Federal Reserve Bank of Chicago

Open Mouth Operations

Jeffrey R. Campbell and Jacob P. Weber

REVISED
January 31, 2019

WP 2018-03

Open Mouth Operations

Jeffrey R. Campbell* Jacob P. Weber†

January 31, 2019

Abstract

We examine forward guidance from the standard New Keynesian economy's Ramsey problem. It makes two instruments available: the path of current and future interest rates, and an "open mouth operation" which selects from the many equilibria consistent with the chosen interest rates. Removing the open mouth operation by imposing a finite commitment horizon yields policy advice that relies on the forward guidance puzzle. Removing it by altering the private sector's forward-looking behavior requires empirically-implausible separation of asset prices from future economic outcomes. Requiring all agents to follow Markovian strategies can yield an empirically-plausible NK economy without effective open mouth operations.

*Federal Reserve Bank of Chicago and CentER, Tilburg University. e-mail: jcampbell@frbchi.org
†University of California, Berkeley. e-mail: jacob_weber@berkeley.edu

We are grateful to Gadi Barlevy, Marco Bassetto, Gauti Eggertsson, Simon Gilchrist and the late Alejandro Justiniano for insightul comments and discussions. Any errors that remain are our own. The views expressed herein are those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

JEL Codes: E12, E52.
Keywords: Equilibrium Multiplicity, Monetary Policy, Forward Guidance.

1 Introduction

Consider a standard New Keynesian economy with an intertemporal-substitution (IS) curve and a forward-looking Phillips curve. Its central banker seeks to minimize deviations of output and inflation from their steady state values following a transitory markup shock. The standard solution to this policy problem first calculates the best outcomes for inflation and output consistent with the sequence of Phillips curves and then calculates the required interest rates from these choices and the sequence of IS curves. This sequence has been interpreted as the central banker's optimal Odyssean forward guidance (Giannoni and Woodford, 2005; Campbell, Evans, Fisher, and Justiniano, 2012).

We demonstrate that this standard exercise does not yield useful policy advice. Writing the Ramsey problem in terms of optimal *instrument settings* instead of choices for inflation and output clarifies that the Ramsey planner has two instruments: the path of current and future interest rates, and direct control over the initial inflation rate. Selecting this rate chooses one equilibrium from the many which are consistent with the selected interest rates. The model gives no operational guidance on how the central banker coordinates private-sector expectations on her desired equilibrium, so we label the communications tool she uses for this task an "open mouth operation" or OMO.

Clarida, Galí, and Gertler (1999) describe this standard Ramsey policy advice as "in many ways in the tradition of the classic Jan Tinbergen (1952) / Henri Theil (1958) (TT) targets and instruments problem."[1] In their analysis, the central bank's instruments are its interest rate choices, while its targets are the output gap and inflation outcomes. This description fails to note that the number of targets exceeds the number of interest-rate instruments and private-sector constraints by one. This error is understandable since all of these are infinite sequences, but it leads policy makers to interpret outcomes driven by OMOs as consequences of interest rate choices alone. In this way, the analysis of standard Ramsey policy advice can diminish rather than enhance understanding. Considering the outcome of the policy problem as an optimal active interest rate rule fails to add clarity. Following King (2000), we parameterize such rules with a time-varying intercept and an "active" response to deviations of inflation from its time-varying target. The adoption of such a rule commits the central banker to participate in an economic disaster with exploding inflation and output gaps if households and firms fail to coordinate on the chosen equilibrium.(Cochrane, 2011) If such a commitment successfully implements the central banker's chosen values for inflation and output, then the central bank's threatened responses to deviations of inflation from its time-varying target never occur. Therefore, "adoption" of an optimal active interest rate

[1]See the first paragraph on their page 1670.

rule requires persuasion rather than action. The constant interest rate projections of Galí (2009) require a similar commitment to an inherently unobservable rule.

In spite of their empirical implausibility, *OMOs are mathematically essential for implementing the policy prescriptions of the standard New Keynesian economy's Ramsey planning problem.* We provide two complementary demonstrations of this essentiality. First, we note the existence of a knife edge case in which OMOs are the *only* instrument which the policy maker changes in response to a cost-push shock. That is, the central banker with perfect commitment chooses not to fight inflation with contractionary interest rate policy, but instead uses only the OMO to achieve the Ramsey allocation. Second, we recast the Ramsey problem from one of choosing outcomes for inflation and output subject to the constraints of private-sector optimality to one of choosing policy instruments directly. Here, the OMO appears as an option to choose the initial inflation rate *given all current and future interest rates.*

Since the equilibrium multiplicity which underlies OMOs requires an infinite horizon economy, imposing a finite commitment horizon on the central banker (after which the output gap and inflation both return to their steady state values) removes it. For a very long commitment horizon, this constraint imposes almost no welfare cost. However, the policy relies on small changes to the final interest rate under the central banker's control to substitute for the OMO. That is, a central banker with a finite horizon leans heavily upon the forward guidance puzzle of Carlstrom, Fuerst, and Paustian (2015) to implement her desired outcomes.

Another class of solutions to the problem of the existence of an OMO is to modify the agents' preferences, information-processing technology, or available markets model's microfoundations so that expected future outcomes have less influence on desired savings. For example, Fisher (2015) and Michaillat and Saez (2019) give households direct utility from financial wealth, while Gabaix (2018) motivates myopic behavior of savers with behavioral considerations. McKay, Nakamura, and Steinsson (2017) refer to such modifications as introducing "discounting" into the IS curve. However, these modifications require empirically-implausible separations of asset prices from future economic outcomes to eliminate the model's OMO.

The next section presents our results characterizing the optimal monetary policy problem in the face of a one-time cost-push shock, as in Giannoni and Woodford (2005). Section 3 demonstrates how selecting an equilibrium with an active rule is equivalent to selecting one via an open mouth operation. Section 4 contains our characterization of the monetary policy problem with a finite horizon. These sections' results warn central bankers against interpreting outcomes in the New Keynesian economy as direct consequences of their current and future interest rate choices, but they provide no more appropriate procedure for monetary policy evaluation. Section 5 evaluates the quantitative potential of the model modifications discussed above to eliminate the OMO, Section 6 demonstrates that requiring all agents to employ Markovian strategies, which limits the central banker's commitment, can both remove open mouth operations from the central banker's toolkit and resolve the forward guidance puzzle. In this paper, we allow the central banker to make *no* commitments. This has the unfortunate direct implication that the model has only trivial implications for Odyssean forward guidance. In a companion paper (Campbell and Weber, 2019) we introduce quasi-commitment into the NK model (Roberds, 1987; Schaumburg and Tambalotti, 2007; Debortoli and Nunes, 2014). There, we show that if the average duration of commitment is low enough, then the economy has a unique Markov-perfect equilibrium. Quantitatively, "low enough" allows for central bank commitments which can last eleven quarters on average.

2 Ramsey Planning

Here we demonstrate how the well-known indeterminacy present in the standard New Keynesian model makes an extra instrument available to the central banker in her corresponding Ramsey problem.[2] The Phillips curve (PC) is

$$\pi_t = \kappa y_t + \beta \pi_{t+1} + m_t \tag{1}$$

with cost-push shock $m_0 \neq 0$ and $m_t = 0$ for all $t > 0$, and the Intertemporal Substitution (IS) curve is

$$y_t = -\frac{1}{\sigma}(i_t - \pi_{t+1} - i^{\natural}) + y_{t+1}. \tag{2}$$

Here i_t, $i^{\natural}$, π_t and y_t denote the nominal interest rate, the natural rate of interest, inflation and the output gap. The parameters satisfy $\sigma, \kappa \in (0, \infty)$ and $\beta \in (0, 1)$. We ignore the possibility of an effective lower bound on nominal interest rates, so i_t can take any value. The central banker seeks to minimize a loss function which is quadratic in current and future output gaps and inflation rates.

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2}\pi_t^2 + \frac{\lambda}{2} y_t^2 \right) \tag{3}$$

[2]See Galí (2008) for a derivation of these now-standard equations.

This can be derived from the representative household's utility function (Woodford, 2003) or from the central banker's legislative environment (Evans, 2011).

A central banker solving the Ramsey problem chooses paths for i_t, π_t, and y_t to minimize her loss in (3) while satisfying the sequences of constraints given by (1) and (2). Since i_t only appears in the IS curve, it can be selected to satisfy that equation given arbitrary values for π_{t+1}, y_t and y_{t+1}. Therefore, the only binding constraints on the central banker's choices of inflation and output gap sequences come from the sequence of PCs. For this reason, the standard approach to solving this Ramsey problem first minimizes the loss function constrained only by that sequence and then backs out the necessary values for i_t using the IS curve.

Figure 1 presents an example solution with $m_0 = 1$, or a one percent shock to the Phillips curve in the initial time period. (All of our numerical examples use the same value of $\beta = 0.99$.) If the central banker did nothing, so that $i_t = i^\natural$ for all t, and agents' inflation expectations for $\pi_1, \pi_2, ... = 0$ remained unchanged, then inflation today would have to increase by one percent. Even while taking future outcomes as given, the central banker can improve on this outcome by raising i_t above $i^\natural$ and thereby reducing both inflation and output. Because the loss function is quadratic, the central banker receives a first order gain from reducing inflation, at the cost of a second order loss from reducing output. The optimal setting of i_0 equates the marginal benefit of reducing inflation with the marginal cost of diminishing output. Given the parameter values used in Figure 1 and no control over expectations of the future, this policy would yield $\pi_0 = .8$ and $y_0 = -.8$.

Since the central banker controls π_1, she can further improve outcomes by promising a small deflationary recession. Mechanically, the promised deflation in period one partially offsets the cost-push shock, allowing the central banker to achieve both lower inflation today and a smaller output gap (e.g. Campbell, 2013). This reduces inflation and increases output today, yielding a first order gain, at the expense of a second order loss from the deflationary recession in the future. Intuitively, such a policy improves outcomes by "spreading the pain" of a transitory shock across multiple future periods. The dark blue line plots the first few periods of the optimal plan for inflation, while the red line plots the same for output. Although it is feasible to close these gaps at any time after $t = 0$, they close only asymptotically. This is because the central banker who closes both gaps by some finite time T can always lower her loss by spreading the pain into $T + 1$. The light blue line plots the price level, which is the accumulation of inflation. Optimal policy eventually undoes all the inflation that was allowed to occur in the initial time period. This is the familiar price level targeting result of Giannoni and Woodford (2005).

The dashed black line plots the interest rate consistent with the IS curve given the optimal

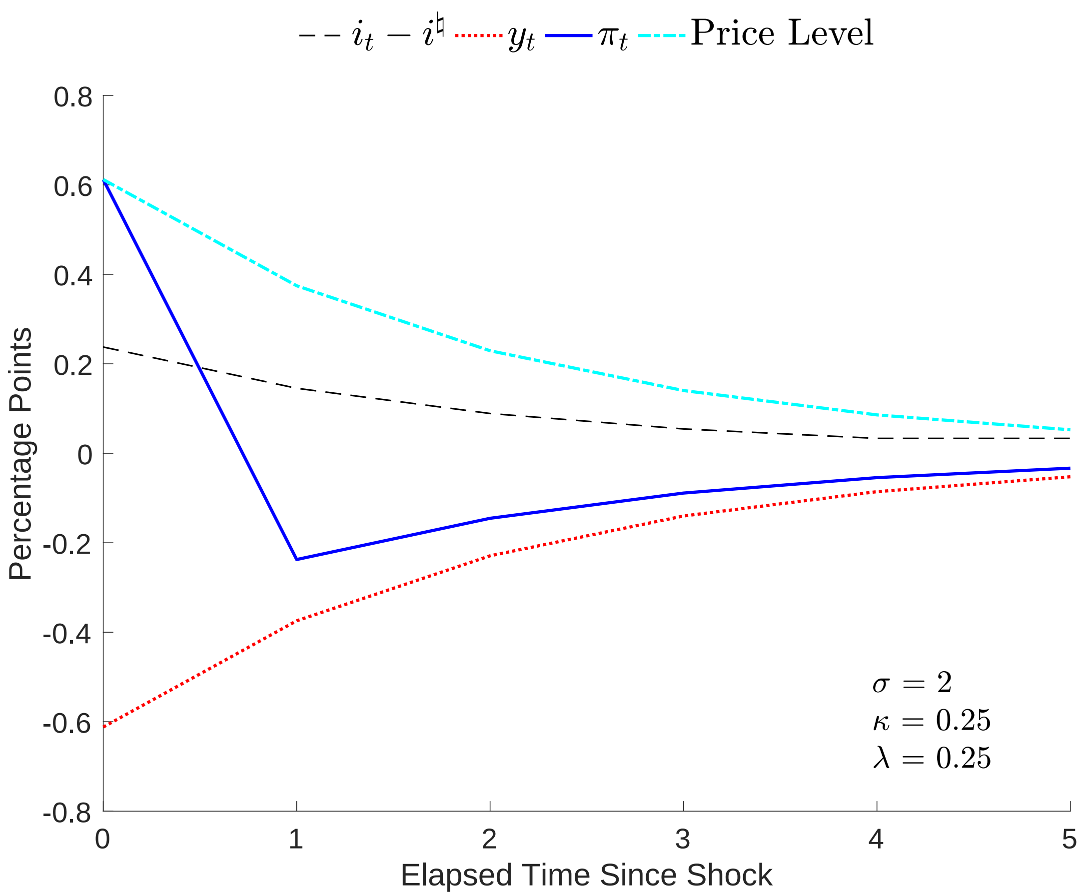

Figure 1: The Standard Solution to the Ramsey Problem: $m_0 = 1$

choices of π_t and y_t. When the central banker has no control over expectations, the optimal initial interest rate jumps to $i^\natural + 1.6$ percent. The Ramsey planner chooses a more modest initial response of $i^\natural + 0.24$ percent, but she keeps the interest rate above $i^\natural$ after the cost-push shock has dissipated.

In the conventional interpretation of the exercise presented in Figure 1, the central banker promises to keep interest rates above the natural rate after the cost-push shock has passed, thereby creating deflationary expectations. However, this explanation fails to describe accurately other, similar forward guidance experiments. To see this, consider Figure 2. This plots the Ramsey solution given the same values for κ and λ but a different value for σ. Since the Phillips curve and loss function are unchanged, the chosen values for π_t and y_t equal those in Figure 1: and they are given by the dark blue and red lines, respectively. With the particular IS curve chosen, the interest rate required by this plan is a constant: $i_t = i^\natural$. That is, we have a deflationary recession without contractionary interest rate policy. This is possible because the central banker's chosen outcome for $t \geq 1$ coincides with one of the many equilibria consistent with $i_t = i^\natural$ always. Indeed, this is the case whenever $\frac{\sigma\kappa}{\lambda} = 1$. Implementing this example's Ramsey planning solution requires no contractionary interest rate policy, but coordinating agents' expectations with an Open Mouth Operation is essential.

We presented the "knife edge" case in Figure 2 to illustrate the use of a tool that is *always* available and used by the central banker. To demonstrate its existence analytically, cast the planning problem in terms of choosing instrument settings instead inflation and output outcomes. We plug the PC (1) into the IS curve (2). The result is a single, second order difference equation:

$$\pi_t - \left(1 + \beta + \frac{\kappa}{\sigma}\right) \pi_{t+1} + \beta \pi_{t+2} = x_t. \tag{4}$$

Here $x_t \equiv -\frac{\kappa}{\sigma}\left(i_t - i^\natural\right) + m_t - m_{t+1}$. Henceforth, we represent interest-rate policy with x_t. The full set of solutions to (4) for a particular path of x_t is given by a linear combination of two homogenous solutions and a particular solution. The rates of decay of the homogenous solutions are each governed by one of the roots (φ, ψ) of the characteristic polynomial:

$$1 - \left(1 + \beta + \frac{\kappa}{\sigma}\right) q + \beta q^2 = 0.$$

which are $\varphi \in (0, 1)$ and $\psi \in (\frac{1}{\beta}, \infty)$. Since π_t is governed by a second order difference equation (with forcing function x_t) we generally need *two* restrictions to pin down a solution.

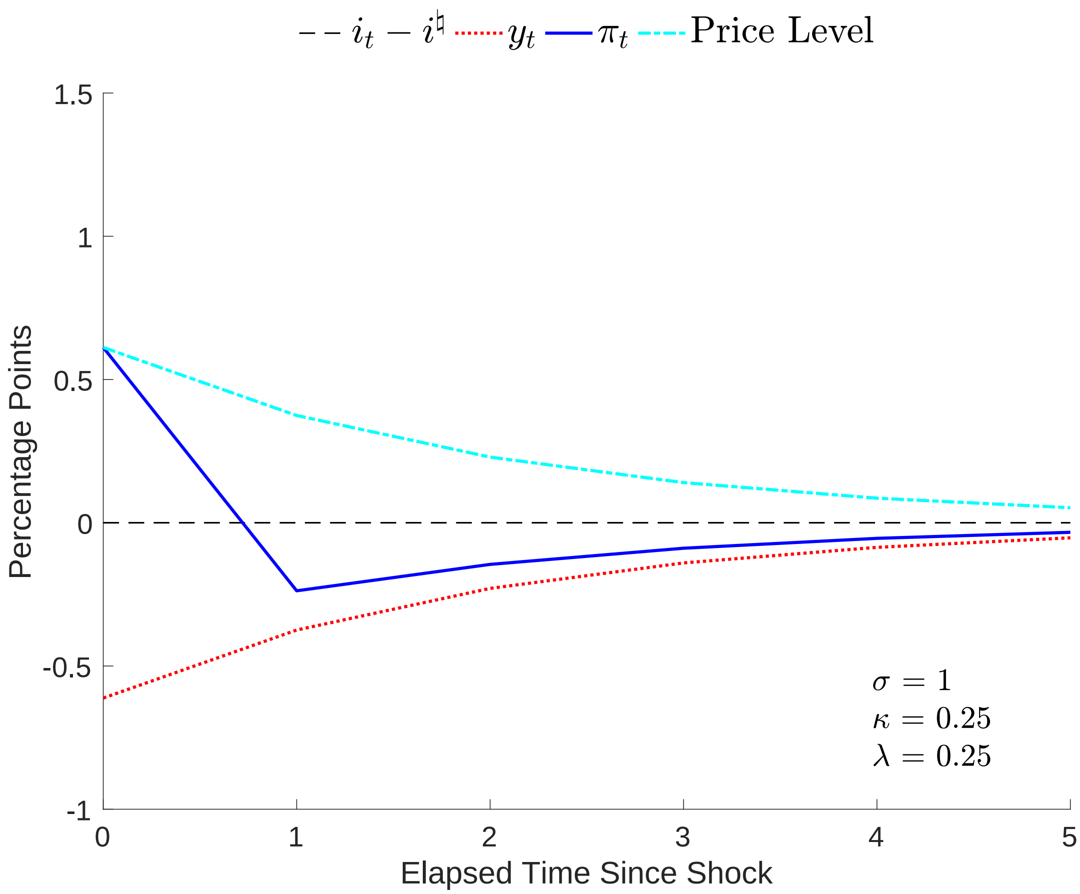

Figure 2: Solution to the Ramsey Problem: $m_0 = 1$ and $\frac{\sigma\kappa}{\lambda} = 1$

We can and do obtain one from this problem's transversality condition.

$$\lim_{t\to\infty} \beta^t \pi_t = 0 \tag{5}$$

Infinitely many nonexplosive possible solutions for $\{\pi_t\}$ are consistent with (4) and (5). Imposing a second restriction that π_0 be fixed at some value and solving the system for $t > 0$ yields

$$\pi_t \;=\; \varphi^t \pi_0 - \sum_{l=0}^{t-1} \varphi^{l+1} \sum_{j=0}^{\infty} \psi^{-j} x_{t+j-l-1}. \tag{6}$$

Combining this solution for π_t in terms of π_0 and $\{x_j\}_{j=0}^{\infty}$ with the Phillips curve allows us to express y_t as a function of these instruments. With this, we can express the central banker's loss in terms of instruments and proceed to its minimization.

From this exercise, it is clear that the Ramsey problem implicitly assumes that the central banker can choose both the path of interest rates, which determines x_t, *and* the level of π_0. The interest rates alone do not determine π_0; the central banker accomplishes this with an open mouth operation.

Figure 3 graphically demonstrates this result: the solid line, which represents optimal desired inflation, is obtained by choosing a particular setting for the OMO. Note that each of the inflation paths in the figure is also consistent with the interest rate path chosen. In particular, the line labelled "Path with No Recession" gives the outcome discussed at the beginning of this section: The central banker leaves interest rates unchanged at $i^\natural$, m_0 passes through one-for-one to π_0, and no other variable moves. The OMO accounts for all of the movement from this outcome to the Ramsey outcome.

Another unsavory implication of equation (6) is that every solution to the two equation model is *history dependent* – not just optimal policy, or the central bankers desired equilibrium, as is well known. This poses a technical challenge for game theoretic analyses wishing to make use of the standard NK model while restricting the strategy space to be Markov, as (6) implies that inflation at time t is determined by a host of variables which are payoff-irrelevant at that date (in particular, the entire history of realized inflation); see Campbell and Weber (2019) for a discussion of a game theoretic setting in which – absent modification – the set of symmetric Markov Perfect Equilibria in a standard New Keynesian economy characterized by (6) is empty.

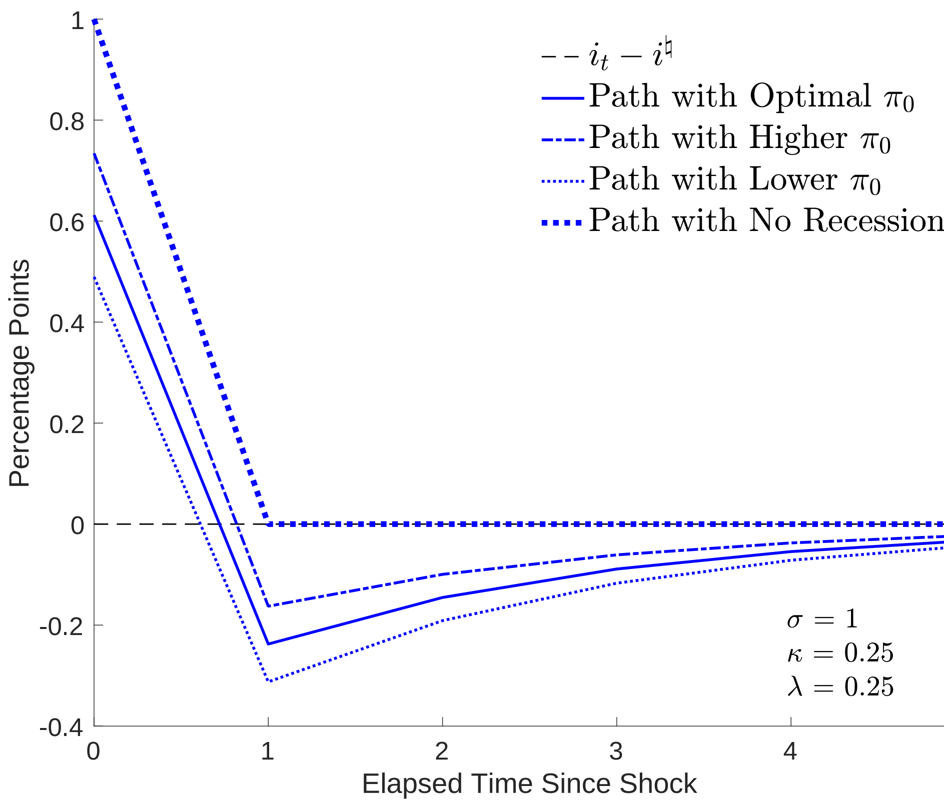

Figure 3: Effect of Changing the Open Mouth Operation

3 Active Rules as "Open Mouth Operations"

Svensson and Woodford (2005) use the solution to the Ramsey problem to calculate an optimal interest rate *rule* for which there is a unique equilibrium consistent with (5). This procedure explicitly presents the method by which the central banker implements her open mouth operation, but it does not constrain its use in any way. To see this, construct such an optimal rule (there are many, indexed by the choice of "active" coefficients) for the current problem. Denote the Ramsey problem's solutions for interest rates and inflation with $i_t^\star$ and $\pi_t^\star$, and consider the rule

$$i_t = i_t^\star + \phi \left(\pi_t - \pi_t^\star \right) \tag{7}$$

with $\phi > 1$. To see how this uniquely implements the Ramsey equilibrium, rewrite (4) in terms of $\tilde{\pi}_t \equiv \pi_t - \pi_t^\star$, which is the deviation of inflation from the central bank's "target," $\pi_t^\star$. Using the fact that the central bank's target solves the original difference equation and then plugging in (7) for $i_t - i_t^\star$ yields a second order homogenous difference equation in terms of $\tilde{\pi}_t$, analagous – but not identical – to (4):

$$\left(1 + \frac{\kappa}{\sigma}\phi\right)\tilde{\pi}_t - \left(1 + \beta + \frac{\kappa}{\sigma}\right)\tilde{\pi}_{t+1} + \beta\tilde{\pi}_{t+2} = 0. \tag{8}$$

It is well known that with $\phi > 1$, both roots of the characteristic polynomial associated with (8) explode when solved forward. Therefore, the trivial solution $\tilde{\pi}_t = 0$ is the unique non-explosive solution to (8). In this sense, the interest rate rule in (7) uniquely implements the Ramsey planning solution. Indeed, such a rule can implement *any* of the non-explosive equilibria given by (6). Allowing the central banker to choose among rules in the class described by (7) is thus equivalent to choosing π_0 and a path of interest rates.

Cochrane (2011) provides three objections to this monetary policy scheme. First, (5) is not a requirement for market clearing or any agent's optimal behavior; so the explosive solutions to (8) satisfy all conditions for competitive equilibria as specified in the original model. We wish to place this issue to one side in this paper and focus on Cochrane's second and third objections. If private agents coordinate on an equilibrium with $\tilde{\pi}_0 \neq 0$, then the interest rate rule commits the central banker to feed an explosive inflation path by repeatedly following the conventional monetary policy prescription which is supposed to *tame* inflation: respond more than one-for-one to deviations of inflation from its chosen level. Furthermore, if adopting (7) successfully coordinates agents on the equilibrium with $\tilde{\pi}_0 = 0$, then the central banker has no opportunity to display her commitment to following that rule. For this reason, the adoption of (7) is an act of pure communication which requires no action from the central banker. Publicly adopting an active interest rate rule communicates a threat to create explosive inflation if central banker's desired outcome does not occur. The specifics of this attempt at persuasion hardly make the original open mouth operation more empirically plausible.

The remainder of this paper examines other modifications to the NK model with the potential to create equilibrium determinacy and thereby remove the OMO from the central banker's instrument set. To make it clear that these do not leverage active interest rate rules, we henceforth specify interest-rate policy as a fixed path.

4 Imposing a Finite Commitment Horizon

Equilibrium multiplicity in the New Keynesian model with a fixed interest rate path depends on the infinite horizon. If we instead suppose that inflation and the output gap after some date $T+1$ are out of the central banker's control, then backward induction from that date yields unique outcomes for any choice of $(i_0, i_1, \ldots, i_T)$. Thus the outcomes of the pivate sector no longer depend on payoff-irrelevant history. Since real-world central bankers lack *perfect* commitment, perhaps the problems presented by OMOs disappear once we limit the central banker's commitment to a finite and deterministic date.

To shed light on this possibility, consider the Ramsey planning problem with the additional constraints that $y_t = \pi_t = 0$ for all $t > T$. Facing this finite-horizon problem, it is *feasible* for the central banker to select the optimal inflation and output gap from the infinite horizon problem for $t = 0, \ldots, T-1$. To see this, note that by selecting $i_T = i^\natural - \frac{\sigma}{\kappa}\pi_T^\star$, the central banker sets $\pi_T = \pi_T^\star$. Then, selecting $i_t = i_t^\star$ for all $t = 0, 1, \ldots, T-1$ and using backwards induction with (4) sets $\pi_t = \pi_t^\star$ for the same periods. The central banker's loss from this outcome in $T+1, \ldots, \infty$ equals zero, and it is identical to the loss from the infinite-horizon solution in $0, \ldots, T-1$. The difference between the two outcomes' losses in period T is bounded above by

$$\frac{\lambda}{2} y_T^2 = \frac{\lambda}{2\kappa} (\pi_T^\star)^2.$$

Putting these results together shows that the welfare cost of imposing a finite planning horizon on the central banker is bounded above by $\varsigma \beta^T (\pi_T^\star)^2$, where ς is an uninteresting constant.

One implication of this construction is entirely foreseen: The welfare loss from imposing a finite planning horizon on the central banker goes to zero as the planning horizon itself becomes long. However, one detail illuminates the nature of optimal monetary policy with a finite horizon well. The central banker can use her final interest rate choice to achieve any otherwise feasible path for inflation and the output gap that she desires. That is, this final rate can substitute for the OMO's absence at a very small cost. Figure 4 illustrates this by plotting the central banker's finite-horizon planning solution (with $T = 5$ and the parameter values from Figure 1) alongside the corresponding infinite horizon solution, which the dashed lines represent. As expected, the values for inflation and the output gap are relatively close to their counterparts from the infinite horizon solution. For $t = 0, \ldots, 4$, the two solutions' interest-rate prescriptions are also very similar to each other. However, $i_5 - i^\natural$ rises to 17 basis points to create the disinflationary value of π_5 required to support the rest of the outcome.

The finite-horizon solution's use of the final interest rate suggests that the central banker relies substantially on the forward guidance puzzle of Carlstrom, Fuerst, and Paustian (2015) to implement her desired inflation and output gap sequences. This speculation can be verified analytically by showing that $\partial\pi_0/\partial i_T$ diverges as T grows. Figure 5 demonstrates this more concretely. It plots the finite horizon planning problem's solution from Figure 4 along with the competitive equilibrium arising from the central bank setting its interest rates equal to those from the infinite-horizon solution for $t = 0, \ldots, T$ and to zero thereafter. Although the two interest rate sequences almost equal each other outside of period 5, they produce quite different outcomes. Removing period 5's interest rate "bump" raises π_0 by 19 basis points and raises the output gap by 23 basis points. This makes sense, because the change substantially moderates the future disinflation and output gap used to "spread the pain" from period zero. The price level in $t = 5$ provides one summary measure of that change. As Giannoni and Woodford (2005) demonstrate, the infinite-horizon solution stabilizes the price level at its original level in the long run. The finite-horizon solution comes close to this benchmark. In $t = 5$ and thereafter, the price level is eight basis points above its original level. However, removing period 5's interest rate bump leaves the long-run price level drift 60 basis points above its starting value. We conclude that imposing a finite planning horizon merely replaces the open mouth operation with the forward guidance puzzle without making the Ramsey problem's prescription for forward guidance more plausible.

5 Different Private-Sector Primitives

An alternative approach to solving the problem of equilibrium indeterminacy is to change the primitives underlying private sector behavior in order to make the NK model less forward looking. More specifically, changes which lower the elasticity of current output with respect to future output in the IS curve can produce a unique equilibrium given a path of chosen interest rates. McKay, Nakamura, and Steinsson (2017) label this effect "discounting" of the IS curve. In this section, we will evaluate the quantitative potential of several such

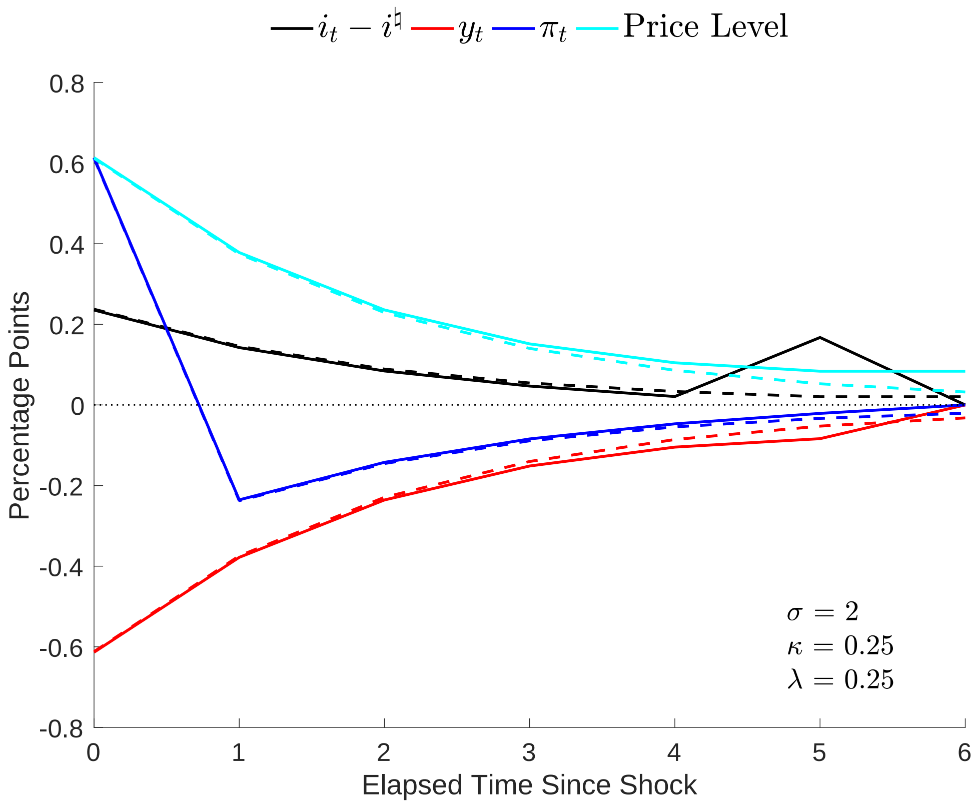

Figure 4: Solutions to the Finite-Horizon and Infinite-Horizon Planning Problems

Note: The solid lines plot the solution to the finite-horizon Ramsey planning problem with $T = 5$. The dashed lines plot the solution to the corresponding infinite-horizon problem.

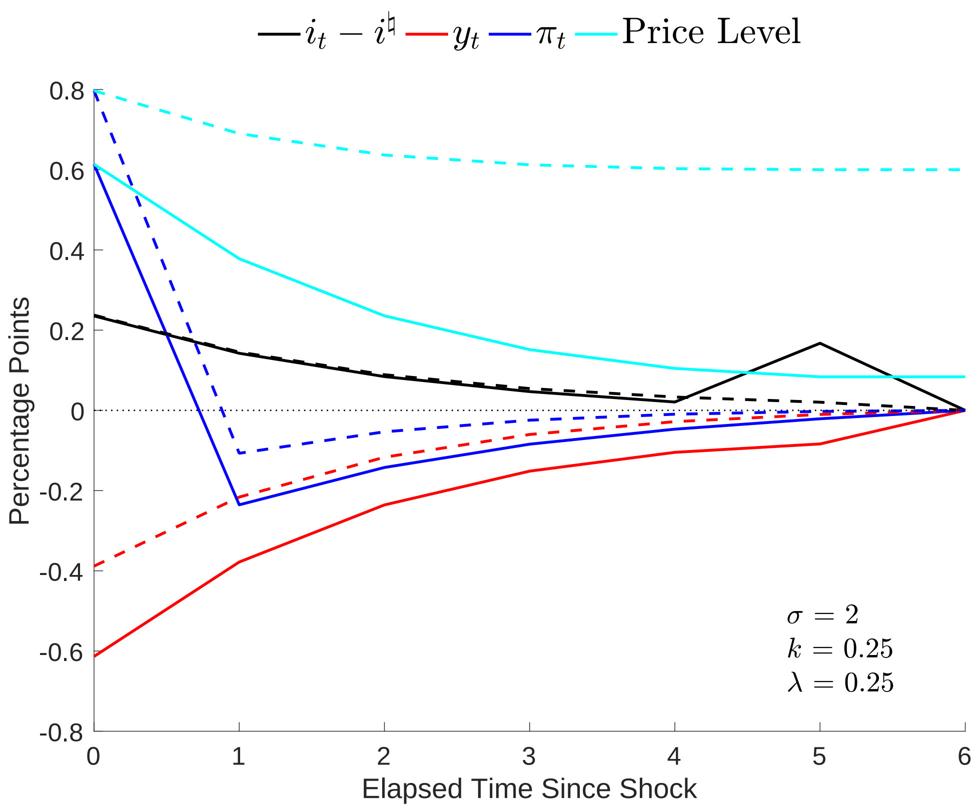

Figure 5: The Forward Guidance Puzzle in the Finite-Horizon Planning Problem

Note: The solid lines plot the solution to the finite-horizon Ramsey planning problem. The dashed lines plot the interest rate from the infinite-horizon planning problem truncated to zero after $T = 5$ and the corresponding competitive equilibrium.

modifications which lead to IS curve discounting given $\kappa/\sigma = 0.01$ and $\beta = 0.99$[3]

5.1 Bonds in the Utility Function

The first modification we consider directly alters the nature of financial markets by putting the household's stock of bonds into the utility function. Fisher (2015) motivates this as a representation of the liquidity services they provide, while Michaillat and Saez (2019) argue that households accumulate wealth for the social status it gives them today rather than for the consumption opportunities it creates for tomorrow.

In such models, the representative household has preferences over consumption, leisure, and the stock of bonds in her portfolio.

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + v(L_t) + X\left(\frac{B_{t+1}}{R_t P_t}\right)\right)$$

Here, R_t is the gross risk-free nominal interest rate for bonds maturing at time $t+1$, P_t is the price level at $t+1$, and $B_{t+1}/(R_t P_t)$ is the time t real value of bonds maturing at time $t+1$ held by the household. The household's budget constraint is standard. Available funds sum labor market earnings, redemption of bonds, and profits rebated from firms. Funds are used for consumption purchases, buying new bonds, and paying lump-sum taxes.

With these preferences, the first-order condition for optimal bond purchases can be written as

$$C_t^{-\sigma} = X'\left(\frac{B_{t+1}}{R_t P_t}\right) + \beta C_{t+1}^{-\sigma} R_t \frac{P_t}{P_{t+1}} \tag{9}$$

Both Fisher (2015) and Michaillat and Saez (2019) assume that bonds are in zero net supply. In this case, the steady-state gross interest rate with zero inflation equals

$$R_\star = \frac{1}{\beta} - \frac{X'(0)}{\beta C_\star^{-\sigma}},$$

where $C_\star$ is steady-state consumption. Thus, the steady state interest rate equals the rate of time preference minus a premium.

To put this into the NK model, we equate consumption with output and log linearize (9) around the steady state. This gives us

$$y_t = \beta R_\star y_{t+1} - \frac{\beta R_\star}{\sigma}\left(i_t - \pi_{t+1} - i^\natural\right) \tag{10}$$

[3]These parameter values match those from Eggertsson and Woodford's (2003) numerical example: $\kappa = 0.02$, $\sigma = 2$, $\beta = 0.99$.

Since $X'(0) > 0$, the IS curve's right-hand side gets "discounted" by $\beta R_\star < 1$. The discount factor equals the ratio of two interest rates, the actual steady-state rate divided by what it would equal in the standard model with $X'(0) = 0$.

After plugging the PC into (10), we obtain the following second order difference equation:

$$\pi_t - \left(\beta R_\star + \beta + \frac{\beta R_\star \kappa}{\sigma}\right)\pi_{t+1} + \beta^2 R_\star \pi_{t+2} = x_t \tag{11}$$

where the forcing function is now $x_t \equiv \frac{\beta R^\star \kappa}{\sigma}(i_t - i^\natural) + m_t - \theta m_{t+1}$. Setting $\beta R^\star = 1$ transforms (11) into (4). We can proceed with a similar analysis as before, since the characteristic polynomial continues to have two real roots.[4] Its derivative evaluated at one equals:

$$\beta R_\star(\beta - 1) + \beta(\beta R_\star - 1) - \frac{\beta R_\star \kappa}{\sigma} < 0.$$

Since the coefficient on the highest order term is positive, its larger root is above one. So if the *value* of the characteristic polynomial when evaluated at one is positive, then the smaller root must also be above one. Manipulating the expression for the characteristic polynomial evaluated at one, $1 - \left(\beta R_\star + \beta + \frac{\beta R_\star \kappa}{\sigma}\right) + \beta^2 R_\star$, we obtain the following necessary and sufficient condition for $\beta R_\star$ so that the smaller root is above one:

$$\beta R_\star < \frac{1-\beta}{1-\beta+\frac{\kappa}{\sigma}} \tag{12}$$

With $\kappa/\sigma = .01$ and $\beta = .99$ (12) says that that we need $\beta R_\star < 0.5$ to achieve determinacy. That is, the *gross* steady-state interest rate equals half of its value in an economy without bonds in the utility function but with the same rate of time preference. Fisher (2015) includes only the short-term securities with prices controlled by the central bank in households' utility, so $\beta R_\star$ can be alternatively identified using the interest rate spread between those assets and otherwise identical bonds which do not provide liquidity services. Generously assuming a 10 percentage point *quarterly* liquidity premium yields $\beta R_\star \approx 0.9$. Michaillat and Saez (2019) calibrate β using an annual rate of time preference equal to 40 percent based on experimental observations. This yields a quarterly value of β equal to 0.9048. Plugging this into (12) along with $\kappa/\sigma = 0.01$ and rearranging yields $R_\star < 1.0001$. Thus, the steady-state nominal interest rate must be counterfactually low to generate determinacy.[5] We conclude that placing bonds into the household's utility function might

[4]It requires some tedious algebra to show that the discriminant can be rewritten as $\left(\theta + \beta + \frac{\theta\kappa}{\sigma}\right)^2 - 4\beta\theta = \left((\theta+\beta) + \frac{\theta\kappa}{\sigma}\right)\left((\theta+\beta) + \frac{\theta\kappa}{\sigma}\right)^2 - 4\theta\beta = \left(\theta - \beta + \frac{\theta\kappa}{\sigma}\right)^2 + 4\beta\frac{\theta\kappa}{\sigma}$, which is clearly always positive.

[5]Michaillat and Saez (2019) use a continuous-time model with logarithmic utility and quadratic costs of

improve some aspects of the NK model, it cannot by itself yield equilibrium determinacy and remove the OMO from the central banker's set of instruments while keeping the price of bonds substantially connected to their expected future payoffs.

5.2 Myopia

Gabaix (2018) introduces discounting by assuming that firms and households are myopic. Specifically, household expectations of a future-dated random variable X_{t+1} equal a factor $M \in [0,1)$ times the rational expectation. Similarly, producers' expectations equal $M^f \in [0,1)$ times the rational expectation. Under these assumptions, the IS curve and PC become

$$\pi_t = \kappa y_t + M^f \beta \pi_{t+1} + m_t \tag{13}$$

$$y_t = -\frac{1}{\sigma}(i_t - M\pi_{t+1} - i^\natural) + M y_{t+1} \tag{14}$$

It is straightforward to show that there exist unique inflation and output sequences satisfying (13) and (14) for a given sequence of interest rates if and only if

$$M \leq \frac{1-\beta M^f}{\kappa/\sigma + 1 - \beta M^f}. \tag{15}$$

This inequality is analogous to that in (12), with M replacing $\beta R_\star$ and βM^f replacing β. If $M^f = 1$, then myopia gives us another microeconomic foundation for the discounted IS curve. The mechanical condition on M required for determinacy is then exactly the same as that placed on $\beta R_\star$ above. With $\beta = 0.99$ and $\kappa/\sigma = 0.01$, determinacy requires $M < 0.5$. However, the interpretation of M is different. Long-run interest rates in the myopia model equal the rate of time preference with no modification. Nevertheless, M dampens the responses of bond prices to future economic outcomes. We are aware of no empirical failure of the standard model which such dampening could remedy.

Besides giving another source of IS equation discounting, the myopia model also changes the discount factor in the PC. Of course, this cannot yield equilibrium determinacy on its own. However, it can provide a quantitative substitute for discounting in the IS curve. Figure 6 illustrates this point by plotting the right-hand side of (15) in the Cartesian plane for M^f and M. All points below the curve give M^f, M pairs for which the myopia model displays equilibrium determinacy. As noted above, determinacy requires a low value of M when M^f is very close to one. However, determinacy can be achieved with much larger values of M

price adjustment. They calibrate the Phillips curve's slope to 0.03. Plugging these values into (12) gives an upper bound on the risk-free interest rate of 36.74 percent. Thus, their calibration does yield determinacy within the annualized version of our discrete-time model.

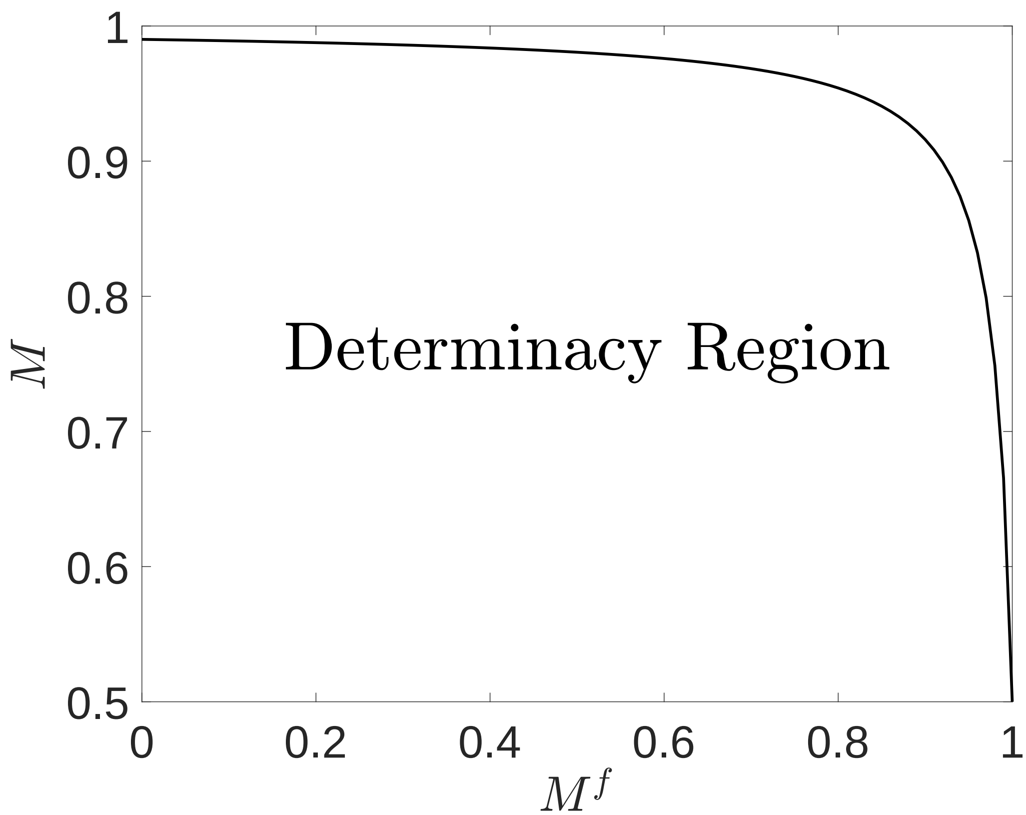

Figure 6: Determinacy in Gabaix's (2018) Myopia Model

Note: The line plots the function on the right-hand side of (15). The coefficients M and M^f are the expectations-adjustment parameters for households and producers, respectively.

when M^f is only somewhat smaller. We conclude from this that including myopic firms in the NK model could make a substantial contribution to resolving its indeterminacy and removing the OMO from the central banker's toolkit. However, this modification's empirical evaluation awaits future work.[6]

5.3 Incomplete Markets

A third class of modifications to the NK model drops the assumption that households trade in complete markets. For example, Del Negro, Giannoni, and Patterson (2015) develop a perpetual youth model where agents live finite lives; however, the discounting in this model cannot deliver quantitatively large amounts of discounting without counterfactually high death rates; see their Table 2 and their model's IS curve, equation (43). Relatedly, McKay, Nakamura, and Steinsson (2016) develop a model where households face uninsurable income risk and borrowing constraints. The resulting precautionary saving motive drives a wedge between bonds' expected future payoffs and their value to households. McKay, Nakamura, and Steinsson (2017) develop a simplified version of this model. Each period, there are "high types" who receive high income (who can borrow, though bonds are in zero net supply) and "low types," who make no decisions and receive exogenous welfare benefits. The "high types" put some weight on the probability that their marginal utility next period will be fixed at the low type level, which introduces a wedge into the Euler equation, similar to the analysis of bonds in the utility function in Section 5.1. They calibrate the discounting in their model to match the effects of forward guidance in McKay, Nakamura, and Steinsson (2016), which yields only a modest amount of discounting in front of the forward looking output term ($\theta = .97$). These initial explorations into incomplete markets in the NK model leave us pessimistic about their ability to resolve equilibrium indeterminacy and thereby remove the OMO from the central banker's toolkit.

6 Monetary Policy Discretion

The modifications to private agent's primitives reviewed above are at best empirically unproven and at worst quantitatively implausible. In this section, we pursue an alternative approach to resolving equilibrium indeterminacy and removing OMOs from central bankers' toolkit: requiring all agents – households, firms, and the central banker – to follow Markov

[6] Gabaix (2018) estimated M and M^f. However, his procedure chooses these values to help the NK model fit aggregate data. It did not confronting the model with more direct evidence of inattention. Indeed, he registers (on page 18) a hope for better measurement of macroeconomic attention parameters like M and M^f.

strategies. That is, their decisions may only be functions of payoff-relevant variables, such as the current cost push shock and the current natural rate of interest. Since the empirical benchmark for monetary policy formation from which we should depart is probably closer to perfect discretion than perfect commitment, this modification arguably improves the model's realism.

In the standard NK model with indeterminacy, the private sector's choices for output and inflation are backward looking. (See equation (6).) Since these equilibria can fairly be described as self-fulfilling prophecies, this makes sense. Fulfilling a prophecy requires conditioning your actions on what you prophesied. Therefore, requiring households and firms to ignore the payoff-irrelevant past can contribute to resolving equilibrium indeterminacy. Putting the same requirement on the central bank completes the refinement. The mechanics of the result are very similar to those in McKay, Nakamura, and Steinsson (2017). In that paper, households ignore macroeconomic outcomes in future states of the world where they are unemployed, and this leaves less room for self-fulfilling prophecies to arise. Here, private agents take their own future actions and those of the central bank as given when choosing current inflation and output. Therefore, there is no room for self-fulfilling prophecies.

We call this game one of complete monetary policy discretion.[7] The sequence of cost-push shocks remains the same, $m_0 \neq 0$ and $m_t = 0$ for all $t \geq 1$. The players consist of households, firms, and a central banker. A Markov strategy for the central banker maps the current value of m_t into a choice for the interest rate, and Markov strategies for the private sector is a pair of mappings from (m_t, i_t). Rather than model the private sector's objectives explicitly, we follow Atkeson, Chari, and Kehoe (2010) by merely requiring these mappings to always satisfy the IS curves and PCs given the central banker's strategy.

Since the central banker and the private sector take future outcomes as given, the central banker's problem is static: Minimize the current loss subject to the current PC with π_{t+1} and y_{t+1} taken as given. Since there is one and only one solution for π_t and y_t given i_t, π_{t+1}, and y_{t+1}, the central bank has only the single instrument i_t. That is, there is no OMO available. Solving this problem yields the central banker's choice of inflation as a function of its future value.

$$\pi_t = -\frac{m_t + \beta\pi_{t+1}}{1 + \kappa^2/\lambda} \tag{16}$$

For $t \geq 1$, $m_t = 0$ and π_t equals a constant. The only possible constant value for inflation which satisfies (16) is $\pi_t = \pi_{t+1} = 0$. This and the sequence of PCs then gives us $y_t = 0$ for $t \geq 1$. The IS curves then require $i_t = i^\natural$ for $t \geq 1$. That is, Markov perfect equilibrium requires the central bank and the private sector to implement together the divine coincidence

[7] Blake and Kirsanova (2012) demonstrate equilibrium uniqueness in essentially the same model considered here. We present the result here for the benefit of readers unfamiliar with their work.

outcome. With this outcome in hand, it is straightforward to show that the central bank sets

$$i_0 = i^\natural + \frac{\sigma\kappa}{\lambda}\frac{m_0}{1+\kappa^2/\lambda}.$$

The resulting inflation and output equal

$$\begin{aligned}\pi_0 &= -\frac{m_0}{1+\kappa^2/\lambda} \text{ and}\\ y_0 &= \frac{\kappa}{\lambda}\frac{m_0}{1+\kappa^2/\lambda}.\end{aligned}$$

This game of complete monetary policy discretion has a single virtue for monetary policy formulation: There exists exactly one private sector outcome for each choice of the policy maker's single instrument. As presented here, this comes at the cost of being able to say nothing meaningful about central bank communication and Odyssean forward guidance. In a companion paper (Campbell and Weber, 2019) we demonstrate that the analytic benefits of Markov perfection can be gained while maintaining a meaningful role for forward guidance. For that, we give the central banker stochastic opportunities for reoptimization. When optimizing, the central banker chooses the entire infinite future path of interest rates knowing that she will discard these promises when the next reoptimization opportunity arrives. Schaumburg and Tambalotti (2007) call this "quasi-commitment" If these opportunities arrive frequently enough, the economy has a unique Markov perfect equilibrium. With $\beta = 0.99$ and $\kappa/\sigma = 0.01$, "frequently enough" means on average once every 11 quarters. Furthermore, the quasi-commitment economy does not display a forward guidance puzzle. Therefore, the quasi-commitment monetary policy game gives us well-defined predictions for output and inflation as functions of interest rates alone while having quantitatively meaningful implications for forward guidance. We refer the reader to that companion paper for more details.

References

Atkeson, A., V. V. Chari, and P. J. Kehoe (2010). Sophisticated monetary policies. *The Quarterly Journal of Economics 125*(1), 47.

Blake, A. P. and T. Kirsanova (2012, October). Discretionary Policy and Multiple Equilibria in LQ RE Models. *The Review of Economic Studies 79*(4), 1309–1339.

Campbell, J. R. (2013, October). Odyssean forward guidance in monetary policy: A primer. *Federal Reserve Bank of Chicago Economic Perspectives*, 130–139.

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012). Macroeconomic Effects of Federal Reserve Forward Guidance. *Brookings Papers on Economic Activity 1*, 1–54.

Campbell, J. R. and J. P. Weber (2019). Discretion rather than rules: Equilibrium uniqueness and forward guidance with inconsistent optimal plans. Federal Reserve Bank of Chicago Working Paper 2018-14.

Carlstrom, C. T., T. S. Fuerst, and M. Paustian (2015, November). Inflation and output in New Keynesian models with a transient interest rate peg. *Journal of Monetary Economics 76*(C), 230–243.

Clarida, R., J. Galí, and M. Gertler (1999, November). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature 37*(4), 1661–1701.

Cochrane, J. H. (2011). Determinacy and identification with Taylor rules. *Journal of Political Economy 119*(3), 565–615.

Debortoli, D. and R. Nunes (2014, November). Monetary Regime Switches and Central Bank Preferences. *Journal of Money, Credit and Banking 46*(8), 1591–1625.

Del Negro, M., M. Giannoni, and C. Patterson (2015). The Forward Guidance Puzzle. Federal Reserve Bank of New York Staff Report 574.

Eggertsson, G. and M. Woodford (2003). The Zero Interest-Rate Bound and Optimal Monetary Policy. *Brookings Papers on Economic Activity 1*, 139–211.

Evans, C. (2011). The Fed's dual mandate: responsibilities and challenges facing US monetary policy. Speech at European Economics and Financial Centre, London, 7 September 2011. Accessed online 12/12/16.

Fisher, J. D. (2015). On the structural interpretation of the Smets-Wouters "risk premium" shock. *Journal of Money, Credit and Banking 47*(2-3), 511–516.

Gabaix, X. (2018). A behavioral New Keynesian model. Unpublished Working Paper, Harvard University.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton, New Jersey, USA: Princeton University Press.

Galí, J. (2009, March). Constant interest rate projections without the curse of indeterminacy: A note. *International Journal of Economic Theory 5*(1), 61–68.

Giannoni, M. and M. Woodford (2005, March). Optimal Inflation-Targeting Rules. In M. Woodford and B. Bernanke (Eds.), *The Inflation Targeting Debate*, pp. 93–172. University of Chicago Press.

King, R. G. (2000). The New IS-LM Model: Language, Logic, and Limits. *Federal Reserve Bank of Richmond Economic Quarterly 86*(3), 45–103.

McKay, A., E. Nakamura, and J. Steinsson (2016). The power of forward guidance revisited. *The American Economic Review 106*(10), 3133–3158.

McKay, A., E. Nakamura, and J. Steinsson (2017). The discounted Euler equation: A note. *Economica 84*(336), 820–831.

Michaillat, P. and E. Saez (2019). A New Keynesian model with wealth in the utility function. *Manuscript*, 34.

Roberds, W. (1987). Models of Policy under Stochastic Replanning. *International Economic Review 28*(3), 731–755.

Schaumburg, E. and A. Tambalotti (2007). An investigation of the gains from commitment in monetary policy. *Journal of Monetary Economics 54*(2), 302–324.

Svensson, L. and M. Woodford (2005, March). Implementing Optimal Policy through Inflation-Forecast Targeting. In M. Woodford and B. Bernanke (Eds.), *The Inflation Targeting Debate*, pp. 19–91. University of Chicago Press.

Theil, H. (1958). *Economic Forecasts and Policy.* North-Holland Publishing Company.

Tinbergen, J. (1952). *On the Theory of Economic Policy.* North-Holland Publishing Company.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, New Jersey, USA: Princeton University Press.