. The light blue line plots the price level, which is the accumulation of inflation. Optimal policy eventually undoes all the inflation that was allowed to occur in the initial time period. This is the familiar price level targeting result of Giannoni and Woodford (2005).

The dashed black line plots the interest rate consistent with the IS curve given the optimal choices of π_t and y_t. When the central banker has no control over future expectations of π_t, y_t, the optimal initial interest rate jumps to $i^\natural + 1.6$ percent. The Ramsey planner chooses a more modest initial response of $i^\natural + 0.24$ percent, but she keeps the interest rate above $i^\natural$ after the cost-push shock has dissipated.

In the conventional interpretation of the exercise presented in Figure 1, the central banker promises to keep interest rates above the natural rate after the cost-push shock has passed, thereby creating deflationary expectations. However, this explanation fails to accurately describe other, similar forward guidance experiments. To see this, consider Figure 2. This plots the Ramsey solution given the same values for κ and λ but a different value for σ. Since the Phillips curve and loss function are unchanged, the chosen values for π_t and y_t equal those in Figure 1: and they are given by the dark blue and red lines, respectively. With the particular IS curve chosen, the interest rate required by this plan is a constant: $i_t = i^\natural$. That is, we have a deflationary recession without contractionary interest rate policy. This is possible because the central banker's chosen allocation for $t \geq 1$ coincides with one of the many equilibria consistent with $i_t = i^\natural$ always. Indeed, this is the case whenever the parameters satisfy $\sigma\kappa/\lambda = 1$. Implementing this example's Ramsey allocation requires no contractionary interest rate policy, but coordinating agents' expectations with an Open Mouth Operation is essential.

We presented the "knife edge" case in Figure 2 to illustrate the use of a tool that is *always* available and used by the central banker. To demonstrate its existence analytically, cast the planning problem in terms of choosing the settings of the instruments instead of allocations. We plug the PC (1) into the IS curve (2). The result is a single, second order difference equation with forcing function x_t:

$$\pi_t - \left(1 + \beta + \frac{\kappa}{\sigma}\right)\pi_{t+1} + \beta\pi_{t+2} = x_t. \tag{4}$$

Where $x_t \equiv -\frac{\kappa}{\sigma}\left(i_t - i^\natural\right) + m_t - m_{t+1}$. Henceforth, we represent interest-rate policy with x_t. The full set of solutions to (4) for a particular path of x_t is given by a linear combination of two homogenous solutions and a particular solution. The rates of decay of the homogenous

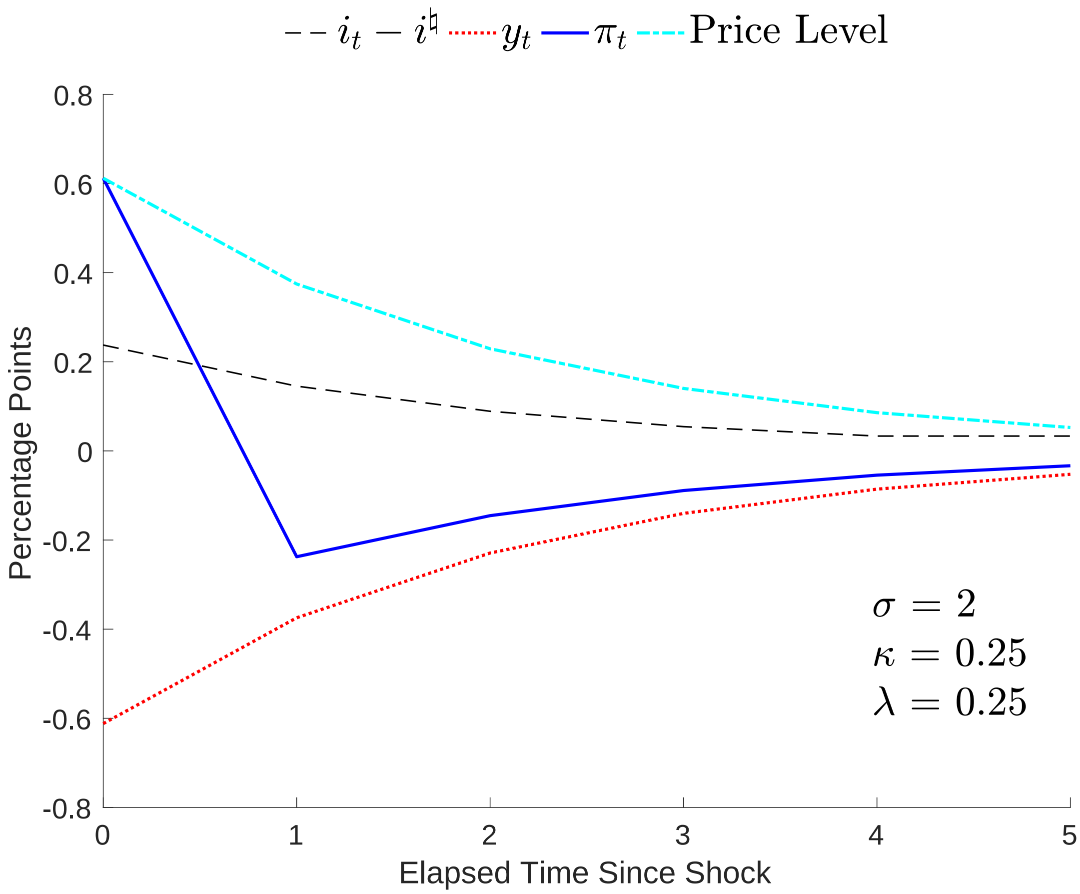

Figure 1: The Standard Solution to the Ramsey Problem: $m_0 = 1$

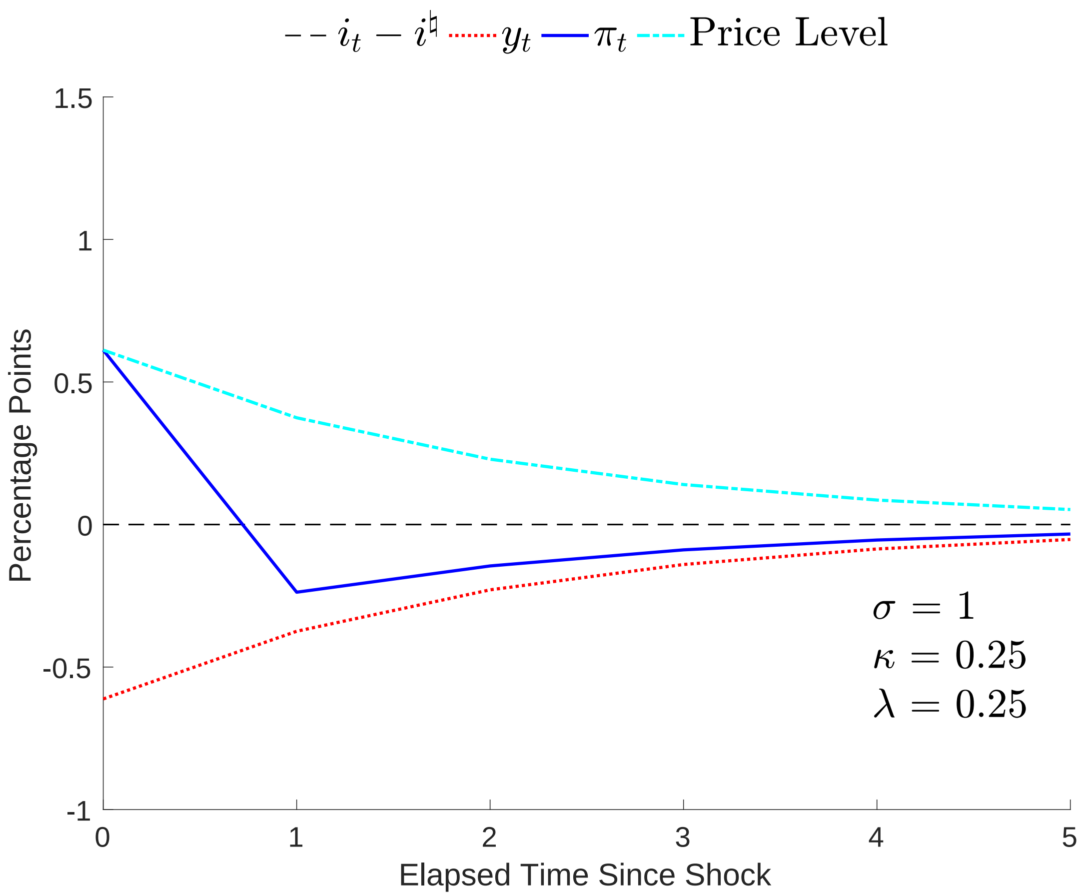

Figure 2: Solution to the Ramsey Problem: $m_0 = 1$ and $\frac{\sigma\kappa}{\lambda} = 1$

solutions are each governed by one of the roots (φ, ψ) of the characteristic polynomial:

$$1 - \left(1 + \beta + \frac{\kappa}{\sigma}\right) q + \beta q^2 = 0.$$

which are $\varphi \in (0, 1)$ and $\psi \in (\frac{1}{\beta}, \infty)$. Since π_t is governed by a second order difference equation (with forcing function x_t) we generally need *two* restrictions to pin down a solution. We can and do obtain one from this problem's transversality condition.

$$\lim_{t\to\infty} \beta^t \pi_t = 0 \tag{5}$$

Infinitely many nonexplosive possible solutions for $\{\pi_t\}$ are consistent with (4) and (5). Imposing a second restriction that π_0 be fixed at some value and solving the system for $t > 0$ yields

$$\pi_t = \varphi^t \pi_0 - \sum_{l=0}^{t-1} \varphi^{l+1} \sum_{j=0}^{\infty} \psi^{-j} x_{t+j-l-1}.$$

From this exercise, it is clear that in solving the Ramsey problem for optimal allocations and assuming that the central banker can pick an entire path for π_t, we implicitly assume that she can choose both the path of interest rates, which governs x_t, *and* the level of π_0. The interest rates alone do not pin down π_0; the central banker accomplishes this with an open mouth operation.

Figure 3 graphically demonstrates this result: the solid line, which represents optimal desired inflation, is obtained by choosing a particular setting for the OMO. Note that each of the inflation paths in the figure is also consistent with the interest rate path chosen, although only the solid line is optimal.

Svensson and Woodford (2005) solve the Ramsey problem as a preliminary step towards calculating an optimal interest rate *rule* for which there is a unique equilibrium consistent with (5). This procedure explicitly presents the procedure by which the central banker implements her open mouth operation, but it does not constrain its use in any way. To see this, construct such an optimal rule (there are many) for the current problem. Denote the Ramsey problem's solutions for interest rates and inflation with $i_t^\star$ and $\pi_t^\star$, and consider the rule

$$i_t = i_t^\star + \phi\left(\pi_t - \pi_t^\star\right) \tag{6}$$

with $\phi > 1$. With this rule, it is a textbook exercise to derive an analogue to (4) in $\tilde{\pi}_t \equiv \pi_t - \pi_t^\star$,

$$\left(1 + \frac{\kappa}{\sigma}\phi\right) \tilde{\pi}_t - \left(1 + \beta + \frac{\kappa}{\sigma}\right) \tilde{\pi}_{t+1} + \beta \tilde{\pi}_{t+2}. \tag{7}$$

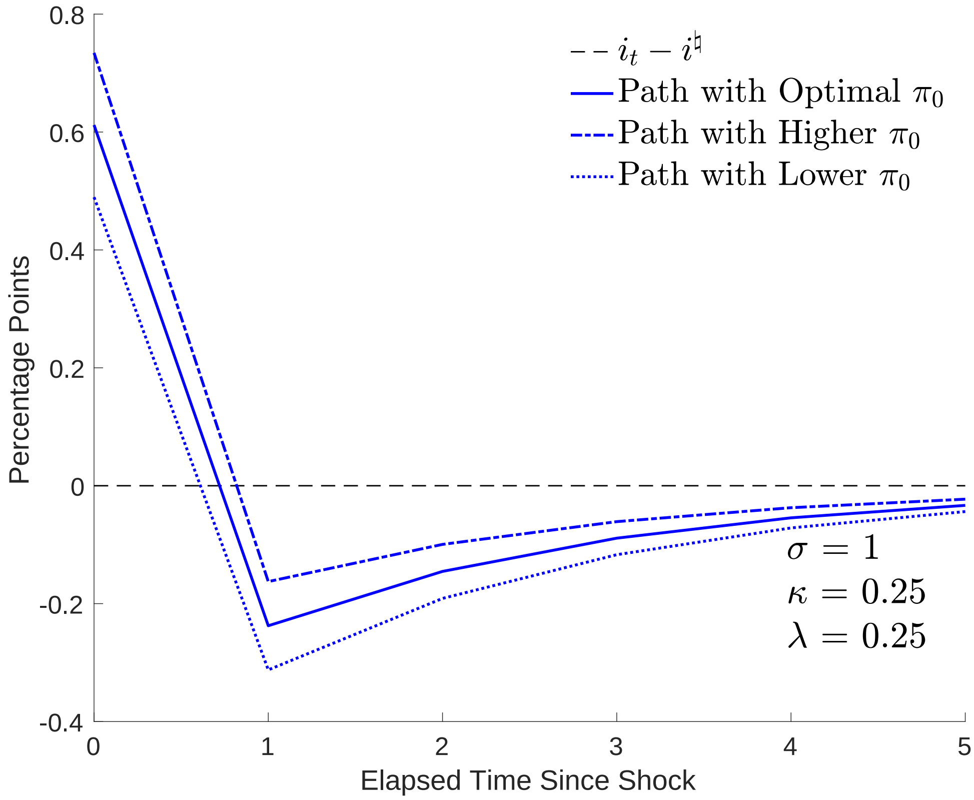

Figure 3: Effect of Changing the Open Mouth Operation

Obviously, the constant sequence $\tilde{\pi}_t = 0$ solves (7). It is well known that since $\phi > 1$, both roots of the characteristic polynomial associated with (7) explode when solved forward. Therefore, $\tilde{\pi}_t = 0$ always is the *only* non explosive solution to (7). In this sense, the interest rate rule in (6) uniquely implements the Ramsey planning solution.

Cochrane (2011) provides three objections to this monetary policy scheme. First, (5) is not a requirement for market clearing or any agent's optimal behavior; so the explosive solutions to (7) satisfy all conditions for competitive equilibria as specified in the original model. We wish to place this issue to one side and focus on Cochrane's second and third objections. If private agents coordinate on an equilibrium with $\tilde{\pi}_0 \neq 0$, then the interest rate rule commits the central banker to feed an explosive inflation path by repeatedly following the conventional monetary policy prescription which is supposed to *tame* inflation: respond more than one-for-one to deviations of inflation from its chosen level. Furthermore, if adopting (6) successfully coordinates agents on the equilibrium with $\tilde{\pi}_0 = 0$, then the central banker has no opportunity to display her commitment to following that rule. For this reason, the adoption of (6) is an act of pure communication which requires no action from the central banker. Publicly adopting an active interest rate rule communicates a threat to create explosive inflation if central banker's desired outcome does not occur. The specifics of this attempt at persuasion hardly make the original open mouth operation more empirically plausible.

3 Imposing a Finite Commitment Horizon

Equilibrium multiplicity in the New Keynesian model with a fixed interest rate path depends on the infinite horizon. If we instead suppose that inflation and the output gap after some date $T+1$ are out of the central banker's control, then backward induction from that date yields unique outcomes for any choice of $(i_0, i_1, \ldots, i_T)$. Since real-world central bankers lack *perfect* commitment, perhaps the problems presented by OMOs disappear once we limit the central banker's commitment to a finite and deterministic date.

To shed light on this possibility, consider the Ramsey planning problem with the additional constraints that $y_t = \pi_t = 0$ for all $t > T$. Facing this finite-horizon problem, it is *feasible* for the central banker to select the optimal inflation and output gap from the infinite horizon problem for $t = 0, \ldots, T-1$. To see this, note that by selecting $i_T = i^\natural - \frac{\sigma}{\kappa}\pi^\star_T$, the central banker sets $\pi_T = \pi^\star_T$. Then, selecting $i_t = i^\star_t$ for all $t = 0, 1, \ldots, T-1$ and using backwards induction with (4) sets $\pi_t = \pi^\star_t$ for the same periods. The central banker's loss from this allocation in $T+1, \ldots, \infty$ equals zero, and it is identical to the loss from the infinite-horizon solution in $0, \ldots, T-1$. The difference between the two allocations' losses in period T is bounded above by

$$\frac{\lambda}{2}y_T^2 = \frac{\lambda}{2\kappa}(\pi_T^\star)^2.$$

Putting these results together shows that the welfare cost of imposing a finite planning horizon on the central banker is bounded above by $\varsigma\beta^T(\pi_T^\star)^2$, where ς is an uninteresting constant.

One implication of this construction is entirely foreseen: The welfare loss from imposing a finite planning horizon on the central banker goes to zero as the planning horizon itself becomes long. However, one detail illuminates the nature of optimal monetary policy with a finite horizon well. The central banker can use her final interest rate choice to achieve any otherwise feasible path for inflation and the output gap that she desires. That is, this final rate can substitute for the OMO's absence at a very small cost. Figure 4 illustrates this by plotting the central banker's finite-horizon planning solution (with $T = 5$ and the parameter values from Figure 1) alongside the corresponding infinite horizon solution, which the dashed lines represent. As expected, the allocations for inflation and the output gap are relatively close to their counterparts from the infinite horizon solution. For $t = 0, \ldots, 4$, the two solutions' interest-rate prescriptions are also very similar to each other. However, i_5 rises to 17 basis points to support the disinflationary value of π_5 required to support the rest of the allocation.

The finite-horizon solution's reliance on the final interest rate suggests that the central banker relies substantially on the forward guidance puzzle of Carlstrom, Fuerst, and Paustian (2015) to implement her desired inflation and output gap sequences. This speculation can be verified analytically by showing that $\partial\pi_0/\partial i_T$ diverges as T grows. Figure 5 exemplifies this more concretely. It plots again the finite horizon planning problem's solution along with the competitive equilibrium arising from the central bank setting its interest rates equal to those from the infinite-horizon solution for $t = 0, \ldots, T$ and to zero thereafter. Although the two interest rate sequences almost equal each other outside of period 5, the allocations they produce differ greatly. Removing period 5's interest rate "bump" raises π_0 by 19 basis points and raises the output gap by 23 basis points. This makes sense, because the change substantially moderates the future disinflation and output gap used to "spread the pain" from period zero. The price level in $T = 5$ provides one summary measure of that change. As Giannoni and Woodford (2005) demonstrate, the infinite-horizon solution stabilizes the price level at its original level in the long run. The finite-horizon solution comes close to this benchmark. In $t = 5$ and thereafter, the price level is eight basis points above its original level. However, removing period 5's interest rate bump lets the long-run price level drift up 60 basis points. We conclude that imposing a finite planning horizon merely replaces the open mouth operation with the forward guidance puzzle without making the Ramsey problem's prescription for forward guidance more plausible.

4 Conclusion

This paper has highlighted a pathology of the standard New Keynesian model's Ramsey planning problem under perfect commitment. With an infinite horizon, the indeterminacy present in the standard New Keynesian model forces us to acknowledge that the central banker selects one of many equilibria when calculating optimal forward guidance. The model gives no operational advice on how the central banker coordinates private-sector expectations on her desired equilibrium, so we label the communications tool she uses for this task an "open mouth operation." Following the arguments of Cochrane (2011), we demonstrate that adopting an "active" interest rate rule also relies only on communications rather than actions; and we show that a central banker facing a finite horizon planning problem relies on the forward guidance puzzle.

Another approach to employing New Keynesian models for monetary policy advice directly specifies an interest rate rule and directly solves the resulting "three-equation" model. The result might be interpreted as Delphic forward guidance (Campbell et al., 2012), because it gives equilibrium sequences of inflation and output along with interest rates that fit the specified interest rate rule by construction. Although we do not directly discuss this procedure, our results nevertheless apply to it. *Given* the interest rate sequence produced from this solution, there are many private-sector equilibria. The central banker must do something to coordinate private agents on the computed equilibrium sequences. By construction, the interest rate sequences satisfy a rule which, if followed after an "incorrect" private-sector choice of π_0, would yield an explosion. In this limited sense, the central banker provides evidence to the public along the equilibrium path that it follows the given rule. However, the central banker's "commitment" to the rule selects the equilibrium only by convincing the public that she will follow the rule mechanically through any ever-intensifying inflation destabilization. Not only does this rely on communication rather than action, the communication's content lacks credibility.

Although imposing a deterministic finite planning horizon on the central banker does not meaningfully address the pathology we highlight, we show in a companion paper (Campbell and Weber, 2018) that an economically similar modification can make the New Keynesian model useful for monetary policy formation. This alteration imposes *stochastic* finite planning horizons by allowing the central banker to reoptimize with a constant probability each period. Schaumburg and Tambalotti (2007) call this "quasi-commitment," and Debortoli

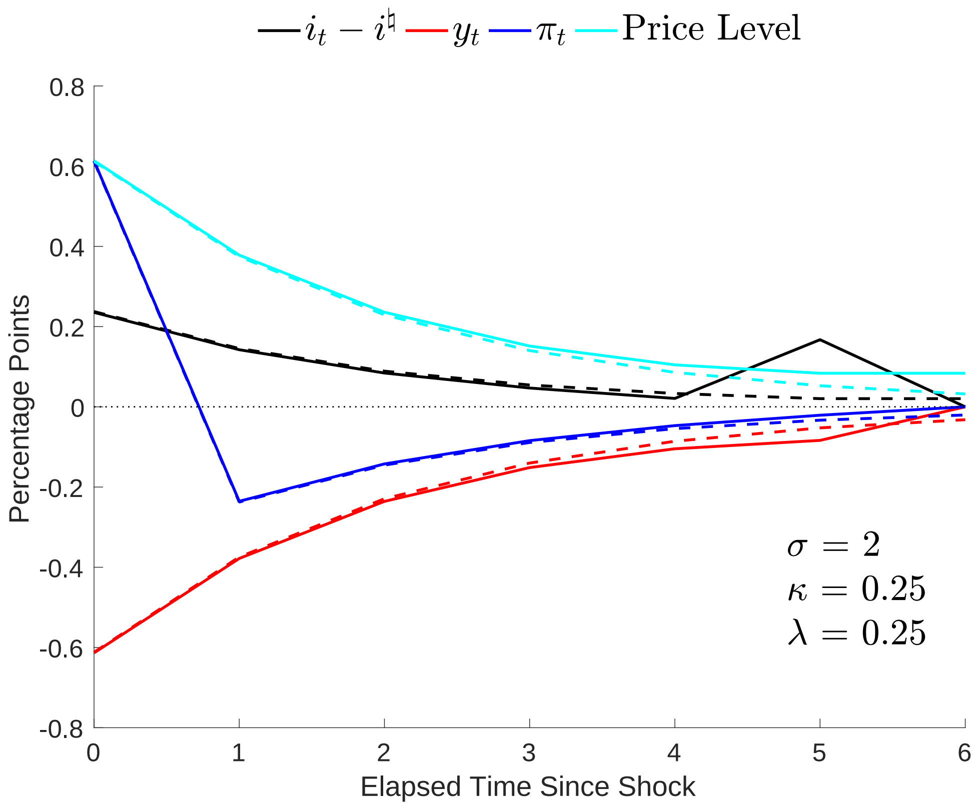

Figure 4: Solutions to the Finite-Horizon and Infinite-Horizon Planning Problems

Note: The solid lines plot the solution to the finite-horizon Ramsey planning problem with $T = 5$. The dashed lines plot the solution to the corresponding infinite-horizon problem.

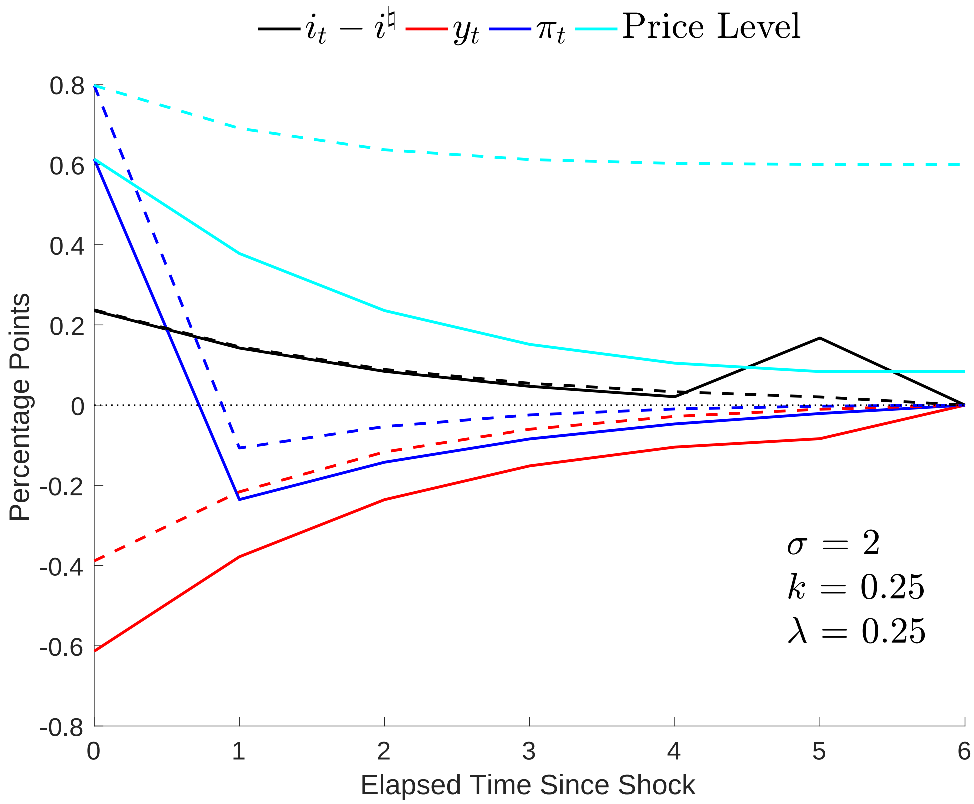

Figure 5: The Forward Guidance Puzzle in the Finite-Horizon Planning Problem

Note: The solid lines plot the solution to the finite-horizon Ramsey planning problem. The dashed lines plot the interest rate from the infinite-horizon planning problem truncated to zero after $T = 5$ and the corresponding competitive equilibrium.

and Lakdawala (2016) apply it to a medium-scale empirical DSGE model. In our companion paper, we demonstrate that if the central banker reoptimizes frequently enough, then there exists a unique equilibrium given the central banker's initially chosen interest rate sequence and expectations of her future choices and private sector responses to them. With reasonable parameter values, "frequently enough" is on average every three to seven quarters. The paper furthermore shows that this analysis produces a unique Markov-perfect equilibrium when we endogenize future central banker actions within a properly-specified game. Therefore, we refer the reader to this companion work for a feasible and empirically-attractive alternative to analyzing monetary policy without open mouth operations.

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