# Mind the Gap: The Market Price of Financial (In)Flexibility

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# Mind the Gap: The Market Price of Financial (In)Flexibility*

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**Abstract**

The empirical connection between financial leverage and equity risk premia is surprisingly weak. We propose a quantitative model linking limited financial flexibility to levered risk premia, where firms make dynamic investment, financing, and default decisions. The model highlights two key variables, *leverage gaps* and *leverage targets*, as drivers of risk premia. Firms partially close the gap toward their target, remaining optimally over- or underlevered. Equityholders of overlevered firms face higher debt costs, as their capital structure becomes more exposed to bankruptcy risk. As a result, leverage gaps amplify asset returns. The "lost" leverage risk premium reappears after controlling for targets.

*Keywords:* firm dynamics, financial flexibility, financial frictions, heterogeneous firms, capital misallocation, cross section of returns, dynamic capital structure, risk premia.

*JEL Classification Numbers:* G12, G32.

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# 1 Introduction

Financial flexibility, or the ability of a business to efficiently adjust its funding, is a key driver of firm dynamics. As highlighted in Graham (2022)'s survey, around 80% of U.S. chief financial officers indicate financial flexibility as the primary driver of corporate debt policy. While Gamba and Triantis (2008) estimate that financial flexibility holds significant value for companies, its impact on stock market valuations for levered firms remains underexplored.

The main takeaway of this paper is that accounting for limited financial flexibility resurrects the expected strong connection between financial leverage and equity risk premia, which is crucial for understanding firms' financing costs and capital allocation decisions.[1] Accordingly, financial flexibility offers a simple explanation for the surprisingly weak relationship previously documented in the empirical literature.

We present a quantitative model of firm dynamics, in the spirit of Gomes and Schmid (2010), to study the equilibrium relationship between limited financial flexibility and expected returns. We calibrate the model to U.S. data, and we simulate artificial panels of firms. The key novel feature of the model, relative to comparable work, is the presence of debt adjustment costs, whose magnitude determines the degree of financial flexibility in the economy. To build intuition, we first develop a two-period version of the model, in which the optimal capital structure can be characterized analytically relative to a leverage target, and equity returns can be linked to leverage and to the leverage target through an expression similar to Proposition 2 of Modigliani and Miller (1958).

Two main predictions emerge. First, when financial flexibility is limited, firms generally face excessive costs to attain their leverage target, and optimally choose to only partially close the gap between their current and target leverage; the latter is the optimal leverage that firms would pursue with full financial flexibility. As a result, firms may be either optimally underlevered, when their leverage is lower than the target, or overlevered, when their leverage exceeds the target. The rate at which firms close their leverage gaps increases with financial flexibility. Second, in equilibrium,

[1] The conventional NPV approach to capital budgeting discounts cash flows at the WACC, whose computation rests on Proposition 2 of Modigliani and Miller (1958). Moreover, David, Schmid, and Zeke (2022) estimate that risk premium dispersion accounts for around 25% of the cross-sectional dispersion in the marginal product of capital among U.S. firms.

every dollar of debt incurs a higher cost for equity holders of overlevered firms than for those of underlevered firms. Optimal leverage is therefore no longer a sufficient statistic for the net cost of debt, and large leverage gaps trigger further amplification of asset return fluctuations. The higher cost of holding debt for equity holders of overlevered firms arises because, at their optimal leverage point, the average bankruptcy costs they bear exceed the average tax benefits. When financial flexibility is perfect, firms reach their leverage targets, and gaps become irrelevant for equity returns.[2] Leverage targets offer an equivalent characterization of the relationship between leverage and returns: under mild conditions, the "lost" leverage risk premium re-emerges when controlling for targets, and high targets are associated with lower risk premia, since firms with high targets are less likely to become overlevered after adverse productivity shocks.

Guided by the model's predictions, we document two new empirical facts. Because leverage targets and leverage gaps are unobservable, we first construct two complementary empirical measures of the target. Our first measure follows the empirical corporate finance literature and is based on the partial-adjustment approach of Flannery and Rangan (2006). Because the mapping between this reduced-form proxy and the notion of target in our model is unclear, we also build a second, model-disciplined measure that exploits a feature of our model: in the simulated economy, leverage targets are observable. We project the model-implied targets nonlinearly onto a small set of variables that are also available in the data; under the baseline calibration, this projection correlates with the model-based target at approximately 95%. We then estimate the same nonlinear specification in the data, using a multinomial logit model of leverage adjustments toward targets. The two measures deliver consistent results. We find, first, that estimated leverage gaps have a strong positive association with stock returns. Second, when leverage gaps are decomposed into observed leverage and the leverage target, stocks of high-leverage firms command a premium, while leverage targets exhibit a negative relationship with returns. These empirical patterns are robust across various specifications.[3]

We perform our quantitative analysis through the lens of a discrete-time, infinite-horizon neoclassical investment model in which firms make investment, financing, and default decisions to maximize

[2] Since investment is endogenous in our model, the leverage amplification mechanism remains active regardless of the asset return differences between mature and young firms emphasized by Gomes and Schmid (2010).

[3] They are robust to both book and market value measures of leverage, and to the inclusion of additional controls such as size, book-to-market, investment, profitability, and cash holdings. We also verify that the results are not driven by firms' investment, disinvestment, or cash accumulation policies.

their value in arbitrage-free markets. The model incorporates idiosyncratic and aggregate productivity shocks, costly external equity, and endogenous default on debt issued by competitive lenders. The model structure follows Gomes and Schmid (2010), to which we add costs associated with adjusting the firm's debt stock as a parsimonious, reduced-form catch-all for the various frictions that limit financial flexibility in practice (Fischer, Heinkel, and Zechner, 1989; Strebulaev, 2007; Acharya, Bharath, and Srinivasan, 2007; Myers, 1977). Although including debt adjustment costs in endogenous default models is conceptually straightforward, it raises non-trivial computational challenges, because it prevents the redefinition of the state space that is commonly used in the literature to reduce dimensionality. We address this challenge with global projection methods.

In the model, we define leverage targets following the "gap approach" to costly adjustments of production factors used in macroeconomics.[4] To extend this approach to economies with multiple frictions affecting dynamic choices, we follow the literature in industrial organization and sovereign default studies (Chatterjee and Eyigungor, 2012; Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021; Chatterjee, Corbae, Dempsey, and Ríos-Rull, 2023) and introduce logistically distributed shocks to shareholders' outside options. This setup nests the standard framework where shareholders default when equity value turns negative, and allows us to derive the firm's investment and debt policy optimality conditions analytically. A natural definition of the dynamic leverage target emerges, with the same structure and economic interpretation as in the two-period case. We use the calibrated model, in which leverage targets are observed, as a laboratory to assess how financial flexibility affects risk premia, and we also implement in the simulated economy the two empirical proxies used in the data. The results are consistent with those obtained from the model-implied definitions, and with our empirical findings.

Quantitatively, the model provides a good fit to the data, matching not only the targeted moments used in the calibration but also a broad set of untargeted moments, including firm-level investment, financing, and profitability dynamics, as well as aggregate stock-market valuations. Through the lens of the calibrated model, we study capital structure dynamics and levered risk premia. Firms adjust their capital structure toward their dynamic targets: overlevered firms tend to reduce leverage, and underlevered firms tend to increase it. Firms close, on average, only a

[4] See, for example, Sargent (1978), Caballero and Engel (1993), Cooper and Willis (2004), King and Thomas (2006), and Bayer (2009).

fraction of their gap each year, consistent with the hysteresis in capital structure documented by Hennessy and Whited (2005) and Lemmon, Roberts, and Zender (2008). Crucially, the calibrated model replicates the empirical patterns in levered risk premia. Leverage gaps are strongly positively related to equity returns, and the traditional positive leverage premium reemerges when we control for leverage targets, which load negatively on returns. The estimated coefficient of leverage in the specification with targets is roughly three times larger than in specifications that omit them.

The quantitative analysis also delivers an array of supplementary results. We simulate counterfactual economies with varying degrees of financial flexibility. The relationship between financial inflexibility and the rate at which firms close their gaps is strongly nonlinear: small departures from full flexibility trigger sharp declines in adjustment speeds, and the relationship between returns and gaps is more pronounced in less flexible economies. Since existing studies, such as Chaderina, Weiss, and Zechner (2022), document risk premia arising from the covariance of costly debt reductions with a counter-cyclical price of risk, we also consider counterfactual economies with different degrees of asymmetry between the costs of debt issuances and reductions. Leverage gaps remain relevant for returns and are sensitive to asymmetries, but no clear monotonic pattern emerges in the unconditional gap-return slope. The intuition is that greater asymmetry reshapes the gap distribution itself: anticipating the higher cost of cutting debt, firms issue less debt ex ante and become more underlevered, while firms that do end up overlevered face larger adjustment costs and cannot easily reduce their gaps. The unconditional slope therefore mixes a price effect with a distributional effect. Consistent with this, the return premia in our model arise even when the stochastic discount factor features a constant price of risk, distinguishing our channel from those that rely on counter-cyclical risk prices and asymmetric adjustment costs.

Finally, we study how the relevance of leverage gaps for stock returns varies with the persistence of profitability shocks, both in the model and in the data. The impact of financial inflexibility on capital structure dynamics and on the gap-return relationship depends on the degree of shock persistence. In the model, a comparative-statics exercise shows that lower persistence leads firms to close smaller fractions of their leverage gaps, and that gaps in turn matter more for equity returns. The intuition is that two forces operate when shocks are transitory: leverage targets become more volatile, generating frequent and sizable deviations, while shorter-lived shocks lower the expected present value of being mis-levered, weakening the incentive to pay debt adjustment costs. Empirically, isolating changes in persistence without affecting other properties of shock dynamics is challenging, and event-based methods are unsuitable since cross-sectional return tests require long time spans. We address this with a within-firm matching design: each year, we estimate profitability persistence at the firm level and match firms in the top tercile of persistence with firms in the lower two terciles, minimizing differences in the level and volatility of profitability. The empirical results support the model's comparative-statics prediction.

**Related Literature.** Our work primarily contributes to three research areas.

*Firm-Level Equity Risk Premia, Corporate Capital Structure, and Anomalies.* Our paper builds on studies that calibrate dynamic models to link equity risk premia with aspects of capital and debt structure. We align most closely with Gomes and Schmid (2010), who also study the weak leverage-return relation. They develop a dynamic model with endogenous investment and financing decisions, predict a quantitatively weak relationship, and emphasize that mature, safer firms tend to be more levered. Our findings suggest that the conventional leverage amplification mechanism remains important once leverage gaps and leverage targets are accounted for. Other papers studying asset pricing implications of dynamic leverage models focus on mechanisms that emphasize asymmetries, particularly the higher cost of downward adjustments, which generate risk by covarying with the counter-cyclical price of risk in the stochastic discount factor. Yamarthy (2020), Chaderina, Weiss, and Zechner (2022), and Friewald, Nagler, and Wagner (2022) examine risk premia arising from debt maturity choices, building on insights from Admati, DeMarzo, Hellwig, and Pfleiderer (2018), Dangl and Zechner (2021), and DeMarzo and He (2021). Chen (2010), Bhamra, Kuehn, and Strebulaev (2010), and Gomes and Schmid (2021) propose credit-risk models to interpret the joint time-series patterns of debt and equity prices over the business cycle.[5] Unlike these studies, the mechanism in our model does not necessarily depend on asymmetries in debt adjustments or on the cyclical behavior of the price of risk: our stochastic discount factor, following Clementi and Palazzo (2019), implies a constant market price of risk. We show that limited financial flexibility modifies the standard leverage amplification mechanism from Modigliani and Miller (1958), with leverage gaps

[5]Other studies focus on pricing firms' credit instruments, such as Bhamra, Fisher, and Kuehn (2011), Kuehn and Schmid (2014), Palazzo and Yamarthy (2022), and Danis and Gamba (2023). In contrast, we examine cross-sectional equity risk premia rather than aggregate risk premia on credit instruments.

and targets playing a central role. Analytical results from our two-period example highlight that the relationship between equity returns, leverage, and leverage gaps holds not only for expected returns but also for realized returns, under any stochastic discount factor. Other recent studies, such as Ozdagli (2012), Choi (2013), Obreja (2013), and Doshi, Jacobs, Kumar, and Rabinovitch (2019), jointly study risk premia related to corporate leverage and book-to-market equity; we find that the sensitivity of returns to book-to-market decreases when leverage gaps are included in cross-sectional return regressions. Finally, contributions such as Friewald, Wagner, and Zechner (2014) and Chen, Hackbarth, and Strebulaev (2022) study the "distress puzzle" of Campbell, Hilscher, and Szilagyi (2008); this puzzle is not the focus of our analysis, which examines the leverage amplification mechanism.

*Firm Dynamics and Financial Flexibility.* Numerous studies calibrate or estimate dynamic models of firm dynamics with frictions in adjusting capital structure. The closest study to ours is Gamba and Triantis (2008), who emphasize the value losses from financial inflexibility, especially for small firms. Differently from them, we use asset pricing data and focus on the puzzling evidence on levered risk premia. Among studies that introduce leverage targets in dynamic economies, Hennessy and Whited (2005) underscore the challenges in defining leverage targets in dynamic investment and financing models, where there is no single optimal capital structure independent of the current state. DeAngelo, DeAngelo, and Whited (2011) recognize that a meaningful leverage target exists in a similar model, and consider a long-term target from which firms occasionally deviate. Leverage targets are also routinely used to characterize $(S, s)$-type refinancing policies in continuous-time models.[6] Our findings complement these studies by characterizing leverage policies through a dynamic leverage target defined using the "gap approach," and by linking that target to the cross-section of equity returns. Consistent with this goal, we adopt a parsimonious approach to financial inflexibility by modeling reduced-form debt adjustment costs as a catch-all for a broad set of frictions, without microfounding their sources.

*Discrete-Time Models with Endogenous Default.* From a methodological perspective, we conduct our quantitative analysis using a discrete-time model with endogenous investment, financing, and

[6] A non-exhaustive list of contributions includes Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), Strebulaev (2007), Morellec and Zhdanov (2008), Hugonnier, Malamud, and Morellec (2015), Bolton, Wang, and Yang (2021), and Benzoni, Garlappi, Goldstein, and Ying (2022).

default, for which we employ a global numerical solution. As discussed in Chen and Frank (2022), the solution of these models presents computational complexity, because the default condition depends on the equity value, which in turn depends on debt prices that are internalized by each firm. Successful quantitative analyses of models of this type include Hennessy and Whited (2005), Gomes and Schmid (2010), Obreja (2013), Kuehn and Schmid (2014), Gomes and Schmid (2021), and Danis and Gamba (2018). Incorporating financial inflexibility into this class of models prevents the redefinition of the state space that the literature commonly uses to reduce dimensionality. We tackle this challenge with projection methods and a non-parametric structure for interpolation.

The paper is organized as follows. Section 2 illustrates the main economic forces at work in a two-period version of the model. Section 3 documents three empirical facts that motivate our analysis. Section 4 presents the infinite-horizon framework. Section 5 reports the quantitative results. Section 6 concludes.

# 2 Economic Mechanism in a Stylized Model

This section introduces a stylized two-date model that illustrates the economic mechanism behind our quantitative analysis. This simplified setting provides analytical solutions that effectively characterize leverage dynamics and levered returns. The main results take a form similar to the first and second propositions of Modigliani and Miller (1958), within a tradeoff economy with limited financial flexibility. In this setup, definitions of targets and gaps in capital structure arise naturally, consistent with the macroeconomic literature on adjustment gaps.

**Technology and Investment.** Time is discrete and there are two dates, $t = 1, 2$. We consider a value-maximizing firm $i$ in a perfectly competitive environment. After-tax operating profits are

$$\Pi_{i,t} = (1 - \tau)\, f(A_t, Z_{i,t}, K_{i,t}),$$

where $\tau \in (0, 1)$ is the corporate tax rate, $K_{i,t}$ is firm $i$'s capital stock, $f(\cdot)$ is strictly increasing, strictly concave, and differentiable in $K$, $A_t$ is an aggregate shock, and $Z_{i,t}$ is a firm-specific shock. Capital chosen at $t = 1$ becomes productive at $t = 2$ and evolves as

$$K_{i,2} = (1 - \delta)K_{i,1} + I_{i,1}, \qquad \delta \in (0, 1).$$

**Financing.** The firm finances investment and payouts with internal funds, net debt issuance, or external equity. Let $B_{i,t}$ denote the stock of one-period debt outstanding at the start of $t$, paying coupon $c_{i,t}$. Debt can be refinanced at $t = 1$ by choosing $B_{i,2}$, incurring a quadratic adjustment cost

$$\Lambda^B(B_{i,2} - B_{i,1}) = \frac{\lambda_B}{2}(B_{i,2} - B_{i,1})^2, \qquad \lambda_B \geq 0.$$

Accordingly, large debt adjustments are disproportionately more costly than small adjustments. Relaxing the assumption of cost symmetry still allows for closed-form solutions, though these offer limited additional insights. Furthermore, in this simple setup, we assume that equity issuance is frictionless. Thus, we focus on a pure tradeoff economy, in which firms are not financially constrained for their investment expenses, but issue debt to trade off their tax benefits and bankruptcy costs.

**Default and Recovery.** First, each firm $i$ defaults at $t = 2$ with exogenous probability $1 - \rho_i$, where $\rho_i \in (0, 1)$ is a constant solvency probability, independent of other shocks in the model and of the stochastic discount factor. We denote default and solvency events as $DEF_2 = 1$ and $DEF_2 = 0$, respectively. Second, the recovery value $L_{i,2}$ (in case of default at $t = 2$) increases at a constant rate $\xi_K > 0$ with residual capital at $t = 2$ and decreases at a linear rate $\xi_B > 0$ with debt; i.e., $\frac{\partial L_{i,2}}{\partial K_{i,2}} = \xi_K$ and $\frac{\partial L_{i,2}}{\partial B_{i,2}} = -\xi_B B_{i,2}$, with $L_{i,2}(0, K_{i,2}) = \xi_K K_{i,2}$. Thus, bankruptcy costs increase more than proportionally with the firms' stock of debt. These two assumptions, which our full model relaxes, enable us to sidestep the analytical computation of the default region while keeping intact the dependency of expected default costs on debt.

**Debt Pricing.** In the absence of arbitrage, there exists a strictly positive stochastic discount factor $M_2$ between $t = 1$ and $t = 2$. Let $c_{i,2}$ denote the coupon rate promised on debt issued at $t = 1$ and due at $t = 2$. If the firm is solvent, creditors receive at $t = 2$ the promised gross payoff $1+c_{i,2}$; if it defaults, they receive the recovery value $L_{i,2}/B_{i,2}$. Competitive pricing in credit markets requires

$$1 \;=\; \rho_i \, \mathbb{E}_1\big[M_2(1 + c_{i,2})\big] \;+\; (1 - \rho_i) \, \mathbb{E}_1\Big[M_2 \frac{L_{i,2}}{B_{i,2}}\Big]. \tag{1}$$

This formula yields the following expression for $c_{i,2}$:

$$1 + c_{i,2} = \frac{1 + r_F}{\rho_i} - \frac{1 - \rho_i}{\rho_i} \frac{L_{i,2}}{B_{i,2}}, \tag{2}$$

where we define the risk-free rate $r_F \equiv \mathbb{E}_1[M_2]^{-1} - 1$.

**Dividends and Cash Flows.** For explanation purposes, we now drop the subscript $i$ and simply refer to "the firm". Bankruptcy costs $BC_2$ are realized only in default and are given by $R_2^A K_2 - L_2$, while tax shields $TS_2 = \tau c_2 B_2$ arise only in solvency. Debt repayments are $(1 + c_2)B_2$ in case of solvency and $L_2$ in case of default. Accordingly, the firm's dividend $D_2^{Sol}$ at $t = 2$ in solvency is

$$D_2^{Sol} = R_2^A K_2 - R_2^{D,Sol} B_2,$$

where

$$R_2^A \equiv 1 + (1 - \tau)\Big(\tfrac{f(A_2,Z_2,K_2)}{K_2} - \delta\Big), \qquad R_2^{D,Sol} \equiv \frac{(1 + c_2)B_2 - TS_2}{B_2}.$$

In default, debt repayments equal $L_2$ and bankruptcy costs are $BC_2$, with $R_2^{D,Def} \equiv \frac{L_2+BC_2}{B_2}$, which implies that equity holders receive zero. Hence $D_2^{Def} = R_2^A K_2 - R_2^{D,Def} B_2 = 0$ in default by construction. For what follows it is useful to define $R_2^D \equiv R_2^{D,Sol} \chi_{\{DEF_2=0\}} + R_2^{D,Def} \chi_{\{DEF_2=1\}}$, where $\chi_{\{\cdot\}}$ denotes an indicator function that equals 1 if the event occurs and 0 otherwise.

At $t = 1$, the dividend is

$$D_1 = R_1^A K_1 - I_1 + B_2 - \big(1 + (1 - \tau)c_1\big)B_1 - \frac{\lambda_B}{2}(B_2 - B_1)^2,$$

where $R_1^A \equiv 1+(1-\tau)\left(\frac{f(A_1,Z_1,K_1)}{K_1} - \delta\right)$. Therefore, the firm chooses $K_2$ and $B_2$ to maximize equity value:

$$\begin{aligned} V_1 &= \max_{K_2,B_2} \; D_1 + \mathbb{E}_1^{\rho}[M_2 D_2] && (3) \\ &= \max_{K_2,B_2} \; D_1 + \rho\mathbb{E}_1\big[M_2 D_2^{Sol}\big] + (1 - \rho)\mathbb{E}_1\big[M_2 D_2^{Def}\big] = \max_{K_2,B_2} \; D_1 + \rho\mathbb{E}_1\big[M_2 D_2^{Sol}\big], && (4) \end{aligned}$$

where the operator $\mathbb{E}_1^{\rho}[\cdot]$ denotes the expectation taken before the default event is realized.

**Optimality Conditions.** The first-order condition with respect to $K_2$ can be written as

$$1 = \rho \mathbb{E}_1 \left[ M_2 \left( 1 + (1-\tau)(f_k(A_2, Z_2, K_2) - \delta) + (1-\tau) \frac{1-\rho}{\rho} \frac{\partial L_2}{\partial K_2} \right) \right]. \tag{5}$$

This condition has an intuitive interpretation. The firm finds the optimal investment at the point where the marginal cost of giving up one unit of capital at time 1 equates to the marginal benefit from the future after-tax marginal productivity of capital net of depreciation, and from a lower future coupon. The term $(1-\tau)\frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2}$ accounts for the lower promised interest payments the firm attains through a higher lender recovery value in default. The constant term $\frac{1-\rho}{\rho}$ captures the idea that, under fair debt pricing, higher recovery values effectively transfer resources from default states (which occur with probability $1-\rho$) to solvency states (which occur with probability $\rho$).

The first-order condition with respect to $B_2$ is

$$\lambda_B(B_2 - B_1) = \rho \mathbb{E}_1 \left[ M_2 \frac{\partial TS_2}{\partial B_2} \right] - (1-\rho)\mathbb{E}_1 \left[ M_2 \frac{\partial BC_2}{\partial B_2} \right]. \tag{6}$$

The optimal debt level equates the marginal adjustment cost from a change in debt at $t = 1$ (left-hand side) with the expected discounted marginal tax shield net of bankruptcy costs (right-hand side). The right-hand-side benefits and costs reflect the shareholders' perspective and realize at $t = 2$, depending on whether the firm remains solvent or goes bankrupt.

We introduce the notation used in the propositions below. Let $(K_2^*, B_2^*)$ denote the optimal choices of capital and debt with debt adjustment costs, and $(K_2^0, B_2^0)$ their frictionless counterparts in the absence of such costs. This notation highlights the gap between actual and frictionless target policies, a key element in the analysis that follows. The following proposition characterizes the optimal financing policy in terms of leverage gaps.

**Proposition 1** ***(Optimal Financing Policy).*** *Given initial conditions $S_1 \equiv (K_1, B_1, c_1, A_1, Z_1)$, the firm optimally:*

*a) Sets its debt stock $B_2^*$ to close a fraction $\lambda_1$ of the gap between its initial debt stock $B_1$ and its target debt stock $B_2^0$, that is*

$$B_2^* - B_1 = \lambda_1(B_2^0 - B_1), \tag{7}$$

*where*

$$\lambda_1 = \frac{(1-\tau)(1-\rho)\xi_B}{\lambda_B(1+r_F)+(1-\tau)(1-\rho)\xi_B}, \tag{8}$$

*and*

$$B_2^0 = \frac{\tau(r_F+1-\rho)}{(1-\tau)(1-\rho)\xi_B}. \tag{9}$$

*b) Sets its leverage ratio* $\frac{B_2^*}{K_2^*}$ *according to*

$$\frac{B_2^*}{K_2^*} - \frac{B_1}{K_1} = \lambda_1\left(\frac{B_2^0}{K_2^0} - \frac{B_1}{K_1}\right) + (1-\lambda_1)\left(\frac{B_1}{K_2^*} - \frac{B_1}{K_1}\right), \tag{10}$$

*with*

$$K_2^0 = K_2^* = f_k^{-1}\left[\frac{1+r_F-\rho+\rho(1-\tau)\left(\delta - \frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)}{(1+r_F)\rho\,(1-\tau)\,\mathbb{E}_1[M_2A_2Z_2]}\right].$$

**Proof.** See Appendix B.

Part a) of Proposition 1 shows that the optimal debt adjustment policy can be understood as a partial adjustment rule. We define the debt gap, $Gap_1^B \equiv B_1 - B_2^0$, as the difference between actual and target debt. Because $\lambda_1 \in [0,1]$, firms always close a fraction of their gap toward the target. When firms have excess debt compared to their target, they decrease their debt stock, and vice versa, as

$$B_2^* - B_1 = -\lambda_1 Gap_1^B.$$

Expression (9) shows that firms target higher debt stock the higher the marginal tax shield in case of solvency. The term that multiplies the tax rate $\tau$ in the numerator captures tax-deductible interest expenses, which include the risk-free rate $r_F$ and the credit spread to compensate for default with probability $1-\rho$. The term $(1-\tau)(1-\rho)\xi_B$ is the marginal bankruptcy cost. Higher values of $\xi_B$ reduce the debt recovery rate in the case of default per dollar of debt, since $\frac{\partial L_2}{\partial B_2^*} = -\xi_B B_2^*$. The factor $1-\tau$ accounts for the indirect tax benefit of bankruptcy costs, which increases the tax-deductible interest in solvency states.

Part b) of Proposition 1 turns to leverage dynamics by looking at adjustments in the firms' debt-to-asset ratios. As for debt stocks, firms close a fraction $\lambda_1$ of the gap $Gap_1^L$ between observed and target leverage, defined as

$$Gap_1^L \equiv \frac{B_1}{K_1} - \frac{B_2^0}{K_2^0}.$$

Firms with positive gaps are *overlevered*, i.e., their observed leverage is above the target, while firms with negative gaps are *underlevered*. Hence, equation (10) can be rewritten as:

$$\frac{B_2^*}{K_2^*} - \frac{B_1}{K_1} = -\lambda_1 Gap_1^L + (1-\lambda_1)\left(\frac{B_1}{K_2^*} - \frac{B_1}{K_1}\right).$$

The term $(1-\lambda_1)\left(\frac{B_1}{K_2^*} - \frac{B_1}{K_1}\right)$ accounts for the indirect effect of investment on debt-to-asset ratios. Firms that increase their scale (i.e., $K_2^* > K_1$) effectively reduce their leverage even without actively issuing (or withdrawing) debt with respect to their initial stock $B_1$. If the firm starts at the efficient level of capital at $t = 1$ (i.e., $K_1 = K_2^*$), then the overlevered firms always reduce their leverage, and the underlevered firms always increase it, similar to the debt policy in (7). Occasionally, the firm can move away from the target by disinvesting enough to increase its debt-to-asset ratio if overlevered, or by expanding enough to reduce it if underlevered.[7] Similarly, the firm can "overshoot" and choose a leverage ratio even higher than the target leverage when overlevered, or even lower than its target leverage when underlevered.[8] In an environment with full financial flexibility, i.e., $\lambda_B = 0$, it follows that $B_2^* = B_2^0$, and $\lambda_1 = 1$. $\lambda_1$ decreases with $\lambda_B$, as financial flexibility is helpful in closing gaps.[9]

**Financial Flexibility and Equity Returns.** We now investigate the impact of limited financial flexibility on the firm's stock return. Since $V_1 = D_1 + \mathbb{E}_1^{\rho}[M_2 D_2] = D_1 + \rho\mathbb{E}_1[M_2 D_2^{Sol}]$ is the cum-dividend equity value of the firm, $P_1 = V_1 - D_1$ is the (ex-dividend) stock price at time $t = 1$, $P_2$ at $t = 2$ is zero because there are no more cash flows after that time, and the realized firm's stock

[7]Formally, this occurs when the "passive" change in leverage due to the indirect effect of the denominator is large enough in absolute value, i.e., $\frac{B_1}{K_2^*} - \frac{B_1}{K_1} > -\frac{\lambda_1}{1-\lambda_1} Gap_1^L$ for overlevered firms, and $\frac{B_1}{K_2^*} - \frac{B_1}{K_1} < -\frac{\lambda_1}{1-\lambda_1} Gap_1^L$ for underlevered firms.

[8]This requires that firms, despite being underlevered, have excess debt, i.e., $B_1 > B_2^0$. Symmetrically, overlevered firms "overshoot" when $B_1 < B_2^0$.

[9]Observe that the dynamics for debt and leverage in Proposition 1 can be alternatively characterized using the gap with respect to the current debt/leverage (in other words, the "residual gap" after adjustment), that is, $Gap_2^B \equiv B_2^* - B_2^0$ and $Gap_2^L \equiv \frac{B_2^*}{K_2^*} - \frac{B_2^0}{K_2^0}$. In this case, one obtains $B_2^* - B_1 = -\frac{\lambda_1}{1-\lambda_1} Gap_2^B$, and $\frac{B_2^*}{K_2^*} - \frac{B_1}{K_2^*} = -\frac{\lambda_1}{1-\lambda_1} Gap_2^L$. Since $\frac{\partial Gap_2^B}{\partial Gap_1^B} > 0$ and $\frac{\partial Gap_2^L}{\partial Gap_1^L} > 0$, the qualitative predictions for equity returns point in the same direction in both cases.

return $R_2^E$ is defined as

$$R_2^E = \frac{D_2}{\rho \mathbb{E}_1[M_2 D_2^{Sol}]}. \tag{11}$$

We formulate Proposition 2 so that $B_2^*$ depends on the initial state $S_1$ through the partial–adjustment rule in Proposition 1, so lagged leverage $B_1/K_1$ influences expected returns indirectly via its effect on the optimal choice at $t = 2$.

**Proposition 2** ***(Financial Flexibility and Equity Returns).*** *Given initial conditions $S_1$:*
*a) realized equity returns of the firm between $t = 1$ and $t = 2$ are related to leverage targets and gaps through the following relationship:*

$$R_2^E = \frac{R_2^A}{\gamma_A(K_2^*)} + \frac{B_2^* \gamma_D(B_2^*)}{K_2^* \gamma_A(K_2^*) - B_2^* \gamma_D(B_2^*)} \left( \frac{R_2^A}{\gamma_A(K_2^*)} - \frac{R_2^D}{\gamma_D(B_2^*)} \right), \tag{12}$$

*where $\gamma_A(K_2^*) \equiv \mathbb{E}_1^\rho[M_2 R_2^A]$, and $\gamma_D(B_2^*) \equiv \mathbb{E}_1^\rho[M_2 R_2^D] = 1 - \rho \mathbb{E}_1 \left[ M_2 \frac{TS_2}{B_2^*} \right] + (1-\rho)\mathbb{E}_1 \left[ M_2 \frac{BC_2}{B_2^*} \right]$. Expected equity returns are given by:*

$$\mathbb{E}_1[R_2^E] = \frac{\mathbb{E}_1[R_2^A]}{\gamma_A(K_2^*)} + \frac{B_2^* \gamma_D(B_2^*)}{K_2^* \gamma_A(K_2^*) - B_2^* \gamma_D(B_2^*)} \left( \frac{\mathbb{E}_1[R_2^A]}{\gamma_A(K_2^*)} - \frac{\mathbb{E}_1[R_2^D]}{\gamma_D(B_2^*)} \right); \tag{13}$$

*b) the shareholders' expected cost of debt financing $\gamma_D(B_2^*)$ is linked to leverage gaps as follows:*

$$\gamma_D(B_2^*) = 1 + \kappa_1 Gap_1^L + \kappa_2 \frac{B_1}{K_1} + \rho \mathbb{E}_1 \left[ M_2 \Delta TS^{M-A} \right] - (1-\rho)\mathbb{E}_1 \left[ M_2 \Delta BC^{M-A} \right], \tag{14}$$

*where $\kappa_1 \equiv \lambda_B \lambda_1 K_2^* > 0$, $\kappa_2 \equiv -\lambda_B \lambda_1 (K_2^* - K_1)$, and $\Delta TS^{M-A} \equiv \frac{\partial TS_2}{\partial B_2}(B_2^*) - \frac{TS_2}{B_2^*}$ and $\Delta BC^{M-A} \equiv \frac{\partial BC_2}{\partial B_2}(B_2^*) - \frac{BC_2}{B_2^*}$ are, respectively, the differences between marginal and average tax shields and bankruptcy costs.*

**Proof.** See Appendix B.

Part a) of Proposition 2 resembles the second proposition of Modigliani and Miller (1958), which relates to the special case of (13) with $\gamma_D(B_2^*) = 1$ and $\gamma_A(K_2^*) = 1$.[10] As in the case of Modigliani

[10] As in the original second proposition of Modigliani and Miller, leverage would appear as the debt-to-equity ratio $\frac{B_2^*}{K_2^* - B_2^*}$ in this limiting case. As the debt-to-equity ratio is a monotone increasing function of the debt-to-asset ratio, this definition does not alter the qualitative leverage-returns relationship.

and Miller (1958), the relationship between equity returns and capital structure also holds ex post for realized returns, as shown in (12). Part b) instead establishes a link between the equity premia of the left-hand side of (13) and financial flexibility through leverage gaps.

The expression for the expected cost of debt, $\gamma_D(B_2^*)$, from the shareholders' perspective in part (a) highlights how the textbook positive relationship between leverage and expected returns breaks down in our setup. Due to the presence of bankruptcy costs and tax shields, which, unlike debt adjustment costs, do not need to be removed to define a target, the cost of each dollar of debt for the company $R_2^D$ is not equal to the market return on debt for debt holders $1 + r_2^D$. The net cost to leverage (e.g., Korteweg, 2010), which is the difference between the average bankruptcy costs and tax shields, is denoted by the term $\frac{(1-\rho)\mathbb{E}_1[M_2 BC_2] - \rho\mathbb{E}_1[M_2 TS_2]}{B_2^*}$ in $\gamma_D(B_2^*)$. Since $\gamma_D(B_2^*)$ multiplies $B_2^*$ in (13), an amplification effect is created if $\gamma_D(B_2^*) > 1$ and average bankruptcy costs exceed average tax shields in correspondence of optimal leverage choices. Vice versa, a dampening effect arises if $\gamma_D(B_2^*) < 1$ and average tax shields are larger than average bankruptcy costs. As in Modigliani and Miller (1958), capital structure propagates both good and bad cash flow outcomes (asset risk) to equity payouts. In contrast, the relation between leverage and returns is influenced not only by the amount of debt $B_2^*$, but also by the net cost that equity holders bear per dollar of debt.[11]

The conditions for $\gamma_D(B_2^*) = \gamma_A(K_2^*) = 1$ are intuitive. For $\gamma_D(B_2^*) = 1$, tax shields and bankruptcy costs must be absent. Specifically, $\gamma_D(B_2^*)$ equals one when the average present value (per dollar of debt) of tax shields matches the present value of bankruptcy costs. Because tax shields are linear in debt and bankruptcy costs are convex, equal marginal values — i.e., in the absence of debt adjustment costs — still generate surplus on inframarginal units. This surplus affects firm value and expected returns. As in the Modigliani-Miller framework, capital structure becomes irrelevant. For $\gamma_A(K_2^*) = 1$, two conditions must hold: (i) the effect of capital on bankruptcy costs must be eliminated, which follows from the conditions that yield $\gamma_D(B_2^*) = 1$; and (ii) the production function must be homogeneous of degree one in capital, i.e., $f(A_2, Z_2, K_2^*) = f_k(A_2, Z_2, K_2^*)K_2^*$. Under these assumptions, equation (5) simplifies to $\mathbb{E}_1\left[M_2 R_2^A\right] = 1$, implying $\gamma_A(K_2^*) = 1$.[12]

[11] As the present values of tax shields and bankruptcy costs in $\gamma_D(B_2^*)$ are computed using the pricing kernel $M_2$, their risks are priced as they cannot be diversified away.

[12] This parallels the neoclassical q theory result: with constant returns to scale and no capital adjustment costs, average q equals marginal q, and the firm earns no surplus on inframarginal units.

Part b) of Proposition 2 characterizes the relationship between capital structure and expected returns under limited financial flexibility. It links the expected cost of debt, $\gamma_D(B_2^*)$, to inflexibility through leverage gaps. This result helps interpret the findings in Section 3 and Section 5, which support the empirical and quantitative relevance of the characterization.

Expression (14) implies a positive relation between equity returns and leverage gaps through the coefficient on $\kappa_1 Gap_1^L$, holding the remaining terms constant. This prediction aligns with the empirical evidence reported in Table 1. Intuitively, the optimality condition (6) shows that optimal capital structure choices balance the marginal costs of capital structure rebalancing with the wedge between marginal tax shields and bankruptcy costs. Thus, changes in leverage impact the expected cost of debt, as financial flexibility, as captured by $\Lambda^B(\Delta B_1)$, depends on debt adjustments $\Delta B_1$. As Proposition 1 establishes, optimal leverage dynamics entail partially closing leverage gaps. As a consequence, leverage gaps enter the expected cost of debt $\gamma_D(B_2^*)$ in (14). All else being equal, shareholders of overlevered firms (positive gaps) bear larger costs per dollar of debt than underlevered firms (negative gaps), controlling for observed leverage.

The relation between observed leverage and returns remains ambiguous, even after controlling for leverage gaps. The terms $\Delta TS^{M-A}$ and $\Delta BC^{M-A}$ help capture these effects. They measure the difference between marginal and average tax shields and bankruptcy costs at the optimal capital structure associated with $Gap_1^L$. Appendix B shows that $\Delta TS^{M-A}$ is U-shaped and $\Delta BC^{M-A}$ is increasing in leverage. Marginal tax shields, $\frac{\partial TS_2}{\partial B_2}$, increase linearly with leverage, $\frac{B_1}{K_1}$. By contrast, average tax shields, $\frac{TS_2}{B_2}$, are increasing and concave in $B_2$, and therefore increase at a decreasing rate with lagged leverage $\frac{B_1}{K_1}$ through the optimal debt choice $B_2^*$.

Their expected discounted difference decreases at low leverage, when the firm issues little debt, each dollar is well collateralized, and expected default losses remain negligible. The expected discounted difference increases at high leverage, when higher coupons generate larger tax shields.

Marginal bankruptcy costs increase linearly with leverage because liquidation losses are convex in debt. As leverage rises, each additional dollar of debt raises expected fire-sale and distress losses by more than the previous dollar. This convexity implies that the marginal cost of debt increases proportionally with leverage. Average bankruptcy costs per dollar of debt are U-shaped in leverage because two forces move in opposite directions. At low leverage, each dollar of debt is backed by substantial going-concern value relative to liquidation value. As leverage increases, this surplus spreads across more claims, so the average cost per dollar initially falls (collateral dilution). At high leverage, convex liquidation losses dominate, and the average cost per dollar rises. The interaction between dilution at low leverage and convex distress at high leverage generates the U-shape. The difference between marginal and average bankruptcy costs, $\Delta BC^{M-A}$, increases with leverage because it reflects curvature. It measures how much more costly the next dollar of debt is relative to the average dollar already outstanding. Since liquidation losses are convex, marginal distress increases faster than average distress as leverage rises. Therefore, the gap between marginal and average costs widens monotonically.

The effect of leverage on the expected cost of debt to the firm therefore depends on the parameterization, which determines the gap between average tax shields and bankruptcy costs at the firm's optimal choice of $B_2^*$.

Finally, the term $\kappa_2 \frac{B_1}{K_1}$ in $\gamma_D(B_2)$, which equals zero when $K_2^* = K_1$, captures the effect of asset expansion on the cost of debt from the shareholders' perspective. This implication motivates the robustness checks on investment and leverage changes driven by its denominator, discussed in Section 3. Relatedly, the heterogeneity in asset returns $R_2^A$ can also confound the relationship between leverage and gaps and expected equity returns.[13] As standard in the literature, the reduced-form evidence in Table 1 includes variables that capture differences in the riskiness of firms' assets in place and growth options, namely size and book-to-market equity (e.g., Gomes and Schmid, 2010).[14]

Which effects are empirically more relevant is a quantitative question, on which our analysis in Section 5 offers insights. The reduced-form evidence in Table 1 suggests that the ambiguity in the returns-leverage relationship resolves when target leverage is included as a predictor. Since the leverage gap $Gap_1^L$ is the difference between observed leverage and target leverage, the expected

[13]In Appendix B, we show that $\gamma_A(K_2^*)$ is constant in $K_2^*$ in the two-period model when the production function is homogeneous of degree $\alpha$ in $K_2^*$, as in the Cobb-Douglas case with decreasing returns to scale. Notice that, in this case, $\mathbb{E}_1[M_2 R_2^A]$ is not equal to one because of decreasing returns to scale. This is the case only when $\alpha = 1$, which however is incompatible with an interior solution for $K_2^*$.

[14]Hou, Xue, and Zhang (2015) interpret these variables in a neoclassical production economy with endogenous investment. Table A3 shows the results in Table 1 are robust to the inclusion of investment and profitability.

cost of debt in (14) can be equivalently expressed as

$$\gamma_D(B_2^*) = 1 + \kappa_1 \left( \frac{B_1}{K_1} - \frac{B_2^0}{K_2^0} \right) + \kappa_2 \frac{B_1}{K_1} + \rho E_1 \left[ M_2 \Delta TS^{M-A} \right] - (1-\rho) E_1 \left[ M_2 \Delta BC^{M-A} \right]. \quad (15)$$

A comparison of the expressions for $\gamma_D(B_2^*)$ in (14) and (15) suggests that, controlling for leverage targets instead of leverage gaps, helps revive the classic leverage amplification effect on equity returns. Although the relationship between leverage and returns is still formally ambiguous, the term $\frac{B_1}{K_1} - \frac{B_2^0}{K_2^0}$, in lieu of $Gap_1^L$, adds an additional dimension through which leverage loads positively on the expected cost of debt. In this specification, the leverage target is negatively related to returns, as it loads negatively on $\gamma_D(B_2^*)$. Intuitively, leverage is less risky for firms with high leverage targets, as their capital structure yields high tax shields compared to bankruptcy costs, as Proposition 1 shows. Hence the two-period model highlights the competing forces at work but, ultimately, a quantitative resolution of this ambiguity requires the infinite-horizon model, where default probabilities and premia are solved jointly.

# 3 Empirical Patterns in Levered Returns

This section presents three empirical facts that motivate our quantitative analysis. First, we replicate the well-known puzzle that leverage and equity returns are weakly related after controlling for standard return predictors, namely size and book-to-market. Second, we show that empirical proxies of leverage gaps are strongly positively related to stock returns. Third, when we decompose leverage gaps into target leverage and actual leverage, the expected positive relation between leverage and returns reappears, while the coefficient on target leverage turns negative.

## 3.1 Data and Sample

Accounting variables are from the CRSP/COMPUSTAT merged annual database over the period 1965-2020. Our sample includes all companies listed on AMEX, NYSE, and NASDAQ. We exclude financial firms (SIC codes 6000-6999), utility companies (SIC codes 4900-4999), and firms that are not incorporated in the United States. All variables used in the analysis are defined in Appendix A.

We match accounting variables to monthly stock prices and returns from the Center of Research in Security Prices (CRSP). We drop observations for which the trading status is halted or suspended and we include delisting returns in the time series of returns. To avoid look-ahead biases, we follow Fama and French (1992) and conservatively require a minimum gap of six months between fiscal year-ends and realized returns. To do so, for each firm, we match data from the latest fiscal year ending in calendar year $t-1$ to returns on common shares from July of calendar year $t$ to June of calendar year $t+1$.

## 3.2 Empirical Proxies of Leverage Targets and Gaps

Our analysis involves reduced-form measures of leverage targets and gaps, which need to be estimated. We construct two such measures: one based on the empirical corporate finance literature, and another derived from the quantitative analysis in Section 5. First, we follow the empirical model of Flannery and Rangan (2006), which is widely used in the corporate finance literature to produce proxies of leverage targets.[15] We implement rolling estimates of leverage targets $LT_{i,t}$ for firm $i$ in year $t$ to avoid look-ahead biases when including them in our asset pricing tests. Years from 1965 to 1979 are used as a "burn-in" period to obtain sensible initial estimates. We then continue on a rolling basis using accounting information available until year $t$. To be included in the analysis, firms are required to have at least five consecutive years of observations.

In Flannery and Rangan (2006)'s model, firms partially adjust their leverage $L_{i,t}$ over time toward the target level $LT_{i,t}$ with a "speed of adjustment" $\lambda$, i.e.,

$$L_{i,t} - L_{i,t-1} = \lambda(LT_{i,t} - L_{i,t-1}) + \epsilon_{i,t},$$

where leverage targets $LT_{i,t} = \beta X_{i,t-1}$ are linear functions of firm-level characteristics $X_{i,t-1}$, and vary both over time and across firms. Using this linear approximation, one obtains the following estimable regression equation:

$$L_{i,t} = (\lambda\beta)X_{i,t-1} + (1-\lambda)L_{i,t-1} + \epsilon_{i,t}. \tag{16}$$

[15] The validation of alternative reduced-form proxies of leverage targets is outside the scope of our model-based analysis. In a review article, Flannery and Hankins (2013) assess different estimation procedures for leverage targets in dynamic panels. The evidence on their performance is mixed, but we choose Flannery and Rangan (2006)'s approach primarily for its computational efficiency in large datasets.

The control variables that Flannery and Rangan (2006) include in $X_{i,t-1}$ are profitability, the market-to-book value of assets, depreciation, total assets, a measure of tangibility, R&D expenses, an indicator variable for missing values of R&D expenses, industry-median leverage, and a firm fixed effect. We first estimate (16) to obtain the coefficients $\lambda\beta$ on $X_{i,t-1}$ and $(1-\lambda)$ on $L_{i,t-1}$. We then recover $\lambda$ and $\beta$ and estimate $LT_{i,t}$. Finally, we compute leverage gaps as the difference between observed leverage and leverage targets, i.e., as $L_{i,t-1} - LT_{i,t}$ .

Second, we estimate a multinomial logit model for leverage adjustments. Leverage targets are approximated using a functional form obtained by projecting the policy functions from the calibrated dynamic model onto a set of observable variables. For brevity, we refer to this proxy as the "projection target." As detailed in Section 5.2, we choose a functional form that closely approximates the model-implied leverage targets, achieving a correlation of approximately 95%.[16]

To capture nonlinearities in adjustment behavior without imposing strong functional assumptions, we define a discrete variable $Adj_{i,t}$: it takes the value 1 if a firm increases its leverage $L_{i,t}$ relative to the previous year's leverage ($L_{i,t-1}$), -1 if it decreases it, and 0 if it remains stable, all within a ±1% band. Based on the analysis in Section 5.2, we model leverage targets as:

$$LT_{i,t} = \frac{1}{1 + e^{-\gamma Z_{i,t}}},$$

where $\gamma$ is a vector of coefficients to be estimated, and $Z_{i,t}$ includes a second-order polynomial in the logs of total assets, median leverage, and market-to-book ratio, and profitability (i.e., these variables along with their interaction terms and squared terms), and a firm fixed effect.

We model the probabilities of leverage adjustments using a multinomial logit function of the leverage gap $Gap^L_{i,t} \equiv L_{i,t-1} - LT_{i,t}$:

$$\mathbb{P}(\mathrm{Adj}_{i,t} = 0) = \frac{1}{1 + e^{\beta_{\mathrm{inc}}\mathrm{Gap}^L_{i,t}} + e^{\beta_{\mathrm{dec}}\mathrm{Gap}^L_{i,t}}},$$
$$\mathbb{P}(\mathrm{Adj}_{i,t} = 1) = \frac{e^{\beta_{\mathrm{inc}}\mathrm{Gap}^L_{i,t}}}{1 + e^{\beta_{\mathrm{inc}}\mathrm{Gap}^L_{i,t}} + e^{\beta_{\mathrm{dec}}\mathrm{Gap}^L_{i,t}}},$$
$$\mathbb{P}(\mathrm{Adj}_{i,t} = -1) = \frac{e^{\beta_{\mathrm{dec}}\mathrm{Gap}^L_{i,t}}}{1 + e^{\beta_{\mathrm{inc}}\mathrm{Gap}^L_{i,t}} + e^{\beta_{\mathrm{dec}}\mathrm{Gap}^L_{i,t}}},$$

[16]A related approach — often referred to as empirical policy functions — has been used in Bazdresch, Kahn, and Whited (2018) and Nikolov, Schmid, and Steri (2021).

where $\beta_{\text{inc}}$ and $\beta_{\text{dec}}$, to be estimated, capture how sensitive adjustment behavior is to the leverage gap. Each firm-year observation contributes to the total log-likelihood $\log(\mathcal{L})$ as:

$$\log \mathcal{L}_{i,t} \equiv \begin{cases} \log \mathbb{P}(\text{Adj}_{i,t} = 0) & \text{if } Adj_{i,t} = 0, \\ \log \mathbb{P}(\text{Adj}_{i,t} = 1) & \text{if } Adj_{i,t} = 1, \\ \log \mathbb{P}(\text{Adj}_{i,t} = -1) & \text{if } Adj_{i,t} = -1, \end{cases}$$

where $\log(\mathcal{L}) = \sum_{i,t} \log \mathcal{L}_{i,t}$. We estimate the parameters $\{\gamma, \beta_{\text{inc}}, \beta_{\text{dec}}\}$, and leverage targets and gaps, by maximizing this likelihood. As before, we use a rolling estimation procedure to avoid look-ahead bias. Appendix A provides further details.

As is standard in the literature on dynamic models linking corporate policies to asset returns (see Gomes and Schmid, 2010; Bolton, Chen, and Wang, 2011; Kuehn and Schmid, 2014; Gomes, Jermann, and Schmid, 2016, among others), firms differ only through ex-post realizations of productivity shocks. That is, the model does not explicitly include firm fixed effects in parameterizations. To map model-based policies to the data, the quantitative literature typically removes firm fixed effects from the data. In some cases, studies add noise to simulations or explicitly model ex-ante firm heterogeneity. These approaches appear in, for example, Cooper and Ejarque (2003), Hennessy and Whited (2005), Hennessy and Whited (2007), and Gamba and Saretto (2020). In our empirical analysis, both measures of leverage targets include firm fixed effects to account for this heterogeneity and to improve comparability with the simulated model economies.

## 3.3 Empirical Facts

Table 1 summarizes our key empirical facts. Table 1 reports estimates from cross-sectional regressions of stock returns on leverage variables, size, and book-to-market. Reported coefficients are time-series averages of the estimates from monthly cross-sectional regressions. Newey-West standard errors with lag-length of 4 are in parentheses. In Appendix E, we report a series of robustness checks and sensitivity analyses, which we summarize in Subsection 3.4. The empirical analysis of levered returns involves several variables, such as controls for dispersion in asset returns, size and book-to-market equity, and multiple leverage variables. Cross-sectional regressions, as an empirical procedure, facilitate the simultaneous inclusion of multiple predictors and control variables in the analysis. In contrast, portfolio sorts become impractical when handling more than a few characteristics concurrently. This is due to the limited number of stocks available to populate a sufficient number of diversified portfolios each month.

**Fact #1: Leverage is Weakly Related to Average Returns.** The leftmost column shows that leverage has a positive coefficient, which is statistically insignificant at the conventional levels after controlling for size and book-to-market equity. This result illustrates the well-known "leverage puzzle" that emerges in the empirical literature and that Gomes and Schmid (2010) highlight. In fact, several studies have explored the relationship between leverage and equity returns. Contributions in this area include Bhandari (1988), Fama and French (1992), Penman, Richardson, and Tuna (2007), George and Hwang (2010), Caskey, Hughes, and Liu (2012), and Doshi, Jacobs, Kumar, and Rabinovitch (2019). In particular, Caskey, Hughes, and Liu (2012), like our study, refer to overlevered and underlevered firms based solely on variations in marginal tax benefits, as measured by the "kink" proxy of Graham (2000).

**Fact #2: Leverage Gaps are Positively Related to Average Returns.** Columns (2) and (4) of Table 1 relate returns to leverage gaps, using the Flannery-Rangan and projection targets, respectively. In both columns, the coefficients are positive and significant at the 1 percent level. Introducing leverage gaps reduces the coefficient on book-to-market equity, though the change is not statistically significant relative to column (1). This result may not be particularly surprising, and it is intuitive to include growth options, proxied by book-to-market, in leverage targets. However, the resulting effect is not straightforward, as book-to-market enters both the linear target specification and the nonlinear projection model alongside other variables, through different functional forms and estimation methods. When observed leverage and the leverage gap are included in columns (3) and (5), the gap coefficient remains positive and significant at the 1 percent level. The coefficient on leverage also remains positive, but is less than one standard error from zero and thus not statistically significant at conventional levels.

**Fact #3: Controlling for Leverage Targets, Leverage is Positively Related to Average Returns.** Columns (1) and (2) of Appendix Table A2 include both observed leverage and leverage targets in the same regression. These specifications are equivalent to those in columns (3) and

(5) of Table 1, since the leverage gap is defined as the difference between observed leverage and the target. As a result, the coefficients on the targets have the same magnitude but the opposite sign of the leverage gap coefficients in columns (3) and (5) of Table 1. The coefficient on observed leverage corresponds to the sum of the coefficients on leverage and the gap. Despite these mechanical relationships, Appendix Table A2 provides an alternative view of the results. When both leverage variables are included, the positive relation between observed leverage and returns reappears, while leverage targets are negatively related to returns. Both coefficients are statistically significant at the 1 percent level across all specifications.

### 3.4 Robustness and Sensitivity Analyses

In Appendix E, we implement a series of robustness checks and sensitivity analyses. Table A1 shows that the results in Table 1 continue to hold qualitatively when leverage is measured using book values or net of cash, as these alternatives occasionally affect results in the literature.[17] Table A3 includes investment and profitability factors as additional controls for heterogeneous asset returns, as suggested by the q-factor model of Hou, Xue, and Zhang (2015) and the five-factor model of Fama and French (2016). Although some coefficients are smaller in magnitude, they retain the same signs as in Table 1 and remain statistically significant.

Table A4 examines whether changes in leverage driven by asset sales (disinvestments) or, more broadly, by changes in assets, influence the gap-return relationship. The baseline results remain robust when controlling for leverage changes driven by asset variation in the denominator, holding debt constant, and for a negative investment indicator. Both variables are positively associated with returns and statistically significant at the one percent level. However, after controlling for profitability and investment (columns 3, 4, 7, and 8), the leverage change due to asset variation loses both statistical and economic significance. The disinvestment indicator remains significant at the ten percent level, though with a smaller economic effect. Table A5 benchmarks our results against firms with zero leverage and negative net leverage, which have received attention in the literature (e.g., Bates, Kahle, and Stulz, 2009; Strebulaev and Yang, 2013). The results align with

[17] See, for example, George and Hwang (2010) and Ozdagli (2012). The analysis of market versus book measures of leverage for the mapping between data and structural models is beyond the scope of this paper. For a detailed discussion, see Bretscher, Feldhütter, Kane, and Schmid (2020).

the baseline. Controlling for the industries in which firms operate reduces sensitivity to leverage gaps, but does not fully absorb the gap risk premia. However, gaps lose relevance for firms with persistent zero or negative net leverage over multiple fiscal years.

Finally, we note that since book-to-market and size are used as controls in the cross-sectional return regressions — and analogous variables (e.g., book-to-market of assets) also enter the construction of leverage targets — this overlap could drive the results. However, the relationship is not straightforward. In the Flannery-Rangan target, book-to-market enters through a linear combination of several variables, while in the projection target, it is included in a more flexible, nonlinear specification. In our sample, the correlation between the leverage target and book-to-market is 0.12 for the Flannery-Rangan target and –0.07 for the projection target. For size, the corresponding values are –0.09 and 0.30, respectively.

# 4 The Model

In this section, we lay out a dynamic model, which serves as a basis for the quantitative analysis in Section 5. In the model, as is standard in the literature (e.g., Gomes, 2001; Gamba and Triantis, 2008; Gomes and Schmid, 2010), firms are ex-post heterogeneous due to idiosyncratic productivity shocks. Unlike ex-ante heterogeneity in parameters, this feature endogenously propagates and amplifies shocks through the model's structure, helping to discipline the analysis. We also introduce aggregate shocks, which affect firms' profitability and discount rates, to better study the role of differences in leverage targets across firms. Every period, firms make endogenous financing and investment decisions. Moreover, firms are financially constrained as external equity is costly and firms can endogenously default on their debt. Finally, firms have limited financial flexibility as they bear costs of adjusting their capital structure.

## 4.1 Setup

**Technology and Investment.** Time is discrete. We consider the problem of a value-maximizing firm $i$ in a perfectly competitive environment. In each period $t$, the after-tax operating profits $\Pi_{i,t}$ are given by

$$\Pi_{i,t} = (1-\tau) f(A_t, Z_{i,t}, K_{i,t}), \tag{17}$$

**Table 1**
EMPIRICAL FACTS ON LEVERAGE AND RETURNS

The table reports coefficient estimates of Fama-MacBeth cross-sectional regression of monthly stock returns on leverage variables and controls. The sample includes all Compustat firms traded on NYSE, AMEX and NASDAQ between 1965 and 2020 and covered by the Center of Research in Security Prices (CRSP). Financial firms and utilities are excluded. De-listing returns are included in monthly returns. Leverage is market debt ratio (MDR). FR Gap is the leverage gap based on the Flannery-Rangan target. Proj Gap is the leverage gap based on the projection (MLE) target. Controls are log market capitalization (Size) and log book-to-market equity (Be/Me). Newey-West t-statistics (4 lags) are in parentheses. The symbols ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

| Dependent Variable: Monthly Stock Return | | | | | |
|---|---|---|---|---|---|
| | | Flannery-Rangan Target | | Projection Target | |
| | (1) | (2) | (3) | (4) | (5) |
| Leverage | 0.24 | | 0.18 | | 0.27 |
| | (0.67) | | (0.55) | | (0.85) |
| Lev. Gap | | 0.96*** | 0.79*** | 0.57*** | 0.55*** |
| | | (2.92) | (2.75) | (4.31) | (4.09) |
| Size | -0.21*** | -0.18*** | -0.18*** | -0.19*** | -0.19*** |
| | (-4.54) | (-4.41) | (-4.49) | (-4.76) | (-4.82) |
| Be/Me | 0.16** | 0.11 | 0.10 | 0.14* | 0.12* |
| | (2.41) | (1.57) | (1.62) | (1.84) | (1.87) |
| $R^2$ | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| N. Obs | 1,221,746 | 880,452 | 880,452 | 761,294 | 761,294 |

where $\tau \in (0,1)$ is the corporate tax rate, $K_{i,t}$ is firm $i$'s capital stock, $f(A_t, Z_{i,t}, K_{i,t})$ is a production function, $A_t$ is an exogenous aggregate shock, and $Z_{i,t}$ is a firm-specific shock. The variables $A_t$ and $Z_{i,t}$ can be interpreted as shocks to demand, input prices, or productivity. We assume that $f(\cdot)$ is strictly increasing, strictly concave, and differentiable in capital $K_{i,t}$. $A_t$ and $Z_{i,t}$ have bounded support $\mathcal{A} = [\underline{A}, \overline{A}]$ and $\mathcal{Z} = [\underline{Z}, \overline{Z}]$ , respectively. The law of motion of $A_t$ is described by a Markovian transition function $Q_A(A_t, A_{t+1})$. Similarly, $Z_{i,t}$ is Markovian with transition function $Q_Z(Z_{i,t}, Z_{i,t+1})$.

At the beginning of each period, firms can scale operations by choosing investment $I_{i,t}$ in physical capital. Next period's capital stock $K_{i,t+1}$ satisfies the standard law of motion for capital accumulation

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}, \tag{18}$$

where $\delta \in (0,1)$ is the depreciation rate of capital.

**Financing.** Investment and distributions to shareholders can be financed with either the internal funds generated by operating profits or new issues. The latter can take the form of new debt (net of repayments) or external equity.

We denote the firm's debt stock as $B_{i,t}$. Outstanding one-period debt pays a coupon $c_{i,t}$ per unit of time. As we detail below, the coupon is set in competitive credit markets to compensate expected bankruptcy costs in case of default, in which lenders recover a liquidation value $L_{i,t}$. Firms are allowed to refinance their debt stock by issuing a net amount $\Delta B_{i,t} \equiv B_{i,t+1} - B_{i,t}$.

Similar to Croce, Kung, Nguyen, and Schmid (2012) and Belo, Lin, and Yang (2014), firms incur costs $\Lambda^B(\Delta B_{i,t}) \geq 0$ of adjusting their debt stock. In our quantitative analysis, we choose a flexible parameterization for $\Lambda^B(\Delta B_{i,t})$ to allow for asymmetries and possibly large marginal costs for small adjustments. Having a flexible reduced-form functional form allows us to keep the model tractable without committing to a specific microfoundation. In fact, debt adjustment costs can arise from several quantitatively relevant sources, including underwriting and management spreads (e.g., Fischer, Heinkel, and Zechner, 1989; Strebulaev, 2007), seniority issues that prevent firms from making large changes in the capital structure (e.g., Acharya, Bharath, and Srinivasan, 2007), costs associated with the liquidation of short-term debt (Diamond, 1991), and agency costs associated with long-term debt (e.g., debt overhang and underinvestment as in Myers, 1977).

Firms can also raise external finance through seasoned equity offerings (SEOs). Let $E_{i,t}$ denote the equity issuances. Following the extensive existing literature, we consider equity issuance costs $\Lambda^E(E_{i,t}) \geq 0$. We interpret negative values for $E_{i,t}$ as equity payouts.

Investment financing decisions must satisfy the firm's budget constraint, which takes the form of the following accounting identity between uses and sources of funds:

$$\Pi_{i,t} + \Delta B_{i,t} + E_{i,t} + \tau\delta K_{i,t} + \tau c_{i,t} B_{i,t} = I_{i,t} + c_{i,t} B_{i,t} + \Lambda^B(\Delta B_{i,t}), \tag{19}$$

where the terms $\tau\delta K_{i,t}$ and $\tau c_{i,t}B_{i,t}$ reflect the tax deductibility of depreciation and interest expenses, respectively. Net distributions to shareholders, $D_{i,t}$, are then defined as equity payout net of issuance costs, i.e.,

$$D_{i,t} = -E_{i,t} - \Lambda^E(E_{i,t}). \tag{20}$$

**Valuation.** We assume the absence of arbitrage opportunities in financial markets, which implies the existence of a stochastic discount factor $M_t > 0$ (e.g., Harrison and Kreps, 1979). Defining $V_{i,t}$ as the firm's equity value, we assume that shareholders strategically default on their debt obligations if $V_{i,t} < \zeta_{i,t}$, where $\zeta_{i,t}$ is a random variable capturing uncertainty in shareholders' outside options. We further assume that $\zeta_{i,t}$ follows a logistic distribution with mean zero and scale parameter $s \geq 0$ and is independent of other shocks in the model. This specification generalizes the standard assumption in the literature that the outside option is deterministic and equal to zero. Specifically, for small values of $s$, the outside option converges to the mean of $\zeta_{i,t}$.

From a technical perspective, introducing shocks to outside options $\zeta_{i,t}$ allows for the analytical derivation of optimality conditions, which prove useful in Subsection 4.2 when defining a dynamic leverage target based on the gap literature (e.g., Bayer, 2009). In our framework, this approach helps address the challenge of characterizing optimal dynamic capital structure choices when various frictions – such as equity issuance and bankruptcy costs – affect firms' decisions, whereas models in the gap literature typically consider only convex adjustment costs. This technique has long been used in economics to formulate differentiable dynamic discrete choice problems that yield smooth decision rules (e.g., McFadden, 1977), including applications in industrial organization and, more recently, sovereign default studies. Recent relevant contributions include Chatterjee and Eyigungor (2012), Dvorkin, Sánchez, Sapriza, and Yurdagul (2021), and Chatterjee, Corbae, Dempsey, and Ríos-Rull (2023), among others.

Accordingly, interest payments $c_{i,t}$ are determined endogenously as follows:

$$B_{i,t+1} = \mathbb{E}_t\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}\chi_{\{V_{i,t+1}>\zeta_{i,t+1}\}} + L_{i,t+1}\chi_{\{V_{i,t+1}\leq\zeta_{i,t+1}\}}\right)\right], \tag{21}$$

where $\chi_{\{\cdot\}}$ is an indicator function that takes: value 1 when the event $\{\cdot\}$ happens; value 0 when the event $\{\cdot\}$ does not happen.

In our baseline infinite-horizon economy, the firm's maximization problem can be formulated recursively using the beginning-of-period value function, before outside option shocks $\zeta_{i,t}$ are realized. Equity holders choose (i) investment $I_{i,t}$, (ii) debt issuance $\Delta B_{i,t}$, and (iii) default decisions $DEF_{i,t} \in \{0,1\}$, where 1 indicates default and 0 solvency. Let $S_{i,t} \equiv \{K_{i,t}, B_{i,t}, c_{i,t}, Z_{i,t}, A_t\}$ denote the state vector of firm $i$ at time $t$. The continuation value function $V^C(S_{i,t})$, conditional on the firm not defaulting in the current period, is defined as

$$V^C(S_{i,t}) \equiv \max_{\Delta B_{i,t}, I_{i,t}} D_{i,t} + \mathbb{E}[M_{t+1}V(S_{i,t+1})|S_{i,t}]. \tag{22}$$

Equity holders decide whether to default before time-$t$ outside option shocks by choosing

$$DEF_{i,t} = \begin{cases} 1 & \text{if } \zeta_{i,t} > V^C(S_{i,t}) \\ 0 & \text{if } \zeta_{i,t} \leq V^C(S_{i,t}). \end{cases}$$

Under this intra-period timing, the beginning-of-period value function $V(S_{i,t})$ relates to the continuation value as[18]

$$\begin{aligned} V(S_{i,t}) &= \mathbb{E}^\zeta[\max\{\zeta_{i,t}, V^C(S_{i,t})\}] \\ &= \mathbb{E}^\zeta\left[\max_{DEF_{i,t}\in\{0,1\}} (1-DEF_{i,t})V^C(S_{i,t}) + DEF_{i,t}\zeta_{i,t}\right], \end{aligned} \tag{23}$$

where the second line follows from the definition of $DEF_{i,t}$, and the expectation operator $\mathbb{E}^\zeta[\cdot]$ is taken with respect to $\zeta_{i,t}$, formally defined as

$$\mathbb{E}^\zeta[g(\zeta)] \equiv \int_{-\infty}^{\infty} g(x)dF^\zeta(x),$$

for any integrable function $g(\zeta)$ and cumulative distribution function $F^\zeta(\zeta)$. Substituting $V^C(S_{i,t})$ from (22) into (23), we obtain the recursive formulation:

$$V(S_{i,t}) = \mathbb{E}^\zeta[\max\{\zeta_{i,t}, \max_{\Delta B_{i,t}, I_{i,t}} D_{i,t} + \mathbb{E}[M_{t+1}V(S_{i,t+1})|S_{i,t}]\}], \tag{24}$$

[18]For any two numbers $a$ and $b$, we can write $\max\{a,b\} = \max_{d\in\{0,1\}}(1-d)b + da$. Hence, the second line of equation (23) simply expresses the binary nature of the default choice, with $DEF_{i,t} = 1$ denoting default and $DEF_{i,t} = 0$ continuation.

subject to: (i) the law of motion for capital (18), (ii) the firm's budget constraints (19) and (20), and (iii) the debt pricing equation (21). Note that leverage, leverage gaps, and leverage targets are ultimately determined by the model's state variables $S_{i,t}$. However, when we analyze our quantitative results and compare them to the data, we avoid depending on this direct mapping to derive a more insightful characterization and empirical predictions. As elaborated on in Section 4.2, this approach aligns well with established practices in the literature on levered risk premia, and it is also commonly seen in other contexts, like when corporate investment is linked to Tobin's Q (e.g., Hayashi, 1982).

## 4.2 Defining a Dynamic Target

**The Gap Approach.** In the simplified setup from Section 2, the definitions of targets and gaps in capital structure naturally characterize firms' optimal policies. These definitions align with the "gap approach", which is widely used to describe costly adjustments in production factors such as capital and labor (e.g., Sargent, 1978; Caballero and Engel, 1993; Cooper and Willis, 2004; King and Thomas, 2006; Bayer, 2009). These studies approximate firm dynamics by modeling how firms adjust production factors toward a dynamic target, known as the frictionless target. This target represents the level a production factor would reach in the absence of changes in stochastic variables. More formally, it is defined as the policy function obtained when adjustment costs are removed for a single period.

Unlike, for example, Bayer (2009), which considers only convex adjustment costs in labor decisions, our dynamic model incorporates additional frictions, such as equity issuance costs and default risk. In this section, we exploit the differentiability of the firm's problem in (24) to extend the definitions from the two-period example – motivated by the gap approach – to our dynamic economy.

**The Firm's Problem.** The expression in (23) can be rewritten as

$$\begin{aligned} V(S_{i,t}) &= \mathbb{E}^{\zeta}[\max\{\zeta_{i,t}, V^C(S_{i,t})\}] \\ &= \int_{-\infty}^{V^C(S_{i,t})} V^C(S_{i,t}) dF^{\zeta}(\zeta_{i,t}) + \int_{V^C(S_{i,t})}^{\infty} \zeta_{i,t} dF^{\zeta}(\zeta_{i,t}) \\ &= P^S_{i,t} V^C(S_{i,t}) + (1 - P^S_{i,t}) \mathbb{E}[\zeta_{i,t} | DEF_{i,t} = 1, S_{i,t}] \\ &= V(S_{i,t}) = V^C(S_{i,t}) + s \cdot \ln\left(1 + e^{-\frac{V^C(S_{i,t})}{s}}\right). \end{aligned} \quad (25)$$

The solvency probability $P^S_{i,t}$ is

$$P^S_{i,t} \equiv \Pr[V^C(S_{i,t}) \geq \zeta_{i,t} | S_{i,t}] = F^{\zeta}(V^C(S_{i,t})) = \frac{1}{1 + e^{-\frac{V^C(S_{i,t})}{s}}}, \quad (26)$$

which, by the properties of the logistic distribution, is differentiable in $V^C(S_{i,t})$. Likewise, the expected value of the outside option in case of default, $(1 - P^S_{i,t})\mathbb{E}[\zeta_{i,t}|DEF_{i,t} = 1, S_{i,t}]$, is also differentiable in $V^C(S_{i,t})$. See Appendix C for further details.

**Optimality Conditions.** By taking first-order conditions and applying the Benveniste-Scheinkman conditions, we derive the following optimality conditions for debt:

$$\begin{aligned} 1 - \frac{\partial \Lambda^B(B_{i,t+1} - B_{i,t})}{\partial B_{i,t+1}} = \mathbb{E}\Bigg[ & M_{t+1} \cdot P^S_{i,t+1} \cdot \left( \frac{1 + \frac{\partial \Lambda^E(E_{i,t+1})}{\partial E_{i,t+1}}}{1 + \frac{\partial \Lambda^E(E_{i,t})}{\partial E_{i,t}}} \right) \\ & \cdot \left( 1 + (1-\tau)\left( c_{i,t+1} + B_{i,t+1} \frac{\partial c_{i,t+1}}{\partial B_{i,t+1}} \right) - \frac{\partial \Lambda^B(B_{i,t+2} - B_{i,t+1})}{\partial B_{i,t+1}} \right) \Bigg| S_{i,t} \Bigg], \end{aligned} \quad (27)$$

and capital:

$$1 = \mathbb{E}\left[ M_{t+1} \cdot P^S_{i,t+1} \cdot \left( \frac{1 + \frac{\partial \Lambda^E(E_{i,t+1})}{\partial E_{i,t+1}}}{1 + \frac{\partial \Lambda^E(E_{i,t})}{\partial E_{i,t}}} \right) \cdot \left( \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}} + 1 - \delta(1-\tau) - \frac{\partial c_{i,t+1}}{\partial K_{i,t+1}} B_{i,t+1}(1-\tau) \right) \Bigg| S_{i,t} \right]. \quad (28)$$

To define a dynamic target, it helps to compare these optimality conditions with those in the two-period example from Section 2, where the target definition naturally emerges. In the two-period case, the optimality conditions for debt (6) and capital (5) can be rewritten as:

$$1-\frac{\partial\Lambda^B(B_2-B_1)}{\partial B_2}=\mathbb{E}\left[M_2\cdot P_2^S\cdot\left(1+(1-\tau)\left(c_2+\frac{\partial c_2}{\partial B_2}B_2\right)\right)\middle|\,S_1\right], \tag{29}$$

and:

$$1=\mathbb{E}\left[M_2\cdot P_2^S\cdot\left(\frac{\partial\Pi_2}{\partial K_2}+1-\delta(1-\tau)-\frac{\partial c_2}{\partial K_2}(1-\tau)B_2\right)\middle|\,S_1\right]. \tag{30}$$

Equations (29) and (30) share the same structure and economic interpretation as (28) and (27) when $t=1$. However, the simplifying assumptions in Section 2 (e.g., exogenous default implies $P_2^S=\rho$) yield closed-form solutions in Proposition 1 and Proposition 2, whereas the dynamic model does not have a closed-form solution. The terms $c_{i,t+1}+B_{i,t+1}\frac{\partial c_{i,t+1}}{\partial B_{i,t+1}}$ and $\frac{\partial c_{i,t+1}}{\partial K_{i,t+1}}$ capture how interest on debt responds to financing and investment decisions. The expression for $c_{i,t+1}$ follows from the debt pricing condition (21).

The differences between the two sets of optimality conditions are intuitive. First, the one-period-ahead adjustment cost term $\frac{\partial\Lambda^B(B_{i,t+2}-B_{i,t+1})}{\partial B_{i,t+1}}$ does not appear in the two-period model, aligning with the "gap approach," which removes current-period adjustment costs. In the two-period model, removing adjustment costs for one period or permanently is equivalent.[19] Second, the term $\frac{1+\frac{\partial\Lambda^E(E_{i,t+1})}{\partial E_{i,t+1}}}{1+\frac{\partial\Lambda^E(E_{i,t})}{\partial E_{i,t}}}$ is absent from the two-period case because, for analytical convenience, we assume no financing constraints. Later in this section, we discuss an extension of the two-period example to include costly equity issuance, further clarifying the parallel with the dynamic model.

**Computing Dynamic Targets and Gaps.** In what follows, we use the superscript 0 to indicate that all endogenous variables are computed in the absence of current-period adjustment costs, i.e., in the frictionless benchmark. By contrast, starred variables such as $(K^*_{i,t+1}, B^*_{i,t+1})$ denote the

[19] Adda and Cooper (2003) offer another interpretation of the frictionless target debt, which extends to linear-quadratic problems and applies to the simplified setup in Section 2. Specifically, the frictionless target is a fixed point of the debt policy function. It is immediate to verify that this is the case in (7), as $B_2^0=B_2^*$ only when adjustment costs of capital structure are zero and $\lambda_1=1$. The frictionless target generally differs from the static target, defined as the debt level that would arise if there were never any costs of adjustments (e.g., Cooper and Willis, 2004; Bayer, 2009), although they coincide in the two-period illustration of Section 2.

corresponding optimal choices when adjustment costs are present. This distinction parallels the two-period example in Section 2 and allows us to clearly define dynamic targets and gaps as the difference between actual (frictional) and frictionless policies.

In our setup, the frictionless target debt stock $B^0_{i,t+1}$ satisfies the optimality condition for debt (27) without current-period debt adjustment costs:

$$1 = \mathbb{E}\left[ M_{t+1} \cdot P^{S,0}_{i,t+1} \cdot \left( \frac{1 + \frac{\partial \Lambda^E(E^0_{i,t+1})}{\partial E^0_{i,t+1}}}{1 + \frac{\partial \Lambda^E(E^0_{i,t})}{\partial E^0_{i,t}}} \right) \right.$$
$$\left. \cdot \left( 1 + (1-\tau)\left( c^0_{i,t+1} + B^0_{i,t+1} \frac{\partial c^0_{i,t+1}}{\partial B^0_{i,t+1}} \right) - \frac{\partial \Lambda^B(B^0_{i,t+2} - B^0_{i,t+1})}{\partial B^0_{i,t+1}} \right) \;\middle|\; S_{i,t} \right]. \tag{31}$$

In the two-period example, the target defined in Proposition 1 analogously satisfies (29) when $\Lambda^B(B^*_{i,2} - B_{i,1}) \equiv 0$.

As discussed in Section 4.3, our numerical solution method does not directly rely on optimality conditions. Instead, we compute the target debt stock as the solution to the Bellman equation:

$$V^0(S_{i,t}) = \mathbb{E}^\zeta[\max\{\zeta_{i,t}, \max_{I^0_{i,t}, \Delta B^0_{i,t}} D^0_{i,t} + \mathbb{E}[M_{t+1}V(S_{i,t+1})|S_{i,t}]\}], \tag{32}$$

which yields $B^0_{i,t+1} = B_{i,t} + \Delta B^0_{i,t}$. The equity payout $D^0_{i,t}$ excludes current-period debt adjustment costs, i.e., $\Lambda(\Delta B^0_{i,t}) \equiv 0$. The continuation value $V(S_{i,t+1})$ is computed using the value function $V_{i,t}$ in (24), as adjustment costs are set to zero only for the current period.

We define the target leverage ratio as $\frac{B^0_{i,t+1}}{K^0_{i,t+1}}$, where $K^0_{i,t+1} = (1-\delta)K_{i,t} + I^0_{i,t}$. The leverage gap is then given by $Gap^L_{i,t} = \frac{B_{i,t}}{K_{i,t}} - \frac{B^0_{i,t+1}}{K^0_{i,t+1}}$. Notice that the dynamic model does not impose mechanical convergence to the target. Instead, the calibrated model serves as a lab to quantify the extent to which firms adjust their capital structure toward the target.

In what follows, we work entirely with the problem including adjustment costs. Since the frictionless counterpart never enters this analysis, there is no need to introduce the superscript 0. All variables in this subsection should therefore be read as actual (frictional) choices, without additional notation.

**Defining Targets: Which Costs to Remove?** The leverage target defined above removes only debt adjustment costs. One might ask whether alternative definitions could better capture capital structure dynamics. In particular, removing equity issuance costs for one period may seem appealing. However, this approach could also create an unrealistic benchmark for firm behavior. Without equity issuance costs, firms would not just gain flexibility in adjusting their capital structure – they would be able to raise external financing at no cost.

To explore this issue, it is helpful to rewrite the optimality conditions for debt (27) and capital (28) as follows:

$$\mathbb{E}\left[ M^F_{i,t+1} R^b_{i,t+1} \middle| S_{i,t} \right] = 1 - \frac{\partial \Lambda^B (B_{i,t+1} - B_{i,t})}{\partial B_{i,t+1}}, \tag{33}$$

$$\mathbb{E}\left[ M^F_{i,t+1} R^k_{i,t+1} \middle| S_{i,t} \right] = 1. \tag{34}$$

Following prior work (e.g., Love, 2003; Rampini and Viswanathan, 2013), we define

$$M^F_{i,t+1} \equiv M_{t+1} P^S_{i,t+1} \frac{1 + \frac{\partial \Lambda^E (E_{i,t+1})}{\partial E_{i,t+1}}}{1 + \frac{\partial \Lambda^E (E_{i,t})}{\partial E_{i,t}}}$$

as the firm's stochastic discount factor. This term incorporates expected marginal equity issuance costs, which influence the discount rate from the equityholders' perspective. The terms $R^b_{i,t+1}$ and $R^k_{i,t+1}$ represent the marginal cost of debt and the marginal product of capital, respectively, both from the shareholders' perspective:

$$R^b_{i,t+1} \equiv 1 + (1-\tau)\left( c_{i,t+1} + B_{i,t+1} \frac{\partial c_{i,t+1}}{\partial B_{i,t+1}} \right) - \frac{\partial \Lambda^B (B_{i,t+2} - B_{i,t+1})}{\partial B_{i,t+1}}, \tag{35}$$

$$R^k_{i,t+1} \equiv \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}} + 1 - \delta(1-\tau) - \frac{\partial c_{i,t+1}}{\partial K_{i,t+1}} B_{i,t+1}(1-\tau). \tag{36}$$

Two remarks are in order. First, the optimality conditions reflect the shareholders' perspective. For this reason, the term $\frac{1 + \frac{\partial \Lambda^E (E_{i,t+1})}{\partial E_{i,t+1}}}{1 + \frac{\partial \Lambda^E (E_{i,t})}{\partial E_{i,t}}}$ appears symmetrically in the conditions for both debt and capital. As captured by the definition of $M^F_{i,t+1}$, equity issuance costs primarily affect the shareholders' discount rate. Combining (35) and (36) yields

$$\mathbb{E}\left[ M^F_{i,t+1} R^e_{i,t+1} \middle| S_{i,t} \right] = \frac{\partial \Lambda^B (B_{i,t+1} - B_{i,t})}{\partial B_{i,t+1}}, \tag{37}$$

where the excess return $R^e_{i,t+1} \equiv R^k_{i,t+1} - R^b_{i,t+1}$ measures the difference between the marginal return on investment and the marginal cost of external financing sources from the shareholders' perspective (i.e., debt). The adjustment costs on the right-hand side of (37) prevent the returns on the use of funds and the cost of external financing from aligning, except at the target. Appendix B extends the two-period example in Section 2 to include equity issuance costs at $t = 1$. While the extension does not yield a closed-form solution, it provides modified versions of Propositions 1 and 2, where leverage targets are defined analogously.

Second, modeling debt adjustment costs in reduced form aligns with the quantitative focus of this paper. It enables the definition of a target without tying the analysis to a specific friction, such as debt maturity (e.g., Chaderina, Weiss, and Zechner, 2022), that may limit financial flexibility. In our setup, as is standard in the literature, equity issuance costs are also modeled in reduced form, and shareholders' and managers' incentives are aligned. The model also does not differentiate between current and future shareholders. Because firm dynamics are shaped by the shareholders' perspective, removing equity adjustment costs does not yield a benchmark for analyzing debt dynamics in this framework. More generally, since the appropriate definition of a target depends on its ability to capture firm behavior, other setups – for example, agency models with multiple shareholder groups – may call for different definitions. Quantitatively, and consistent with intuition, the model fit suggests that debt acts as the primary adjustment margin in response to shocks. This reflects the fact that equity adjustment costs still limit the firm's ability to fully reoptimize in most periods. Empirically, this is also consistent with the infrequent use of equity issuance by U.S. corporations, as documented in several studies (e.g., Hennessy and Whited, 2007).

Finally, although the baseline model does not include capital adjustment costs, it is natural to ask how their inclusion would affect the definition of the target. Appendix C examines an extension of the dynamic model that incorporates these costs. In this case, equation (37) becomes

$$\mathbb{E}\left[ M^F_{i,t+1} R^e_{i,t+1} \middle| S_{i,t} \right] = \frac{\partial \Lambda^B (B_{i,t+1} - B_{i,t})}{\partial B_{i,t+1}} + \frac{\partial \Lambda^K (I_{i,t})}{\partial K_{i,t+1}},$$

where $\Lambda^K(I_{i,t})$ denotes the capital adjustment cost function, the derivative $\frac{\partial \Lambda^K(I_{i,t})}{\partial K_{i,t+1}}$ is taken through the law of motion $I_{i,t} = K_{i,t+1} - (1-\delta)K_{i,t}$, and $R^e_{i,t+1}$ is redefined in Appendix C to include one-period-ahead capital adjustment costs. In this setting, defining a meaningful target for capital structure dynamics requires removing both debt and capital adjustment costs. Appendix B extends the two-period example from Section 2 to include capital adjustment costs, further supporting this conclusion.

Finally, one can remove equity issuance costs and still obtain meaningful leverage gaps and targets to characterize firms' policies, as in the two-period model in Section 2. However, quantitatively, the literature shows that these costs are sizable and affect corporate policies (e.g., Hennessy and Whited, 2005; Hennessy and Whited, 2007). For this reason, we include them in the quantitative analysis.

### 4.3 Functional Forms and Model Solution

**Functional Forms.** To carry out our quantitative analysis, we add structure to the model with the following functional forms. The production function is a Cobb-Douglas $f(A_t, Z_{i,t}, K_{i,t}) = A_t Z_{i,t} K_{i,t}^{\alpha} - F$, where $\alpha \in (0,1)$, $A_t$ is an exogenous aggregate shock, $Z_{i,t}$ is a firm-specific shock, and $F > 0$ is a fixed production cost. We assume $A_t$ and $Z_{i,t}$ follow AR(1) processes such that:

$$\begin{aligned} \log A_t &= \mu_A(1-\rho_A) + \rho_A \log A_{t-1} + \sigma_A \varepsilon_t^A, \\ \log Z_{i,t} &= \rho_Z \log Z_{i,t-1} + \sigma_Z \varepsilon_{i,t}^Z. \end{aligned} \tag{38}$$

As $\varepsilon_t^A$ and $\varepsilon_{i,t}^Z$ are truncated standard normal variables, both $A_t$ and $Z_{i,t}$ are lognormal, with mean $\mu_A \in (-\infty, \infty)$ and 0, persistence $\rho_A \in (0,1)$ and $\rho_Z \in (0,1)$, and volatility $\sigma_A \in (0,\infty)$ and $\sigma_Z \in (0,\infty)$, respectively.[20]

We choose a flexible linear-exponential (LINEX) functional form (e.g., Varian, 1975; Kim and Ruge-Murcia, 2009; Aruoba, Bocola, and Schorfheide, 2017) for the capital structure rebalancing costs, i.e.,

$$\Lambda^B(\Delta B_{i,t}) = \lambda_B(e^{\gamma_B \Delta B_{i,t}} - \gamma_B \Delta B_{i,t} - 1), \tag{39}$$

with $\lambda_B \in (0,\infty)$ and $\gamma_B \in (-\infty,\infty)$. In our context, the LINEX adjustment cost function is attractive as it parsimoniously captures limited financial flexibility arising from different sources we do not explicitly model. Varying only two parameters, the "scale" $\lambda_B$ and the "asymmetry"

[20] As common in the literature, we set the mean of firm-specific shocks to zero (e.g., Hennessy and Whited, 2007; Gomes and Schmid, 2010).

$\gamma_B$, $\Lambda(\Delta B_{i,t})$ spans a broad spectrum of functional forms.[21] As previous studies provide limited guidance on functional forms and economic magnitudes for the overall debt adjustment costs firms face, we exploit data restrictions to calibrate them to realistic values in the context of our model. Altınkılıç and Hansen (2000) provide empirical support for the convex nature of debt issuance costs in the context of straight bond underwriting fees. Their findings instead suggest that fixed costs constitute a minor portion, specifically 10.4% of underwriter spreads.[22]

Following Hennessy and Whited (2007) and Gomes and Schmid (2021), lenders recover a fraction $\xi \in (0,1)$ of firm $i$'s capital stock in bankruptcy, that is, $L_{i,t} = \xi K_{i,t}$. Different from the illustration model in Section 2 with exogenous default, we do not explicitly link ex-post realized bankruptcy costs to the stock of debt $B_{i,t}$. Since default in now endogenous, however, expected bankruptcy costs are increasing with debt.

Several studies, for example, Gomes (2001) and Falato, Kadyrzhanova, Sim, and Steri (2022), choose equity issuance costs $\Lambda(E_{i,t})$ with both a fixed and a proportional component, i.e.,

$$\Lambda^E(E_{i,t}) = (\lambda_0 + \lambda_1 E_{i,t})\chi_{\{E_{i,t}>0\}}, \tag{40}$$

with $\lambda_0 \in [0,\infty)$ and $\lambda_1 \in [0,\infty)$. Considering the extensive adoption of this simple functional form for the quantitative analysis of corporate equity issuance, we utilize it instead of pursuing a more flexible approach like the one for debt adjustment costs.

Following studies in cross-sectional production-based asset pricing (e.g., Berk, Green, and Naik, 1999; Zhang, 2005), we parameterize the stochastic discount factor without explicitly modeling the investor's problem, remaining agnostic about the specific sources of risk aversion. However, the qualitative results in 2 require only the existence of a stochastic discount factor and are consistent with arbitrage-free general equilibrium economies. As in Clementi and Palazzo (2019) and similarly to Zhang (2005) and Jones and Tuzel (2013), we assume the stochastic discount factor follows:

$$\log M_{t+1} = \log \beta + \gamma_0(A_t - \mu_A) + \gamma_1(A_{t+1} - \mu_A), \tag{41}$$

[21] The LINEX functional form nests the quadratic form as an approximation for $\gamma_B \to 0$.

[22] Other studies, such as Jungherr, Meier, Reinelt, and Schott (2022), also assume convex functional forms.

where $\beta \in (0,1)$, $\gamma_0 \in (0,\infty)$, and $\gamma_1 \in (-\infty, 0)$. The specification in Clementi and Palazzo (2019), similar to Gomes and Schmid (2010), results in a constant price of risk. This differentiates the economic mechanism in our model from studies where countercyclical risk premia play a central role, such as Zhang (2005) and Chaderina, Weiss, and Zechner (2022).

**Model Solution.** As the model admits no closed-form solution, we resort to numerical dynamic programming in our quantitative analysis. The model solution is computationally challenging for two main reasons. First, the problem features a large number of state variables and policy functions. Equity and debt values are mutually dependent since the default condition affects the debt pricing equation. In a similar context, Gomes and Schmid (2010) reduce the dimensionality of the state space using total debt commitments as a state variable. However, their approach is not viable in our case because of the presence of debt adjustment costs. Hence, we need to keep track of all five state variables $S_{i,t} \equiv \{K_{i,t}, B_{i,t}, c_{i,t}, z_{i,t}, A_t\}$ separately, for each firm $i$. Thus, we need to solve jointly for four policy functions: (i) default decisions $DEF_{i,t} = DEF_{i,t}(S_{i,t})$, (ii) new level of physical capital $K_{i,t+1} = K(S_{i,t})$ after investment, (iii) new debt issuance $B_{i,t+1} = B(S_{i,t})$, and (iv) interest rate schedule $c_{i,t+1} = c(S_{i,t})$. We reduce the dimensionality by projecting the value function $V(S_{i,t})$ on a non-parametric structure for interpolation. Second, problem (24) contains a forward-looking constraint, i.e., the debt's pricing equation (21), which pins down the interest rate schedule $c_{i,t+1} = c(S_{i,t})$. Hence, since $c_{i,t+1} = c(S_{i,t})$ enters the future default choices directly, its solution is significantly sensitive to the initial guess of the value function (especially in proximity of the default region). Appendix D details our numerical solution method.

# 5 Quantitative Analysis

The illustration in Section 2 provides intuition for our key results. However, it is too stylized to serve as a basis for a quantitative investigation of its predictions. In this section, we calibrate the quantitative model to U.S. data and present a host of quantitative results.

## 5.1 Calibration and Model Fit

Table 2 summarizes our baseline calibration. The calibration frequency is monthly. Details about the computation of the model-based and data variables are provided in Appendix A. The model features 17 parameters, plus the normalization $\mu_z = 0$. The first nine entries in the table are on the technology side. We set the curvature of the profit function, $\alpha$, to 0.4, to roughly match the capital share from the Bureau of Economic Analysis (BEA). This value is similar, for example, to the ones used by Kydland and Prescott (1982), Gomes (2001), and Gomes, Jermann, and Schmid (2016). The depreciation rate $\delta$ is set to be 0.01. This is a fairly common value in the literature, as it implies an annual rate of roughly 12%. This value is in line with the empirical estimates in Cooper and Haltiwanger (2006) and comparable to those used by previous studies. For example, Gomes (2001) uses a depreciation rate of 0.145, while Hennessy and Whited (2005) estimate a value of 0.10. We choose the persistence of the aggregate productivity process, $\rho_A$, and its volatility, $\sigma_A$, to be $0.95^{\frac{1}{3}}$ and 0.007/3, respectively. These monthly values correspond to 0.95 and 0.007 at the quarterly frequency, consistent with several studies, including Cooley and Prescott (2021), Zhang (2005), and Gomes, Jermann, and Schmid (2016). We normalize the average aggregate productivity, $\mu_A$, to -2. $\mu_A$ is purely a scaling constant that determines the long-run average scale of the economy. As in Zhang (2005), we calibrate the persistence $\rho_Z$ and volatility $\sigma_Z$ of the idiosyncratic shock process to 0.97 and 0.1, respectively. The fixed cost of operation, $F$, is chosen to approximately match average profitability in our sample, which leads to a value of 0.18 (or 2.25% of the average capital stock).

The next three parameters describe the dynamics of the pricing kernels. We choose $\beta$, $\gamma_0$, and $\gamma_1$ to minimize mean square errors with respect to three aggregate data moments, namely the average Sharpe ratio, the average risk-free rate, and its volatility. This procedure yields $\beta = 0.9978$, $\gamma_0 = 52.79$, and $\gamma_1 = -53.00$.

The remaining six parameters are on the financing side. We pick the fixed and proportional equity flotation costs, $\lambda_0$ and $\lambda_1$, to be 0.5 and 0.025. Like, for example, Kuehn and Schmid (2014) and Bolton, Wang, and Yang (2021), we choose $\lambda_0$ to approximately match the frequency of equity issuance in the data. For the proportional component $\lambda_1$, we pick the same value as in Gomes and Schmid (2010), who also study levered returns. This is also close to Gomes (2001), who chooses 0.028, based on regressions of flotation costs on amount issued. We choose the recovery rate parameter $\xi$ to be 0.125 of the firm's capital stock. This implies an average debt recovery rate of 53.9%, which is close to the 51% recovery rate for creditors when the firm defaults in Huang and Huang (2012).

We calibrate the debt adjustment cost parameters $\lambda_B$ and $\gamma_B$ to approximately target the average debt issuance and the frequency of default in our sample, respectively. The scale parameter $\lambda_B$ affects the marginal cost of issuing debt and, in the model, discourages the use of external debt financing. Instead, positive values of $\gamma_B$ imply that issuing debt is more costly than withdrawing debt. Thus, the lower $\gamma_B$ (i.e., negative large values), the more likely firms default due to their inability to reduce leverage following negative shocks. These values imply an average cost of issuing debt, as a share of the total amount of debt issued, that is toward the lower end of the range used by Strebulaev (2007). The average costs of reducing debt, as a share of the total amount of debt withdrawn, are roughly 30% larger than issuance costs. Overall, the magnitude of debt adjustment costs suggests that, as in Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Strebulaev (2007), relatively small adjustment costs significantly affect leverage dynamics. Finally, following Nikolov and Whited (2014), we choose the tax rate $\tau$ to be 0.20. This is an approximation of the statutory corporate tax rate relative to personal tax rates.

Panel A of Table 3 summarizes the overall model fit under the parameterization reported in Table 2. The table compares model-implied moments, which are tabulated in the first column, with their empirical counterparts, which are tabulated in the second column. Overall, the model does a reasonable job at matching key variables describing the financing policies of U.S. firms. The model matches quite closely the Sharpe ratios, the annual risk-free rate, and the rate's volatility. The model produces a sizable equity premium, which we do not target in the calibration. Both in the model and in the data, this moment is around 7%. The second set of moments in the table refers to firms' real and financial policies. On the real side, the model matches fairly closely average profitability, the volatility and autocorrelation of profitability, and investment ratios. The model does a reasonable job of reproducing average leverage, an untargeted quantity. The model-implied leverage is 17%, close to its data counterpart of 16%. Our parameterization also produces default rates and book-to-market ratios with comparable magnitudes to the data. The third set of moments in the table describe firms' capital structure rebalancing. Debt issuance is 17% in the model, and 25% in the data. Although we target this moment in our calibration, we report model-implied and data moments that describe the relative frequency of positive and negative debt adjustments. The model-implied magnitudes of these additional non-targeted moments are also close to the data. Finally, the frequency of equity issuance in the model is 5%, fairly close to its data value of 4%.

**Table 2**
PARAMETER VALUES

The table reports parameter choices for the calibrated model. The frequency of calibration is monthly.

| Category | Description | Symbol | Value |
|---|---|---|---|
| Technology | Capital share | $\alpha$ | 0.4 |
| | Depreciation | $\delta$ | 0.01 |
| | Persistence of aggregate shock | $\rho_A$ | $0.95^{1/3}$ |
| | Standard deviation of aggregate shock | $\sigma_A$ | 0.007/3 |
| | Mean of aggregate shock | $\mu_A$ | -2 |
| | Persistence of idiosyncratic shock | $\rho_z$ | 0.97 |
| | Standard deviation of idiosyncratic shock | $\sigma_z$ | 0.1 |
| | Mean of idiosyncratic shock | $\mu_z$ | 0 |
| | Fixed cost of operations | $F$ | 0.18 |
| Pricing Kernel | Time discount | $\beta$ | 0.9978 |
| | "Risk aversion" parameters | $\gamma_0$ | 52.79 |
| | | $\gamma_1$ | $-53.00$ |
| Financing | Fixed equity flotation cost | $\lambda_0$ | 0.5 |
| | Proportional equity flotation cost | $\lambda_1$ | 0.025 |
| | Debt recovery in bankruptcy | $\xi$ | 0.125 |
| | Debt adjustment cost ("scale") | $\lambda_B$ | 0.4 |
| | Debt adjustment cost ("asymmetry") | $\gamma_B$ | $-0.2$ |
| | Corporate tax rate | $\tau$ | 0.2 |

Overall, the model provides a reasonably good fit both for moments that serve as targets in the calibration, and for untargeted key statistics.

Panel B of Table 3 evaluates model fit for target leverage measures, including the Flannery-Rangan and projection targets, in both the model and the data. We obtain data estimates from the model in Flannery and Rangan (2006), estimated using (16), and from the multinomial logit procedure described in Section 3.2. To construct model-based counterparts, we estimate target leverage in simulated data using reduced-form proxies, since targets and gaps are not directly observable. Prior studies, including Flannery and Rangan (2006), rely on such proxies. However, it remains unclear how well they capture the dynamic targets defined in the model. We construct two proxies to assess the validity of the reduced-form measures used in the empirical analysis in Section 3. First, we build a version of the measure in Flannery and Rangan (2006), following equation (16). Some variables in the empirical specification lack model counterparts, such as R&D expenses and tangibility. Therefore, $X_{i,t-1}$ includes total assets, profitability, median leverage, market-to-book of assets, and a firm fixed effect, as defined in Appendix A.

Second, we use the model's policy functions to build an empirical counterpart of leverage targets, which informs the functional form used to construct the projection targets in Section 3. Leverage targets are observable in the model. Thus, we project them onto a set of state and control variables available in both the data and the model. Specifically, we regress the model-implied targets, computed as in (32), on a second-order polynomial in the logs of total assets, median leverage, and market-to-book assets, along with profitability, their interaction terms, and squared terms. In the data, where targets are not observed, we estimate the parameters $\gamma$, $\beta_{inc}$, $\beta_{dec}$ by maximum likelihood based on three bins of leverage adjustments towards the target to be estimated. Under the baseline calibration, the correlation between the model-based and projected targets is approximately 95%.

We compute the average speed of adjustment as the average fraction of closed gaps, that is, $\frac{\Delta L_{i,t+1}}{-Gap^L_{i,t}}$, where $\Delta L_{i,t+1} = L_{i,t+1} - L_{i,t}$. Notice that the negative sign in front of $Gap^L_{i,t}$ captures the fact that adjustments toward the target imply that positive values of $Gap^L_{i,t}$ are associated with negative values of $\Delta L_{i,t+1}$, and vice versa. This is because overlevered firms would tend to reduce their leverage (and vice versa).

Panel B of Table 3 shows that average target leverage in the model and in the data is similar for both the Flannery-Rangan and projection measures, with values between 16% and 20.4%. The two target measures are highly correlated, both in the model and in the data, with correlations above 70%. Since the calibration does not target these moments, these results suggest that the empirical proxies fit the data well. Adjustment speeds are also similar in the model and in the data. Firms close between 22.1% and 30% of the gap to target leverage each year. These estimates align with empirical evidence, including Flannery and Rangan (2006) and Lemmon, Roberts, and Zender (2008), which report adjustment speeds between 0.2 and about 0.35.

Taken together, the results in Table 3 suggest that the Flannery-Rangan and projection targets are close to their data counterparts and highly correlated with each other. In addition, firms adjust their capital structure toward a dynamic target leverage ratio, consistent with Proposition 1.

**Table 3**

Model Fit

Panel A reports model-implied and data moments for the baseline calibration of Table 2. Panel B reports the fit of reduced-form leverage targets estimated within the model and in the data. Model-implied moments and targets are computed on simulated market leverage from 5000 firms and 5000 periods. Data targets are estimated using the full Compustat sample (1963–2020). FR denotes the Flannery–Rangan partial adjustment target. Data figures are obtained with the measure of target leverage obtained from the estimation of the model of Flannery and Rangan (2006) in Equation (16), and with the projection target, denoted as "Projection", computed with the procedure in Section 3.2. The adjustment speed is the average fraction of the gap closed per year. The data source for the Sharpe Ratio and risk-free rate moments is Zhang (2005). The average annual equity return is computed using data from Kenneth French's data library. Default rates are taken from Covas and Den Haan (2011). The remaining data moments are computed from our sample of nonfinancial, unregulated firms from the annual Compustat dataset. All moments are annualized. Details on model and data variable definitions are provided in Appendix A.

Panel A: Moments

| | Model | Data |
|---|---|---|
| Average Sharpe Ratio | 0.43 | 0.43 |
| Average risk-free rate | 0.02 | 0.02 |
| Volatility of risk-free rate | 0.03 | 0.03 |
| Average equity premium | 0.07 | 0.06 |
| | | |
| Average profitability | 0.16 | 0.15 |
| Volatility of profitability | 0.08 | 0.09 |
| Autocorrelation of profitability | 0.65 | 0.77 |
| Average investment | 0.16 | 0.13 |
| Average leverage | 0.17 | 0.16 |
| Frequency of default | 0.01 | 0.02 |
| Average book-to-market ratio | 0.37 | 0.56 |
| | | |
| Average debt issuance | 0.17 | 0.25 |
| Frequency of positive debt adjustments | 0.66 | 0.60 |
| Frequency of negative debt adjustments | 0.34 | 0.40 |
| Frequency of equity issuance | 0.05 | 0.04 |

Panel B: Target Leverage Fit

| | Model | Data |
|---|---|---|
| Mean target (FR) | 0.187 | 0.187 |
| Mean target (Projection) | 0.160 | 0.204 |
| Corr(FR, Projection) | 0.752 | 0.729 |
| | | |
| Adjustment Speed: FR | 0.221 | 0.300 |
| Adjustment Speed: Projection | 0.237 | 0.262 |

## 5.2 Revisiting Levered Returns

In Table 4, we present cross-sectional regression estimates of equity returns on various leverage variables, along with size and book-to-market ratios. These estimates are derived from simulated data using the baseline parameterization. Overall, the results suggest that our model is capable of reproducing the empirical patterns highlighted in Section 3.

Table 4 reports the results. Columns (1) to (3) use model-based targets and gaps, as defined in Section 4.2. Columns (4) and (5) rely on proxies from Flannery and Rangan (2006), while columns (6) and (7) report results for projection targets. All coefficients are averages from 100 bootstrap simulations under the baseline calibration in Table 2, as described in Appendix D. Bootstrapped standard errors appear in parentheses. Column (1) shows a positive relation between leverage and returns, controlling for firm size and book-to-market. However, the coefficient lies within two standard errors of zero and is not statistically significant at the 5 percent level. Columns (2), (4), and (6) report coefficients on leverage gaps. For model-based and projection gaps, these coefficients are positive and statistically significant, exceeding two standard errors from zero. When both leverage and gaps enter the regression, in columns (3), (5), and (7), gap coefficients exceed leverage coefficients (0.88 vs. 0.54 for model-based gaps, and 0.83 vs. 0.20 for projection gaps). With Flannery-Rangan targets, the leverage coefficient is close to zero (0.05) and not statistically significant, while the gap coefficient remains positive and significant at the 1 percent level. Overall, columns (1) to (7) support Proposition 2 and align with the empirical evidence in Table 1. The relation between leverage and returns is weak and not robust, while leverage gaps show a strong positive association with equity returns.

In summary, the findings in Table 4 suggest that financial inflexibility is key to understanding the empirical patterns in levered returns. Costly capital structure rebalancing generates leverage gaps, which mediate the link between leverage and returns. The regression coefficients are not explicitly targeted in the model calibration, and the connection between reduced-form proxies (Section 3) and model targets is still imperfect. Nevertheless, the consistency in sign and relative magnitude of the leverage and gap coefficients is reassuring.

**Table 4**

Empirical Patterns on Leverage and Returns - Model

The table reports estimated coefficients from cross-sectional regressions of stock returns on leverage ("Leverage"), leverage gaps ("Lev. Gap"), market capitalization ("Size"), and book-to-market equity ("Be/Me") across the entire simulated economy. In Columns (1) to (3), leverage targets and gaps are computed using the model-based approach described in Section 4.2. Columns (4) and (5) report results based on leverage targets and gaps estimated by mimicking the Flannery–Rangan approach on simulated data, as detailed in Section 5.2. Columns (6) and (7) use leverage targets obtained by projecting model-based targets onto a set of variables common to both the data and the model, also discussed in Section 5.2. The baseline specification in column (1) regresses returns on leverage, size, and book-to-market only and is therefore common to all target definitions. All coefficients are averages from 100 simulations of 100 randomly drawn firms over 5000 time periods under the baseline calibration of Table 2. Bootstrapped standard errors are reported in parentheses. Note that returns are monthly and expressed in percentage (multiplied by 100), and leverage gaps, leverage targets, and leverage are in levels (not in percentage). Size and Book-to-Market are expressed in log levels. All variables are defined in Appendix A.

| Dependent Variable: Monthly Stock Return | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Model-Based Target | | | Flannery-Rangan Target | | Projection Target | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Leverage | 0.52 | | 0.54 | | 0.05 | | 0.20 |
| | (0.37) | | (0.37) | | (0.46) | | (0.28) |
| Lev. Gap | | 0.86 | 0.88 | 0.58 | 0.57 | 0.90 | 0.83 |
| | | (0.30) | (0.30) | (0.16) | (0.24) | (0.27) | (0.29) |
| Size | -0.09 | -0.18 | -0.09 | -0.35 | -0.34 | -0.17 | -0.14 |
| | (0.07) | (0.06) | (0.07) | (0.06) | (0.12) | (0.06) | (0.06) |
| Be/Me | 0.18 | 0.07 | 0.19 | 0.21 | 0.22 | 0.06 | 0.11 |
| | (0.10) | (0.04) | (0.10) | (0.05) | (0.10) | (0.03) | (0.08) |
| $R^2$ | 0.01 | 0.01 | 0.01 | 0.03 | 0.03 | 0.02 | 0.02 |

In addition, Table A6 assesses the amplification and dampening effects of leverage gaps on equity returns. As Proposition 2 shows, whether leverage gaps capture an increase or a decrease in the cost of carrying debt depends on the sign of $\gamma_D(\cdot)$, which is unobservable. The new Table A6 reports the effect of leverage gaps on equity returns separately for overlevered and underlevered firms, as

$\gamma_D(\cdot)$ increases with leverage gaps. The results show that, both in the model and in the data, dampening effects are larger on average. In other words, leverage gaps affect stock returns mainly through underlevered firms, which face a lower cost per dollar of debt due to their lower expected bankruptcy costs.[23]

## 5.3 Effects of Changing Financial Flexibility

Table 5 compares counterfactual economies that differ in the degree and in the symmetry of financial inflexibility. We conduct this comparison by varying the financial inflexibility parameter $\lambda_B$ and the asymmetry parameter $\gamma_B$ around their baseline calibrated values, resolving the model for each case.

Panel A reports model-implied adjustment speeds. As expected, higher values of $\lambda_B$ lead to slower adjustments. In the benchmark case with full flexibility (column 1), firms fully close their leverage gaps because adjustment costs are absent. When $\lambda_B$ exceeds the baseline value (column 2), adjustment speeds fall relative to the baseline calibration value (0.32). The relationship between $\lambda_B$ and adjustment speed is nonlinear. Increases in $\lambda_B$ do not yield proportionally smaller reductions in the fraction of the leverage gap firms close. This finding suggests that firms encounter significant levels of financial inflexibility, aligning with previous studies that highlight the presence of substantial hysteresis in corporate capital structures (e.g., Hennessy and Whited, 2005; Lemmon, Roberts, and Zender, 2008).

Columns (3) to (6) show adjustment speeds under increasing asymmetry in debt reduction costs, with more negative values of $\gamma_B$. The larger the magnitude of $\gamma_B$, the more costly it is for firms to reduce their debt. As expected, when $\gamma_B$ becomes more negative, overlevered firms close a smaller portion of their leverage gaps. As the cost of reducing debt increases, the gap distribution shows two effects. The median gap (p50 in the table) does not change significantly. In contrast, both tails of the distribution shift. The 90th percentile increases as $\gamma_B$ becomes more negative. This pattern indicates that highly overlevered firms face larger gaps, since they find it harder to reduce leverage. This result aligns with the evidence in Covas and Den Haan (2011), who report that net borrowing

[23] In Table A7, we assess the sensitivity of the return-gap relation to these costs. A $\pm$ 10% change in equity issuance costs leaves the relation strong and positive, and it becomes stronger when issuance costs increase.

declines sharply in recessions and that the distribution shifts toward deleveraging, especially for financially constrained firms. At the same time, the 10th percentile decreases. Firms anticipate higher future deleveraging costs and remain more underlevered to hedge against these costs.

Panel B presents regression coefficients from cross-sectional analyses of stock returns on leverage and leverage gaps, controlling for size and book-to-market. In columns (1) and (2), average returns increase with leverage gaps, while the direct effect of leverage is smaller. In the full flexibility benchmark (column 1), leverage gaps have no explanatory power, as they are zero by construction. Columns (3) to (6) show that leverage gaps remain relevant for returns across different values of $\gamma_B$. Although the stochastic discount factor in the model implies a constant price of risk, the estimated coefficients remain sensitive to the degree of asymmetry in debt adjustment costs. This sensitivity reflects the frictions firms face when reducing leverage, as in Chaderina, Weiss, and Zechner (2022). However, unlike in studies that emphasize countercyclical risk premia and leverage adjustments, the pattern here is not perfectly monotonic.[24]

Panel B also reports coefficients from a regression of returns on the standardized gap, defined as the gap minus its mean and divided by its standard deviation, along with size and book-to-market. Since Panel A shows that changes in $\gamma_B$ affect the gap distribution, the standardized gap captures the importance of gaps for expected returns. In contrast to the non-monotonic pattern of the coefficients on gaps as $\gamma_B$ decreases, the coefficients on standardized gaps remain stable and show no statistical or economic differences. This pattern is consistent with the two effects in Panel A, where some firms become more underlevered while others remain overlevered and cannot reduce their gaps.

In summary, the results indicate that firms face substantial financial inflexibility, which limits their ability to adjust capital structure. Financial flexibility is priced in the stock market. The return-gap relationship strengthens when financial flexibility is limited and varies with the degree of asymmetry in adjustment frictions. The Modigliani-Miller amplification effect of leverage and gaps on returns remains important across different levels of asymmetry.

[24]However, for large deviations of $\gamma_B$ from the baseline, the overall magnitude of adjustment costs is also affected.

**Table 5**

COUNTERFACTUALS: INFLEXIBILITY PARAMETERS

The table describes capital structure adjustments and levered returns for different degrees of the financial inflexibility parameter $\lambda_B$ ("Magnitude") and of the asymmetry parameter $\gamma_B$ ("Asymmetry"). Panel A tabulates model-implied estimates of adjustment speeds for the full sample of firms, computed as the average fraction of closed gaps. "Adj. Speed (Down)" denotes the adjustment speed conditional on firms being overlevered. The gap distribution reports the 10th, 50th, and 90th percentiles of the leverage gap. Panel B reports the estimated coefficients from cross-sectional regressions of stock returns on leverage ("Leverage"), leverage gaps ("Lev. Gap"), market capitalization ("Size"), and book-to-market equity ("Be/Me") on the simulated economy. "Gap (Standardized)" is the coefficient from a regression of returns on the standardized gap, defined as the gap minus its mean divided by the standard deviation, size, and book-to-market. Leverage targets and gaps are computed using the model-based approach described in Section 4.2. All coefficients are averages from 100 simulations of 100 randomly drawn firms over 5,000 time periods. Bootstrapped standard errors are reported in parentheses. Note that returns are monthly and expressed in percentage (multiplied by 100), and leverage gaps, leverage targets, and leverage are in levels (not in percentage). Size and Book-to-Market are expressed in log levels. All variables are defined in Appendix A.

| | Magnitude | | | Asymmetry | | | |
|---|---|---|---|---|---|---|---|
| | $\lambda_B = 0.0$ | $\lambda_B = 0.5$ | | $\gamma_B = -0.18$ | $\gamma_B = -0.19$ | $\gamma_B = -0.21$ | $\gamma_B = -0.30$ |
| | (1) | (2) | | (3) | (4) | (5) | (6) |
| **Panel A: Capital Structure Dynamics** | | | | | | | |
| Adj. Speed (Total) | 1.00 | 0.23 | Adj. Speed (Down) | 0.25 | 0.24 | 0.19 | 0.15 |
| | | | | Gap Distribution | | | |
| | | | 10th Perc. (p10) | -0.0392 | -0.0394 | -0.0404 | -0.0416 |
| | | | 50th Perc. (p50) | -0.0002 | -0.0001 | -0.0001 | -0.0000 |
| | | | 90th Perc. (p90) | 0.0450 | 0.0471 | 0.0484 | 0.1069 |
| **Panel B: Market Price of Flexibility** | | | | | | | |
| Dependent Variable: Monthly Stock Return | | | | | | | |
| Leverage | 0.39 | 0.48 | | 0.51 | 0.59 | 0.49 | 0.63 |
| | (0.31) | (0.32) | | (0.34) | (0.36) | (0.31) | (0.35) |
| Lev. Gap | - | 0.83 | | 0.89 | 0.92 | 0.73 | 0.83 |
| | | (0.36) | | (0.34) | (0.34) | (0.37) | (0.36) |
| Size | -0.17 | -0.07 | | -0.18 | -0.17 | -0.07 | -0.04 |
| | (0.07) | (0.07) | | (0.07) | (0.08) | (0.07) | (0.06) |
| Be/Me | 0.18 | 0.22 | | 0.21 | 0.23 | 0.23 | 0.25 |
| | (0.08) | (0.09) | | (0.10) | (0.10) | (0.09) | (0.09) |
| Gap (Standardized) | - | 0.05 | | 0.04 | 0.04 | 0.04 | 0.05 |
| | | (0.02) | | (0.02) | (0.02) | (0.02) | (0.02) |

## 5.4 Profitability Shocks and Levered Returns Dynamics

With persistence in stochastic profitability shocks, firms that experience positive realizations today expect favorable conditions to continue tomorrow. This implies that the relevance of leverage targets for stock returns may significantly depend on the persistence of shocks.

Table 6 examines how shock persistence affects leverage targets and gaps using simulated data. The table considers different values of $\rho_z$, the persistence of idiosyncratic profitability shocks. As the autoregressive process in (38) implies, lower values of $\rho_z$ make shocks less predictable. In the limiting case of i.i.d. shocks ($\rho_z = 0$), firms cannot use $z_{i,t}$ to forecast future conditions; hence, current shocks $z_{i,t}$ do not affect expectations about future profitability. Although current shocks $z_{i,t}$ do not affect expectations about future profitability, they still affect after-tax operating profits, which in turn affect dividends through the budget constraint (19). In general, in our setting, dividends enter the first-order conditions (27) and (28). Hence, even when $\rho_z = 0$, $Z_{i,t}$ still affects the resources available for investment and financing decisions. As a result, even in this limiting case, the model's policy functions remain stochastically dependent on $Z_{i,t}$, generating non-trivial dynamics for leverage targets and gaps.

Panel A shows that leverage targets are more volatile when persistence is low. Transitory shocks lead firms to revise their optimal capital structure frequently as profitability changes quickly. The mean absolute leverage gap is also larger when shocks are less persistent. Volatile targets and large gaps arise because targets move often and by large amounts when profitability is nearly i.i.d., while adjustment costs prevent firms from keeping pace. When shocks are highly persistent, targets change slowly and firms track them more closely, which leads to smaller gaps. Panel B reports the average fraction of the initial leverage gap that remains after one month, one year, and two years, based on regressions of $h$-month-ahead gaps on current gaps. Gap dynamics differ sharply across persistence levels. When persistence is low, gaps remain highly persistent: 93% of the initial gap remains after one month, 78.7% after one year, and 66.5% after two years. When persistence is high, firms absorb gaps much faster at all horizons.

These results link shock persistence to financial inflexibility. When shocks are transitory, two forces generate large and persistent leverage gaps. First, targets are volatile, which creates frequent and sizable deviations. Second, shocks are short-lived, so the expected present value of being overlevered or underlevered is low, which weakens the incentive to pay adjustment costs. As persistence increases, both effects reverse: targets stabilize and the cost of being overlevered or underlevered rises, which induces faster adjustment.[25]

**Table 6**

Persistence of Profitability Shocks: Targets and Gaps

The table reports leverage gap statistics and gap closing dynamics across different values of $\rho_z$, the persistence of idiosyncratic profitability shocks. Panel A reports the cross-sectional standard deviation of leverage targets (Std Target) and the mean absolute leverage gap (Mean |Gap|). Panel B reports the average fraction of the initial gap remaining at horizons of 1 month, 1 year, and 2 years, estimated as: $\text{Gap}_{i,t+h} = \alpha_h + \rho_h \cdot \text{Gap}_{i,t} + \varepsilon_{i,t+h}$. All other parameters follow the baseline calibration in Table 2.

| | Comparative Statics | | |
|---|---|---|---|
| | Shock Persistence ($\rho_z$) | | |
| | Low (0.1) | Medium (0.5) | High (0.9) |
| **Panel A: Leverage Gap Statistics** | | | |
| Std Target | 0.35 | 0.31 | 0.10 |
| Mean \|Gap\| | 0.15 | 0.04 | 0.02 |
| **Panel B: Fraction of Gap Remaining ($\rho_h$)** | | | |
| $h = 1$ month | 0.930 | 0.633 | 0.556 |
| $h = 1$ year | 0.787 | 0.326 | 0.182 |
| $h = 2$ years | 0.665 | 0.171 | 0.150 |

Table 7 turns to the effect of leverage gaps on equity returns. Columns (1) to (4) of Table 7 explore the effect of firm-level profitability persistence using a matching approach. For each firm, we compute the autoregressive coefficient of profitability, the standard deviation of AR(1) residuals, and average profitability. These variables are estimated each year $t$ using a rolling window from the start of the sample through year $t$, requiring at least five observations per firm. Each year, we form a control sample by matching each firm in the top tercile of profitability persistence ("High Persistence") with a firm from the bottom two terciles ("Low Persistence"), using

[25] Appendix Figure A2 illustrates, using simulated data, a sample response of leverage targets and returns to a sequence of persistent idiosyncratic productivity shocks $z_{i,t}$. We focus on idiosyncratic rather than aggregate shocks, as aggregate shocks would also affect the stochastic discount factor.

nearest neighbor matching based on Euclidean distance in residual standard deviation and average profitability. Although high- and low-persistence firms may differ along other observable and unobservable dimensions, this procedure generates a meaningful spread in profitability persistence without large differences in mean profitability or residual volatility. High-persistence firms have an average persistence of 0.84, an average profitability of 8.5%, and a residual standard deviation of 4.7%. Low-persistence firms have an average persistence of 0.34, an average profitability of 8.8%, and a residual standard deviation of 5.2%.

The estimates of Table 7 show that leverage gaps are more relevant for cross-sectional return regressions among low-persistence firms. Despite the loss of statistical power due to the significantly smaller sample size from the matching procedure, returns are more sensitive to leverage gaps for low-persistence firms, using both the Flannery-Rangan targets (columns 1 and 2) and the projection targets (columns 3 and 4). The coefficients on leverage gaps are economically large and statistically significant at least at the five-percent level for low-persistence firms. For high-persistence firms, the coefficients remain positive but are smaller and not statistically significant at the five-percent level. Overall, these results suggest that leverage gaps have a stronger impact on returns when profitability shocks are less persistent, and that firms adjust their capital structure more slowly.

Finally, columns (5) and (7) of Table 7 present a comparative statics exercise with respect to the model parameter $\rho_z$. This exercise should be interpreted as a qualitative comparison to the empirical results in the four leftmost columns. Nonetheless, the model results in columns (5) and (7) align qualitatively with the empirical patterns. In simulated economies with less persistent profitability shocks, firms close smaller fractions of their leverage gaps, and the coefficients on gaps in return regressions are economically larger than in economies with higher persistence.

**Table 7**

PERSISTENCE OF PROFITABILITY SHOCKS: STOCK RETURNS

The table presents levered returns by profitability persistence in the data and by different values of $\rho_z$, the persistence of idiosyncratic profitability shocks in the model. Columns (1) to (4) ("Matched Compustat Sample") report results for firms listed on the NYSE, AMEX, and NASDAQ between 1965 and 2020, covered by both Compustat and the Center for Research in Security Prices (CRSP). For each firm, we compute the autoregressive coefficient of profitability, the standard deviation of AR(1) residuals, and average profitability, defined as the ratio of EBIT to total assets. These variables are estimated each year $t$ using a rolling procedure from the start of the sample through year $t$, requiring a minimum of five observations per firm. Each year, we create a control sample by matching each firm in the top tercile of profitability persistence ("High Persistence") with a firm from the bottom two terciles ("Low Persistence"), using nearest neighbor matching based on Euclidean distance in residual standard deviation and average profitability. Columns (1) and (2) use leverage gaps estimated with the Flannery–Rangan approach ("Flannery-Rangan Target"). Columns (3) and (4) use model-based targets projected onto observable variables ("Projection Target"). Columns (5), (6), and (7) show results from simulated data with $\rho_z$ values of 0.94, 0.95 and 0.96. The table presents coefficients from cross-sectional regressions of stock returns on leverage ("Leverage"), leverage gaps ("Lev. Gap"), market capitalization ("Size"), and book-to-market equity ("Be/Me"). For Compustat data, annual accounting variables are matched to monthly returns from July 1980 to December 2020, following Fama and French (1992). Newey-West standard errors with a lag length of four are reported in parentheses. For simulated data, coefficients are averaged over 100 simulations of 100 firms across 5,000 periods, with bootstrapped standard errors in parentheses. Size and Book-to-Market are expressed in log levels. All variables are defined in Appendix A.

| | Matched Compustat Sample | | | | Comparative Statics | | |
|---|---|---|---|---|---|---|---|
| | Flannery-Rangan Target | | Projection Target | | Model $\rho_z$ | | |
| | Low Persistence | High Persistence | Low Persistence | High Persistence | Low Persistence | Medium Persistence | High Persistence |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| | Dependent Variable: Monthly Stock Return | | | | | | |
| Leverage | -0.25 | 0.11 | -0.14 | 0.26 | 0.50 | 0.01 | 0.50 |
| | (0.38) | (0.34) | (0.37) | (0.33) | (0.27) | (0.25) | (0.35) |
| Lev. Gap | 1.29 | 0.67 | 0.69 | 0.31 | 1.31 | 1.03 | 0.62 |
| | (0.55) | (0.40) | (0.22) | (0.17) | (0.34) | (0.26) | (0.42) |
| Size | -0.19 | -0.13 | -0.22 | -0.14 | -0.01 | -0.11 | -0.12 |
| | (0.04) | (0.04) | (0.04) | (0.04) | (0.06) | (0.05) | (0.07) |
| Be/Me | 0.10 | 0.03 | 0.13 | 0.02 | 0.41 | 0.16 | 0.27 |
| | (0.08) | (0.06) | (0.08) | (0.07) | (0.08) | (0.08) | (0.10) |

# 6 Conclusions

In this study, we examine the complex relationship between leverage and the dispersion of equity risk premia across firms. To do so, we build a discrete-time, infinite-horizon neoclassical investment model with heterogeneous firms. In the model, firms optimize their investment, financing, and default decisions to maximize their value in arbitrage-free markets.

Our study makes three main contributions. First, we present analytical results that illustrate the interplay between financial flexibility, firm dynamics, and the dispersion of risk premia. These findings indicate that two variables, leverage gaps and leverage targets, can provide valuable insights into levered equity risk premia in a world characterized by limited financial flexibility. Second, we uncover two novel empirical facts that align with the model's predictions: (i) there is a strong positive correlation between estimated leverage gaps and stock returns, and (ii) stocks from high-leverage firms command a premium. This premium becomes apparent when we decompose leverage gaps into leverage targets and observed leverage, with leverage targets negatively impacting risk premia. Third, we assess the model's quantitative implications through calibration.

Overall, our results suggest that limited financial flexibility is instrumental in rationalizing the puzzling empirical patterns surrounding levered returns. The costly rebalancing of capital structures gives rise to leverage gaps, which subsequently mediate the relationship between leverage and returns. Unlike previous studies, the mechanism in our model does not rely on debt adjustment asymmetries or on the cyclical behavior of the price of risk. Instead, limited financial flexibility modifies the standard amplification mechanism from the second proposition in Modigliani and Miller (1958), where leverage magnifies both positive and negative outcomes.

An important avenue for future research is to more directly connect leverage gaps and financial flexibility to distress-related mechanisms and the pricing of distress risk. A growing literature shows that default probabilities, equity betas, and expected returns need not move together. For example, firms with high failure probabilities often exhibit high equity betas but low average returns (Campbell, Hilscher, and Szilagyi, 2008), while measures of distress risk premia based on risk-neutral default probabilities are positively related to returns (Vassalou and Xing, 2004; Friewald, Wagner, and Zechner, 2014). Recent theoretical work further highlights that equity betas in distressed firms may vary systematically over the business cycle, in particular through a negative co-movement with the market risk premium. These findings suggest that the interaction between leverage, distress, and asset pricing operates primarily through the dynamics of equity risk rather than through static measures of default likelihood.

Building on this insight, a natural extension of our framework would be to study how leverage gaps interact with distress risk through their impact on equity betas. This would require combining measures of financial flexibility with empirical proxies for distress risk premia, such as those based on credit spreads or differences between risk-neutral and physical default probabilities, and analyzing their joint implications for the cross section of returns. Such an exercise would also involve decomposing equity risk into underlying asset risk and leverage-induced amplification, as well as accounting for the time variation in betas emphasized by the distress literature. We leave this comprehensive analysis for future work, as it would considerably broaden the scope of the present paper, but we view it as a promising direction for better integrating capital structure dynamics and asset pricing evidence on distress risk.

Our results also carry potential implications for capital allocation across firms. As David, Schmid, and Zeke (2022) show, a significant portion of the dispersion in the marginal product of capital among U.S. firms is driven by risk premia. We leave the task of exploring the general-equilibrium links between limited financial flexibility and capital misallocation to future research.

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# Online Appendix

## A) Variable Definitions and Details on Empirical Procedures

The following table summarizes variable definitions with reference to Compustat and CRSP items.

| Variable | Construction |
|---|---|
| Leverage | $\frac{DLTT+DLC}{AT-BE+PRCC\_F \cdot CSHO}$ |
| Book Leverage | $\frac{DLTT+DLC}{AT}$ |
| Net Market Leverage | $\frac{DLTT+DLC-CHE}{AT-BE+PRCC\_F \cdot CSHO}$ |
| Net Book Leverage | $\frac{DLTT+DLC-CHE}{AT}$ |
| Value of Preferred Stocks ($PS$) | If available, in this order: $PSTKRV$, $PSTKL$, $PSTK$. |
| Book Equity ($BE$) | $CEQ + TXDITC$ (if available) $- PS$ |
| Leverage Growth | $\frac{F.ML-ML}{0.5(F.ML+ML)}$ |
| Market Capitalization | $\log \frac{\lvert PRC \rvert \cdot SHROUT}{1000}$ (in June) |
| Book-to-Market Equity | $\log \frac{BE}{\lvert PRC \rvert \cdot SHROUT/1000}$ |
| Investment | $\frac{AT-L.AT}{L.AT}$ |
| Profitability | $\frac{REVT-COGS}{BE}$ |
| $\Delta$ Lev. from Assets | $\frac{DLTT+DLC}{AT} - \frac{DLTT+DLC}{L.AT}$ |

The projection target is built in two conceptual steps. In the model, we project the model-implied frictionless leverage target onto a set of state and control variables that are observable in both the model and Compustat. This regression delivers the functional form of the projection target. In the data, we use the same functional form to re-estimate the parameters $\gamma, \beta_{inc}, and\beta_{dec}$ to construct a reduced-form proxy of the leverage target. To allow for nonlinear adjustment behavior and to render the units of measurement in the model comparable to the data, the projection target is embedded in a multinomial logit model of leverage changes, estimated by maximum likelihood. We therefore regress the model-implied target $LT_{i,t}$ onto a set of variables common to the model and Compustat.

Let

$$z_{i,t} \equiv \left(\log \widetilde{L}_{j(i),t},\ \log BM_{i,t},\ \text{EBIT}_{i,t}/AT_{i,t},\ \log AT_{i,t}\right), \tag{A1}$$

where $\widetilde{L}_{j(i),t}$ is the industry-year median of market leverage, $BM_{i,t}$ is book-to-market equity, $\text{EBIT}_{i,t}/AT_{i,t}$ is operating profitability, and $AT_{i,t}$ is total assets. Within the simulated panel, the variables in $z_{i,t}$

all have direct counterparts:

$$z_{i,t}^{\text{model}} \equiv \left(\log \text{median}\left[\frac{B_{i,t}}{K_{i,t}}\right],\ \log\frac{K_{i,t}}{B_{i,t}+V_{i,t}-D_{i,t}},\ \frac{\Pi_{i,t}}{K_{i,t}},\ \log K_{i,t}\right), \tag{A2}$$

where $B_{i,t}$ denotes the debt stock, $K_{i,t}$ is the capital stock (the model analogue of total assets), $V_{i,t}$ is firm value, $D_{i,t}$ denotes net distributions to shareholders, and $\Pi_{i,t}$ is after-tax operating profits. Industry is not modeled explicitly, so the industry-year median of market leverage is replaced by the cross-sectional median across all firms in the simulated panel at date $t$. The set of variables $Z_{i,t}$ used to construct the second-order polynomial both in the model and in Section 3.2 is built from the components of $z_{i,t}$ (or $z_{i,t}^{\text{model}}$) and includes their pairwise interactions and squared terms, yielding a full second-order polynomial expansion.

The rolling estimation procedure for the multinomial logit model in Section 3.2 follows the same approach as the one used for the Flannery-Rangan measure. The period from 1965 to 1979 serves as a burn-in window. Estimation then proceeds on a rolling basis using data available up to year $t$. Firms must have at least five consecutive years of observations. To ensure stability in the maximum likelihood estimation, we apply a gradient method 100 times to find an initial parameter vector. We then iterate in blocks of 10 until parameter values converge, with a 1% tolerance in absolute value relative to the previous set of estimates. Firm fixed effects are also estimated on a rolling basis, controlling for the logs of total assets, median leverage, market-to-book ratio, and profitability.

In the model, following Zhang (2005), we define the average Sharpe Ratio as $S_t \equiv \frac{\sigma[M_{t+1}]}{E_t[M_{t+1}]}$ and the risk-free rate as $\frac{1}{E_t[M_{t+1}]}-1$. We define the equity premium as the average value-weighted return in the simulated economy, where realized stock returns are computed ex-dividends as $\frac{V_{i,t}}{V_{i,t-1}-D_{i,t-1}}$. Profitability is the ratio of operating profits $\Pi_{i,t}$ to capital $K_{i,t}$, investment is the ratio of $I_{i,t}$ to capital $K_{i,t}$, leverage is the ratio of $B_{i,t}$ to $B_{i,t}+V_{i,t}-D_{i,t}$, book-to-market is the ratio of $K_{i,t}$ to $B_{i,t}+V_{i,t}-D_{i,t}$, debt adjustments are changes in debt stock $B_{i,t}-B_{i,t-1}$ divided by capital $K_{i,t-1}$, debt issuance is $\max\{0, B_{i,t}-B_{i,t-1}\}$ divided by $K_{i,t-1}$, and equity issuance is $\max\{0, E_{i,t}\}$ scaled by capital $K_{i,t}$.

## B) Two-Period Case: Derivations and Proofs

**Proof of Proposition 1.** Part a). Equations (1) and (6) yield

$$B_2^* = \frac{\lambda_B B_1 + \tau\left(1-\rho\mathbb{E}_1[M_2]\right)}{\lambda_B + (1-\tau)(1-\rho)\mathbb{E}_1[M_2]\xi_B}. \tag{A3}$$

Equation (7) follows after subtracting $B_1$ from both sides of (A3) and collecting $\lambda_1$ as defined in (8). Part b). Equation (7) can be rewritten as

$$\frac{B_2^*}{K_2^*} - \frac{B_1}{K_1} + \frac{B_1}{K_1} - \frac{B_1}{K_2^*} = \lambda_1\left(\frac{B_2^0}{K_2^*} - \frac{B_1}{K_1} + \frac{B_1}{K_1} - \frac{B_1}{K_2^*}\right), \tag{A4}$$

after dividing both sides by $K_2^*$ and summing and subtracting $\frac{B_1}{K_1}$ from both sides. (10) immediately follows after collecting $\frac{B_1}{K_2^*} - \frac{B_1}{K_1}$ on the right-hand side of (A4).

Finally, note that in this two-period setup capital is chosen without adjustment or equity issuance costs. Therefore, the first-order condition for $K_2$ is unaffected by $\Lambda^B(\cdot)$, and the optimal choice coincides with the frictionless benchmark, i.e.

$$K_2^* = K_2^0 = f_k^{-1}\left[\frac{1 + r_F - \rho + \rho(1-\tau)\left(\delta - \frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)}{(1+r_F)\rho\,(1-\tau)\,\mathbb{E}_1[M_2 A_2 Z_2]}\right]. \blacksquare$$

**Proof of Proposition 2.** Part a). Substituting the definition of $D_2$ into the equity return

$$R_2^E = \frac{D_2}{\rho\,\mathbb{E}_1[M_2 D_2^{Sol}]},$$

yields

$$R_2^E = \frac{R_2^A K_2^* - R_2^D B_2^*}{\gamma_A(K_2^*)K_2^* - \gamma_D(B_2^*)B_2^*},$$

where $\gamma_A(K_2^*) \equiv \mathbb{E}_1^\rho[M_2 R_2^A]$, $\gamma_D(B_2^*) \equiv \mathbb{E}_1^\rho[M_2 R_2^D]$. Rearranging the numerator and denominator gives

$$R_2^E = \frac{R_2^A}{\gamma_A(K_2^*)} + \frac{B_2^*\gamma_D(B_2^*)}{K_2^*\gamma_A(K_2^*) - B_2^*\gamma_D(B_2^*)}\left(\frac{R_2^A}{\gamma_A(K_2^*)} - \frac{R_2^D}{\gamma_D(B_2^*)}\right),$$

which is equation (12). Taking expectations yields (13).

Part b). To link $\gamma_D(B_2^*)$ to leverage gaps, note that

$$\begin{aligned}\gamma_D(B_2^*) &\equiv \rho\mathbb{E}_1\left[M_2 R_2^{D,Sol}\right] + (1-\rho)\mathbb{E}_1\left[M_2 R_2^{D,Def}\right] \\ &= \rho\mathbb{E}_1\left[M_2\frac{(1+c_2)B_2^* - TS_2}{B_2^*}\right] + (1-\rho)\mathbb{E}_1\left[M_2\frac{L_2 + BC_2}{B_2^*}\right].\end{aligned}$$

Using the debt pricing equation (1) yields

$$\gamma_D(B_2^*) = 1 - \rho\mathbb{E}_1\left[M_2\frac{TS_2}{B_2^*}\right] + (1-\rho)\mathbb{E}_1\left[M_2\frac{BC_2}{B_2^*}\right]. \tag{A5}$$

Combining (A5) with the first-order condition (6) links $\gamma_D(B_2^*)$ to expected tax shields and bankruptcy costs:

$$\gamma_D(B_2^*) = 1 - \lambda_B(B_2^* - B_1) + \rho E_1\left[M_2\Delta TS^{M-A}\right] - (1-\rho)E_1\left[M_2\Delta BC^{M-A}\right]. \tag{A6}$$

From the partial adjustment relationship (7) we obtain

$$B_2^* - B_1 = -\lambda_1 K_2\left(\frac{B_1}{K_1} - \frac{B_2^0}{K_2^0}\right) + \lambda_1 B_1\left(\frac{K_2^*}{K_1} - 1\right),$$

which, when substituted into (A6), yields (14). ■

**Properties of Marginal and Average Tax Shields and Bankruptcy Costs.**

*i) Proof that* $\mathbb{E}_1\left[M_2\frac{\partial TS_2}{\partial B_2}\right]$ *is strictly increasing and affine in* $B_1/K_1$. Differentiating the coupon in (2) with respect to $B_2$ yields

$$\frac{\partial c_2}{\partial B_2}=\frac{1-\rho}{\rho}\left(\frac{\xi_K K_2}{B_2^2}+\frac{1}{2}\xi_B\right). \tag{A7}$$

Since $TS_2=\tau c_2 B_2$, we have

$$\frac{\partial TS_2}{\partial B_2}=\tau\left(c_2+B_2\frac{\partial c_2}{\partial B_2}\right).$$

Substituting the expressions for $c_2$ and $\partial c_2/\partial B_2$, the terms $\pm\frac{\xi_K K_2}{B_2}$ cancel out, which gives

$$\begin{aligned}c_2+B_2\frac{\partial c_2}{\partial B_2}&=\left[\frac{1-\rho E_1[M_2]}{\rho\mathbb{E}_1[M_2]}-\frac{1-\rho}{\rho}\left(\frac{\xi_K K_2}{B_2}-\frac{1}{2}\xi_B B_2\right)\right]+B_2\cdot\frac{1-\rho}{\rho}\left(\frac{\xi_K K_2}{B_2^2}+\frac{1}{2}\xi_B\right)\\&=\frac{1-\rho\mathbb{E}_1[M_2]}{\rho\mathbb{E}_1[M_2]}+\frac{1-\rho}{\rho}\xi_B B_2.\end{aligned}$$

Therefore,

$$\frac{\partial TS_2}{\partial B_2}=\tau\left(\frac{1-\rho\mathbb{E}_1[M_2]}{\rho\mathbb{E}_1[M_2]}+\frac{1-\rho}{\rho}\xi_B B_2\right).$$

Multiplying by $M_2$ and taking expectations yields

$$\mathbb{E}_1\left[M_2\frac{\partial TS_2}{\partial B_2}\right]=\tau\left(\frac{1-\rho\mathbb{E}_1[M_2]}{\rho}+\frac{1-\rho}{\rho}\xi_B\,\mathbb{E}_1[M_2]\,B_2\right),$$

which shows that $\mathbb{E}_1\left[M_2\frac{\partial TS_2}{\partial B_2}\right]$ is affine and strictly increasing in $B_2$. From (7), we obtain

$$\frac{\partial B_2}{\partial B_1}=\frac{\lambda_B}{\lambda_B+(1-\rho)\xi_B\mathbb{E}_1[M_2](1-\tau)}\in(0,1), \tag{A8}$$

which implies that $B_2$ is affine and strictly increasing in $B_1$. Since $K_1$ is fixed, $B_2$ is also affine and strictly increasing in $B_1/K_1$, and so is $\mathbb{E}_1\left[M_2\frac{\partial TS_2}{\partial B_2}\right]$. ∎

*ii) Proof that* $\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]$ *is strictly increasing and concave in* $B_1/K_1$. From (A7), we obtain

$$\frac{\partial^2 c_2}{\partial B_2^2}=-\frac{1-\rho}{\rho}2\frac{\xi_K K_2}{B_2^3}<0.$$

Since $TS_2=\tau c_2 B_2$,

$$\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]=\mathbb{E}_1[M_2]\tau c_2(B_2),$$

so

$$\frac{\partial}{\partial B_2}\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]=\mathbb{E}_1[M_2]\tau\frac{\partial c_2}{\partial B_2}>0,\qquad\frac{\partial^2}{\partial B_2^2}\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]=\mathbb{E}_1[M_2]\tau\frac{\partial^2 c_2}{\partial B_2^2}<0.$$

Hence, $\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]$ is strictly increasing and strictly concave in $B_2$. Equation (A8) implies that $B_2$ is affine and strictly increasing in $B_1/K_1$. Therefore, $\mathbb{E}_1\left[M_2\frac{TS_2}{B_2}\right]$ is strictly increasing and concave in $B_1/K_1$. ■

*iii) Proof that $\mathbb{E}_1\left[M_2\Delta TS^{M-A}\right]$ is U-shaped in $B_1/K_1$.* By definition,

$$\Delta TS^{M-A} \equiv \frac{\partial TS_2}{\partial B_2} - \frac{TS_2}{B_2} = \tau B_2 \frac{\partial c_2}{\partial B_2},$$

so, from (A7),

$$\Delta TS^{M-A}(B_2) = \tau\frac{1-\rho}{\rho}\left(\frac{\xi_K K_2}{B_2} + \frac{1}{2}\xi_B B_2\right).$$

Since $B_2$ and $K_2$ are deterministic

$$\mathbb{E}_1\big[M_2\Delta TS^{M-A}\big] = \mathbb{E}_1[M_2]\,\tau\frac{1-\rho}{\rho}\left(\frac{\xi_K K_2}{B_2} + \frac{1}{2}\xi_B B_2\right).$$

Differentiating with respect to $B_2$:

$$\frac{\partial}{\partial B_2}\mathbb{E}_1\big[M_2\Delta TS^{M-A}\big] = \mathbb{E}_1[M_2]\tau\frac{1-\rho}{\rho}\left(-\frac{\xi_K K_2}{B_2^2} + \frac{1}{2}\xi_B\right).$$

This derivative is negative for $B_2^2 < 2\xi_K K_2/\xi_B$ and positive for $B_2^2 > 2\xi_K K_2/\xi_B$, and it equals zero only at

$$\widehat{B_2} = \sqrt{\frac{2\xi_K K_2}{\xi_B}}.$$

Moreover,

$$\frac{\partial^2}{\partial B_2^2}\mathbb{E}_1\big[M_2\Delta TS^{M-A}\big] = \mathbb{E}_1[M_2]\tau\frac{1-\rho}{\rho}\,2\frac{\xi_K K_2}{B_2^3} \;>\; 0,$$

so $\mathbb{E}_1[M_2\Delta TS^{M-A}]$ is strictly convex in $B_2$ and has a unique global minimum at $\widehat{B_2}$ (i.e. it is decreasing for $B_2 < \widehat{B_2}$ and increasing for $B_2 > \widehat{B_2}$). By the chain rule,

$$\frac{\partial}{\partial(B_1/K_1)}\mathbb{E}_1\big[M_2\Delta TS^{M-A}\big] = \frac{\partial}{\partial B_2}\mathbb{E}_1\big[M_2\Delta TS^{M-A}\big]\cdot\frac{\partial B_2}{\partial(B_1/K_1)},$$

and $\partial B_2/\partial(B_1/K_1) = K_1\,\partial B_2/\partial B_1 > 0$. Hence the sign of the derivative in $B_1/K_1$ is the sign of

$$-\frac{\xi_K K_2}{B_2^2} + \frac{1}{2}\xi_B,$$

so it is negative when $B_2 < \widehat{B_2}$ and positive when $B_2 > \widehat{B_2}$, with a unique zero at $B_2 = \widehat{B_2}$.

The corresponding threshold value of $B_1/K_1$ is obtained by setting $B_2 = \widehat{B_2}$ in the affine map above:

$$\frac{\widehat{B_1}}{K_1} = \frac{1}{K_1}\cdot\frac{\Big(\lambda_B + (1-\rho)\xi_B\mathbb{E}_1[M_2](1-\tau)\Big)\sqrt{\frac{2\xi_K K_2}{\xi_B}} - \tau(1-\rho\mathbb{E}_1[M_2])}{\lambda_B}.$$

Therefore $\mathbb{E}_1[M_2 \Delta TS^{M-A}]$ is decreasing in $B_1/K_1$ for $B_1/K_1 < \widehat{B_1}/K_1$ and increasing for $B_1/K_1 > \widehat{B_1}/K_1$, i.e. it is U-shaped in $B_1/K_1$.$\blacksquare$

*iv) Proof that $\mathbb{E}_1\left[M_2 \frac{\partial BC_2}{\partial B_2}\right]$ is strictly increasing and affine in $B_1/K_1$.* Since $BC_2 = R_2^A K_2 - L_2$ and $R_2^A K_2$ does not depend on $B_2$,

$$\frac{\partial BC_2}{\partial B_2} = -\frac{\partial L_2}{\partial B_2} = \xi_B B_2.$$

Hence

$$\mathbb{E}_1\left[M_2 \frac{\partial BC_2}{\partial B_2}\right] = \xi_B \mathbb{E}_1[M_2]\, B_2,$$

which is linear and strictly increasing in $B_2$ because $\xi_B \mathbb{E}_1[M_2] > 0$. Equation (A8) implies that $B_2$ is affine and strictly increasing in $B_1/K_1$. Therefore, $\mathbb{E}_1\left[M_2 \frac{\partial BC_2}{\partial B_2}\right]$ is strictly increasing and affine in $B_1/K_1$. $\blacksquare$

*v) Proof that $\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right]$ is U-shaped in $B_1/K_1$ under* $\mathbb{E}_1[M_2(R_2^A - \xi_K)] > 0$. Since

$$\frac{BC_2}{B_2} = \frac{R_2^A K_2 - L_2}{B_2} = \frac{R_2^A K_2 - \xi_K K_2}{B_2} + \frac{1}{2}\xi_B B_2,$$

taking $\mathbb{E}_1[M_2(\cdot)]$ and using that $B_2$ is deterministic at the time of choice yields

$$\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right] = \frac{K_2}{B_2}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right] + \frac{1}{2}\xi_B\, \mathbb{E}_1[M_2]\, B_2.$$

Differentiating with respect to $B_2$,

$$\frac{\partial}{\partial B_2}\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right] = -\frac{K_2}{B_2^2}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right] + \frac{1}{2}\xi_B\, \mathbb{E}_1[M_2],$$

$$\frac{\partial^2}{\partial B_2^2}\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right] = 2\frac{K_2}{B_2^3}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right].$$

Since $\mathbb{E}_1[M_2(R_2^A - \xi_K)] > 0$, the second derivative is strictly positive for all $B_2 > 0$, so $\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right]$ is strictly convex in $B_2$. Moreover, the first derivative crosses zero exactly once at

$$\overline{B_2} = \sqrt{\frac{2K_2\, \mathbb{E}_1[M_2(R_2^A - \xi_K)]}{\xi_B\, \mathbb{E}_1[M_2]}},$$

so the function is decreasing for $B_2 < \overline{B_2}$ and increasing for $B_2 > \overline{B_2}$, i.e. U-shaped in $B_2$.

As $B_2$ is affine and strictly increasing in $B_1$ by (A8), composing a strictly convex function of $B_2$ with an affine increasing map preserves convexity and the unique turning point. Therefore, under $\mathbb{E}_1[M_2(R_2^A - \xi_K)] > 0$,

$$\mathbb{E}_1\left[M_2 \frac{BC_2}{B_2}\right]$$

is U-shaped in $B_1/K_1$, with a unique minimum at the $B_1/K_1$ that implies $B_2 = \overline{B_2}$. ■

*vi) Proof that* $\mathbb{E}_1\left[M_2 \Delta BC^{M-A}\right]$ *is strictly increasing and strictly concave in* $B_1/K_1$ *when* $\mathbb{E}_1[M_2 R_2^A] > \xi_K \mathbb{E}_1[M_2]$. Notice that

$$\Delta BC^{M-A} = \xi_B B_2 - \left(\frac{K_2(R_2^A - \xi_K)}{B_2} + \frac{1}{2}\xi_B B_2\right) = \frac{1}{2}\xi_B B_2 - \frac{K_2(R_2^A - \xi_K)}{B_2}.$$

Taking $\mathbb{E}_1[M_2(\cdot)]$ and using that $B_2$ is deterministic at the time of choice,

$$\mathbb{E}_1[M_2 \Delta BC^{M-A}] = \frac{1}{2}\xi_B\, \mathbb{E}_1[M_2]\, B_2 - \frac{K_2}{B_2}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right].$$

Differentiating with respect to $B_2$ yields

$$\frac{\partial}{\partial B_2}\mathbb{E}_1[M_2 \Delta BC^{M-A}] = \frac{1}{2}\xi_B\, \mathbb{E}_1[M_2] + \frac{K_2}{B_2^2}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right],$$

$$\frac{\partial^2}{\partial B_2^2}\mathbb{E}_1[M_2 \Delta BC^{M-A}] = -2\frac{K_2}{B_2^3}\, \mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right].$$

Since $\mathbb{E}_1\left[M_2(R_2^A - \xi_K)\right] > 0$, the first derivative is strictly positive for all $B_2 > 0$ and second derivative is strictly negative for all $B_2 > 0$. Hence $\mathbb{E}_1[M_2 \Delta BC^{M-A}]$ is strictly increasing and strictly concave in $B_2$. Since $B_2$ is affine and strictly increasing in $B_1$ by (A8) (and therefore in $B_1/K_1$ with $K_1$ fixed), composition with an affine increasing map preserves monotonicity and concavity. ■

**Expected Asset Return for the Firm ($\gamma_A(K_2^*)$).** In the text, we claim that $\gamma_A(K_2^*)$ is constant for a production function such that the elasticity of output with respect to capital is $\alpha$, that is, under the assumption that

$$\frac{f(A_2, Z_2, K_2^*)}{K_2^*} = \alpha^{-1} f_k(A_2, Z_2, K_2^*).$$

Then

$$\begin{aligned}
\gamma_A(K_2^*) &= \mathbb{E}_1\left[M_2 R_2^A\right] \\
&= \mathbb{E}_1\left[M_2\left(1 + (1-\tau)\left(\alpha^{-1} f_k(A_2, Z_2, K_2^*) - \delta\right)\right)\right] \\
&= \alpha^{-1}\mathbb{E}_1\left[M_2\left(1 + (1-\tau)\left(f_k(A_2, Z_2, K_2^*) - \delta\right)\right)\right] \\
&\quad +(1-\alpha^{-1})\mathbb{E}_1\left[M_2\left(1 - \delta(1-\tau)\right)\right] \\
&= \alpha^{-1}\mathbb{E}_1\left[M_2\left(1 + (1-\tau)\left(f_k(A_2, Z_2, K_2^*) - \delta\right)\right)\right] + (1-\alpha^{-1})\frac{1-\delta(1-\tau)}{1+r_F}.
\end{aligned}$$

Combining equations (1) and (5), we get the following analytical expression for $K_2^*$:

$$K_2^* = f_k^{-1}\left[\frac{r_F + 1 - \rho + \rho(1-\tau)\left(\delta - \frac{1-\rho}{\rho}\xi_K\right)}{(1+r_F)\rho\,(1-\tau)\,\mathbb{E}_1[M_2 A_2 Z_2]}\right]. \tag{A9}$$

Because $f(\cdot)$ is increasing and strictly concave, its derivative $f_k(\cdot)$ is strictly decreasing, hence invertible. From (5), one obtains

$$\gamma_A(K_2^*) = \alpha^{-1}\left(\frac{1}{\rho} - \frac{1}{1+r_F}(1-\tau)\frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2}\right) + (1-\alpha^{-1})\frac{1-\delta(1-\tau)}{1+r_F},$$

which is constant because $\frac{\partial L_2}{\partial K_2} = \xi_K$.

**Extension with Equity Issuance Costs.** We consider the following equity maximization problem:

$$V_1 \equiv \max_{K_2^*, B_2^*} D_1 - \Lambda^E(-D_1) + \rho\mathbb{E}_1[M_2 D_2],$$

where firms may issue external equity at $t = 1$, incurring an issuance cost $\Lambda^E(-D_1)$, a differentiable function. The first-order conditions with respect to capital and debt are:

$$1 = \rho\mathbb{E}_1\left[M_2 \cdot \frac{1}{1+\frac{\partial\Lambda^E(-D_1)}{(-D_1)}}\left(1 + (1-\tau)(f_k(A_2, Z_2, K_2^*) - \delta) + (1-\tau)\frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)\right].$$

and

$$\lambda_B(B_2^* - B_1) = \frac{\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)}}{1+\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)}} + \rho\,\mathbb{E}_1\left[M_2 \cdot \frac{1}{1+\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)}}\frac{\partial TS_2}{\partial B_2^*}\right] - (1-\rho)\,\mathbb{E}_1\left[M_2 \cdot \frac{1}{1+\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)}}\frac{\partial BC_2}{\partial B_2^*}\right].$$

Using similar steps as in the proofs of Propositions 1 and 2, the following adjustments apply:

$$\lambda_1 = \frac{(1-\tau)(1-\rho)\xi_B\mathbb{E}_1[M_2^F]}{\lambda_B + (1-\tau)(1-\rho)\xi_B\mathbb{E}_1[M_2^F]},$$

and

$$B_2^0 = \frac{(1+r_F)\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)} + \tau(1+r_F-\rho)}{(1-\tau)(1-\rho)\xi_B},$$

where the firm's stochastic discount factor is defined as $M_2^F \equiv \frac{M_2}{1+\frac{\partial\Lambda^E(-D_1)}{\partial(-D_1)}}$. The optimal capital stock is:

$$K_2^* = f_k^{-1}\left[\frac{1 + r_F - \rho + \rho(1-\tau)\left(\delta - \frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)}{(1+r_F)\rho\,(1-\tau)\,\mathbb{E}_1[M_2^F A_2 Z_2]}\right].$$

Observe that the previous expressions do not provide a fully closed-form solution, as $E_1$ remains endogenous. The firm's expected cost of debt is now given by:

$$\gamma_D(B_2^*) = 1 + \kappa_1^E Gap_1^L + \kappa_2^E \frac{B_1}{K_1} + \rho E_1\left[M_2 \Delta TS^{M-A}\right] - (1-\rho)E_1\left[M_2 \Delta BC^{M-A}\right] + \frac{\partial \Lambda^E(-D_1)}{\partial(-D_1)},$$

where $\kappa_1^E \equiv \lambda_B \lambda_1 K_2^* \left(1 + \frac{\partial \Lambda^E(-D_1)}{\partial(-D_1)}\right)$ and $\kappa_2^E \equiv -\lambda_B \lambda_1 (K_2^* - K_1)\left(1 + \frac{\partial \Lambda^E(-D_1)}{\partial(-D_1)}\right)$.

**Extension with Capital Adjustment Costs.** We introduce the following capital adjustment cost function:

$$\Lambda^K(K_2^* - (1-\delta)K_1) = \frac{\lambda_K}{2}(K_2^* - (1-\delta)K_1)^2.$$

The expression for dividends at $t = 1$ becomes:

$$D_1 = R_1^A K_1 - K_2^* + B_2^* - (1 + c_1(1-\tau))B_1 - \Lambda^B(B_2^* - B_1) - \Lambda^K(K_2^* - (1-\delta)K_1).$$

The first-order condition with respect to capital $K_2^*$ is:

$$1 + \frac{\partial \Lambda^K(K_2^* - (1-\delta)K_1)}{\partial K_2^*} = \rho \mathbb{E}_1\left[M_2\left(1 + (1-\tau)\left(f_k(A_2, Z_2, K_2^*) - \delta + \frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)\right)\right],$$

while the condition with respect to debt $B_2^*$ remains unchanged.

Accordingly, the statements in Propositions 1 and 2 can be amended by defining a new level of capital $K_2^*$ as:

$$K_2^* = g^{-1}\left(\frac{(1+r_F)(1 - \lambda_K(1-\delta)K_1) - \rho + \rho(1-\tau)\left(\delta - \frac{1-\rho}{\rho}\frac{\partial L_2}{\partial K_2^*}\right)}{(1+r_F)\rho(1-\tau)\mathbb{E}_1[M_2 A_2 Z_2]}\right),$$

where the function

$$g(K) \equiv f_k(K) - \frac{\lambda_K}{\rho \mathbb{E}_1[M_2 A_2 Z_2](1-\tau)} K$$

is strictly monotonic and therefore invertible.

## C) Dynamic Model: Derivations and Proofs

In this appendix, we provide derivations for the dynamic model. Since the purpose of these calculations is not to compare the frictional and frictionless problems, we adopt a plain notation throughout. In particular, frictionless counterparts do not appear in this section, so all variables are understood as the relevant baseline choices of the dynamic model.

**Statement.** In what follows we provide further details about the firm's problem with logistic outside option shocks. Consider the following statement.

a) The equity valuation equation (23) and the debt valuation equation (21) can be rewritten as

$$V(S_{i,t}) = V^C(S_{i,t}) + s \cdot \ln\left(1 + e^{-\frac{V^C(S_{i,t})}{s}}\right), \tag{A10}$$

and

$$B_{i,t+1} = \mathbb{E}\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}P^S_{i,t+1} + L_{i,t+1}(1-P^S_{i,t+1})\right)\big|S_{i,t}\right]. \tag{A11}$$

b) As $s \to 0^+$, the firm's problem in (A10), subject to (A11), (18), (19), and (20), converges to

$$V(S_{i,t}) = \max\left\{0, \max_{\Delta B_{i,t}, I_{i,t}} D_{i,t} + \mathbb{E}[M_{t+1}V(S_{i,t+1})|S_{i,t}]\right\}, \tag{A12}$$

subject to

$$B_{i,t+1} = \mathbb{E}\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}\chi_{\{V_{i,t+1}>0\}} + L_{i,t+1}\chi_{\{V_{i,t+1}\leq 0\}}\right)\big|S_{i,t}\right], \tag{A13}$$

along with (18), (19), and (20).

**Derivation.** Part a). To compute the expected outside option in case of default, we evaluate the following indefinite integral:

$$\int \zeta_{i,t} dF^\zeta(\zeta_{i,t}) = -\zeta_{i,t} \cdot \frac{e^{-\zeta_{i,t}/s}}{1+e^{-\zeta_{i,t}/s}} - s\ln\left(1+e^{-\frac{\zeta_{i,t}}{s}}\right) + C, \tag{A14}$$

where $C$ is an integrating constant. Evaluating (A14) from $V^C(S_{i,t})$ to $\infty$ gives

$$\int_{V^C(S_{i,t})}^{\infty} \zeta_{i,t} dF^\zeta(\zeta_{i,t}) = V^C(S_{i,t})\frac{e^{-\frac{V^C(S_{i,t})}{s}}}{1+e^{-\frac{V^C(S_{i,t})}{s}}} + s\ln\left(1+e^{-\frac{V^C(S_{i,t})}{s}}\right).$$

Substituting (6) and (26) into (25) and simplifying yields (A10). To derive (A11), note that $V_{i,t+1} \leq \zeta_{i,t+1}$ if and only if $V^C_{i,t+1} \leq \zeta_{i,t+1}$. Thus,

$$\begin{aligned}
B_{i,t+1} &= \mathbb{E}\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}\mathbb{E}^\zeta\left[\chi_{\{V^C_{i,t+1}>\zeta_{i,t+1}\}}|S_{i,t}\right] + L_{i,t+1}\mathbb{E}^\zeta\left[\chi_{\{V^C_{i,t+1}\leq\zeta_{i,t+1}\}}|S_{i,t}\right]\right)\Big|\, S_{i,t}\right] \\
&= \mathbb{E}\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}\mathbb{E}^\zeta[1-DEF_{i,t}|S_{i,t}] + L_{i,t+1}\mathbb{E}^\zeta[DEF_{i,t}|S_{i,t}]\right)\big|\, S_{i,t}\right] \\
&= \mathbb{E}\left[M_{t+1}\left((1+c_{i,t+1})B_{i,t+1}P^S_{i,t+1} + L_{i,t+1}(1-P^S_{i,t+1})\right)\big|\, S_{i,t}\right].
\end{aligned}$$

Part b). For small $s$, $\zeta_{i,t}$ converges to its mean, which is 0. The probability of solvency approaches

$$\lim_{s\to 0^+} P^S_{i,t} = \begin{cases} 1 & \text{if } V^C(S_{i,t}) > 0 \\ 0.5 & \text{if } V^C(S_{i,t}) = 0 \\ 0 & \text{if } V^C(S_{i,t}) < 0. \end{cases} \tag{A15}$$

Additionally, $\lim_{s\to 0^+}\int_{V^C(S_{i,t})}^{\infty}\zeta_{i,t}dF^{\zeta}(\zeta_{i,t})$ converges to zero for any $V^C(S_{i,t})$. Thus,

$$\lim_{s\to 0^+}\mathbb{E}^{\zeta}[\max\{\zeta_{i,t},V^C(S_{i,t})\}]=\begin{cases} V^C(S_{i,t}) & \text{if } V^C(S_{i,t})>0 \\ 0 & \text{if } V^C(S_{i,t})\le 0. \end{cases}$$

As $s\to 0^+$, (A10) simplifies to

$$V(S_{i,t})=\mathbb{E}^{\zeta}[\max\{\zeta_{i,t},V^C(S_{i,t})\}]=\max\{0,V^C(S_{i,t})\},$$

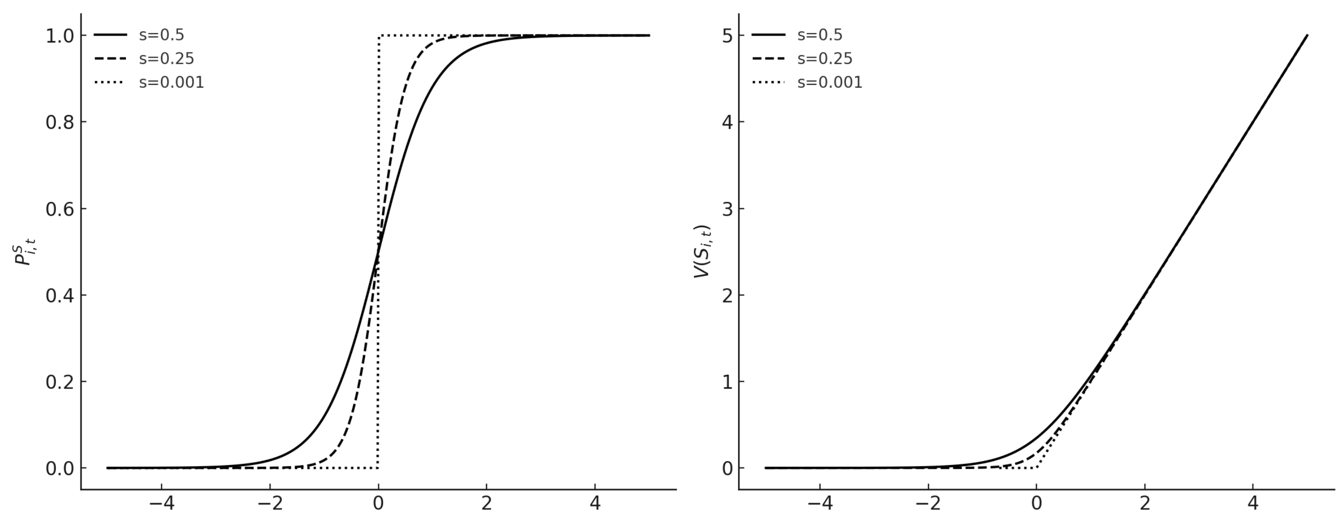
which leads to (A12). The derivation of (A13) follows directly from combining (A11) and (A15). ■

Figure A1 illustrates the intuition behind part b) of the statement, showing that this differentiable problem nests the textbook case where shareholders' outside option is deterministically zero and converges to it as the scale parameter $s$ approaches zero. Panel A of Figure A1 plots the solvency probability $P^S_{i,t}$ as a function of the continuation value $V^C(S_{i,t})$ for different values of $s$, while Panel B depicts the equity value $V(S_{i,t})$ from (A10) against $V^C(S_{i,t})$. The solid, dashed, and dotted lines correspond to progressively smaller values of $s$. While maintaining differentiability due to its "softmax" property, the dashed line shows that for small $s$, the problem collapses to the textbook case with a zero outside option.

**Figure A1**

The Firm's Problem with Shocks to Outside Options

The figure shows the solvency probability $P^S_{i,t}$ (Panel A) and the equity value $V(S_{i,t})$ as a function of the continuation value $V^C(S_{i,t})$ for different values of the scale parameter $s$ in the logistic distribution of outside option shocks $\zeta_{i,t}$. The solid line corresponds to $s=0.5$, the dashed line to $s=0.25$, and the dotted line to $s=0.001$.




In particular, the solvency probability in Panel A (dotted line) shows that for small $s$, firms default deterministically when the continuation value is negative and continue operating otherwise. Formally, this follows from

$$\lim_{s\to 0^+}P^S_{i,t}=\begin{cases}1, & \text{if } V^C(S_{i,t})>0\\ 0, & \text{if } V^C(S_{i,t})<0.\end{cases}$$

Similarly, the dotted line in Panel B shows that the equity value collapses to $\max\{0, V^C(S_{i,t})\}$, since

$$\lim_{s\to 0^+} V(S_{i,t}) = \begin{cases} V^C(S_{i,t}), & \text{if } V^C(S_{i,t}) \geq 0 \\ 0, & \text{if } V^C(S_{i,t}) < 0. \end{cases}$$

**Extension with Capital Adjustment Costs.** We introduce a differentiable capital adjustment cost function $\Lambda^K(I_{i,t})$ into the baseline model. This modifies the budget constraint as follows:

$$\Pi_{i,t} + \Delta B_{i,t} + E_{i,t} + \tau\delta K_{i,t} + \tau c_{i,t}B_{i,t} = I_{i,t} + c_{i,t}B_{i,t} + \Lambda^B(\Delta B_{i,t}) + \Lambda^K(I_{i,t}).$$

Relative to the baseline model, the optimality condition for debt remains unchanged. The optimality condition for capital becomes:

$$1 + \frac{\partial \Lambda^K(I_{i,t})}{\partial K_{i,t+1}} = \mathbb{E}\left[ M_{t+1} \cdot P^S_{i,t+1} \cdot \left( \frac{1 + \frac{\partial \Lambda^E(E_{i,t+1})}{\partial E_{i,t+1}}}{1 + \frac{\partial \Lambda^E(E_{i,t})}{\partial E_{i,t}}} \right) \cdot R^k_{i,t+1} \,\middle|\, S_{i,t} \right],$$

where

$$R^k_{i,t+1} \equiv \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}} + 1 - \delta(1-\tau) + (1-\delta)\frac{\partial \Lambda^K(I_{i,t+1})}{\partial I_{i,t+1}} - \frac{\partial c_{i,t+1}}{\partial K_{i,t+1}} B_{i,t+1}(1-\tau).$$

Intuitively, investment now involves an additional expense due to adjustment costs.

## D) Numerical Solution Method

We solve the model corresponding to $s \to 0^+$ using a combination of value function iteration (VFI) and simulation. Each firm $i$'s problem is characterized by five state variables, i.e., $S_{i,t} \equiv (K_{i,t}, B_{i,t}, c_{i,t}, z_{i,t}, A_t)$. We approximate the value function $V(S_{i,t})$ with piece-wise linear interpolation on a grid $7 \times 7 \times 5 \times 5 \times 3$, respectively. Note that the projection of $V(S_{i,t})$ onto an interpolated structure allows for a precise solution with a relatively parsimonious number of grid points. We check the robustness of our numerical solution by experimenting with finer grids.

Given $S_{i,t}$, each firm faces three continuous choices, i.e., $(K_{i,t+1}, B_{i,t+1}, c_{i,t+1})$, and one discrete choice, i.e., whether to default or not $DEF_{i,t}$. Choices $K_{i,t+1}$ and $B_{i,t+1}$ are evaluated on a grid $30 \times 30$, and we solve numerically at each step of the VFI for $c_{i,t+1}$ given each fix evaluation of $(K_{i,t}, B_{i,t}, c_{i,t}, z_{i,t}, A_t, K_{i,t+1}, B_{i,t+1})$. This requires to solve for $7 \times 7 \times 5 \times 5 \times 3 \times 30 \times 30$ nonlinear equations at each step of the VFI, given the guess of the value function and future default. We solve for the coupon using golden search.

The solution with VFI proceed in two steps. First, we find the equilibrium value function $\tilde{V}(_{i,t})$ associated with an identical model but without endogenous default. Second, we use $\tilde{V}(_{i,t})$ as an initial guess for the model with endogenous default and we progressively increase the fixed cost $F$, re-converge the value function and re-initialize. The convergence criteria on the value function is defined as a max absolute difference between the value functions in two consecutive iterations of the VFI of $10^{-4}$, or lower.

We then use the four policy functions $\{K(S_{i,t}), B_(S_{i,t}), c(S_{i,t}), DEF_{i,t}(S_{i,t})\}$ to perform a long simulation of our economy ($T = 5000$) with a panel of $i = 1, ..., 5000$ firms, given random draws of idiosyncratic shocks $\{\{z_{i,t}\}_{t=0}^{T}\}_{i=1}^{5000}$ and aggregate shocks $\{A_t\}_{t=0}^{T}$. To compute regression coefficients and standard errors in model-based regressions, we bootstrap 100 randomly selected firms over 5,000 time periods across 100 simulations.

## E) Robustness Checks and Sensitivity Analysis

**Table A1**

ROBUSTNESS: ALTERNATIVE LEVERAGE MEASURES

The table reports coefficient estimates of Fama-MacBeth cross-sectional regression of monthly stock returns on leverage variables and controls. Columns (1) to (3) refer to market leverage net of cash ("Net Market Lev."). Columns (4) to (6) refer to book leverage ("Book Lev."). Columns (7) to (9) refer to book leverage net of cash ("Net Book Lev."). The sample includes all Compustat firms traded on NYSE, AMEX and NASDAQ between 1965 and 2020 and covered by the Center of Research in Security Prices (CRSP). Leverage targets and leverage gaps are the results of the rolling estimation procedure described in Section 3. "Lev. Gap (FR)" refers to leverage gaps estimated using the Flannery–Rangan approach. "Lev. Gap (Proj.)" refers to leverage gaps derived from the dynamic model, where model-based targets are projected onto a set of variables observed in the data. The resulting functional form is then used to estimate a multinomial logit model, as discussed in Section 5.2. Years from 1965 to 1979 are used as a "burn in" period. Annual accounting variables are matched to monthly returns from July 1980 to December 2020 following the standard procedure of Fama and French (1992). Newey-West standard errors with lag-length of 4 are in parentheses. $R^2$ and N. Obs denote the cross-sectional R-squared and the number of observations respectively. All variables are described in Appendix A.

| | Dependent Variable: Monthly Stock Return | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Net Market Lev. | | | Book Lev. | | | Net Book Lev. | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Leverage | -0.06 | 0.05 | 0.09 | 0.17 | 0.12 | 0.19 | -0.03 | -0.01 | 0.04 |
| | (0.29) | (0.27) | (0.27) | (0.29) | (0.27) | (0.27) | (0.22) | (0.22) | (0.22) |
| Lev. Gap (FR) | | 0.45 | | | 0.61 | | | 0.55 | |
| | | (0.20) | | | (0.23) | | | (0.18) | |
| Lev. Gap (Proj.) | | | 0.59 | | | 0.56 | | | 0.59 |
| | | | (0.13) | | | (0.13) | | | (0.13) |
| Size | -0.21 | -0.19 | -0.20 | -0.21 | -0.19 | -0.20 | -0.21 | -0.19 | -0.20 |
| | (0.04) | (0.04) | (0.04) | (0.05) | (0.04) | (0.04) | (0.04) | (0.04) | (0.04) |
| Be/Me | 0.18 | 0.13 | 0.13 | 0.18 | 0.13 | 0.13 | 0.17 | 0.13 | 0.12 |
| | (0.07) | (0.07) | (0.06) | (0.08) | (0.07) | (0.07) | (0.06) | (0.06) | (0.07) |
| $R^2$ | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| N. Obs. | 1,221,663 | 880,356 | 761,211 | 1,222,311 | 880,537 | 761,379 | 1,222,228 | 880,441 | 761,296 |

**Table A2**

Specifications with Leverage and Leverage Targets

The table reports coefficient estimates from cross-sectional regressions of monthly stock returns on leverage variables and controls. In the columns labeled "Data", leverage targets result from the rolling estimation procedure described in Section 3. The sample includes all Compustat firms traded on the NYSE, AMEX, and NASDAQ between 1965 and 2020 and covered by the Center for Research in Security Prices (CRSP). In the columns labeled "Model", coefficients are averages from 100 simulations of 100 randomly drawn firms over 5,000 time periods, using the baseline calibration in Table 2. "Lev. Gap (FR)'" refers to leverage gaps estimated using the Flannery–Rangan approach. "Lev. Gap (Proj.)" refers to leverage gaps derived from the dynamic model, where model-based targets are projected onto a set of variables observed in the data. The resulting functional form is then used to estimate a multinomial logit model, as discussed in Section 5.2. In column (3), leverage targets are computed using the model-based approach described in Section 4.2, denoted as "Lev. Target (Model)". Columns (4) and (5) report results based on leverage targets and gaps estimated on simulated data by applying the Flannery–Rangan approach, as detailed in Section 5.2. In the data, the years from 1965 to 1979 serve as a burn-in period. Annual accounting variables are matched to monthly returns from July 1980 to December 2020, following the standard procedure of Fama and French (1992). Newey–West standard errors with a lag length of 4 are reported in parentheses. For the model, bootstrapped standard errors are in parentheses. $R^2$ denotes the cross-sectional R-squared. All data variables are described in Appendix A.

| | Data | | Model | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Leverage | 0.97 | 0.85 | 1.42 | 0.61 | 1.04 |
| | (0.38) | (0.32) | (0.46) | (0.32) | (0.33) |
| Lev. Target (FR) | -0.79 | | | -0.57 | |
| | (0.29) | | | (0.24) | |
| Lev. Target (Proj.) | | -0.54 | | | -0.83 |
| | | (0.13) | | | (0.29) |
| Lev. Target (Model) | | | -0.88 | | |
| | | | (0.30) | | |
| Size | -0.18 | -0.19 | -0.09 | -0.34 | -0.14 |
| | (0.04) | (0.04) | (0.07) | (0.12) | (0.06) |
| Be/Me | 0.10 | 0.08 | 0.19 | 0.22 | 0.11 |
| | (0.06) | (0.06) | (0.10) | (0.10) | (0.08) |
| $R^2$ | 0.02 | 0.02 | 0.01 | 0.03 | 0.02 |

**Table A3**

ROBUSTNESS: CONTROLLING FOR PROFITABILITY AND INVESTMENT

The table reports coefficient estimates of Fama-MacBeth cross-sectional regression of monthly stock returns on leverage variables and controls. The sample includes all Compustat firms traded on NYSE, AMEX and NASDAQ between 1965 and 2020 and covered by the Center of Research in Security Prices (CRSP). Leverage targets and leverage gaps are the results of the rolling estimation procedure described in Section 3. Columns (2) and (3) report results based on leverage gaps estimated using the Flannery–Rangan approach ("Flannery-Rangan Target"). Columns (4) and (5) use leverage targets derived from the dynamic model, where model-based targets are projected onto a set of variables observed in the data ("Projection Target"). The resulting functional form is then used to estimate a multinomial logit model, as discussed in Section 5.2. Years from 1965 to 1979 are used as a "burn in" period. Annual accounting variables are matched to monthly returns from July 1980 to December 2020 following the standard procedure of Fama and French (1992). Newey-West standard errors with lag-length of 4 are in parentheses. $R^2$ and N. Obs denote the cross-sectional R-squared and the number of observations respectively. All variables are described in Appendix A.

| Dependent Variable: Monthly Stock Return | | | | | |
|---|---|---|---|---|---|
| | | Flannery-Rangan Target | | Projection Target | |
| | (1) | (2) | (3) | (4) | (5) |
| Leverage | 0.10 | | 0.02 | | 0.07 |
| | (0.34) | | (0.31) | | (0.31) |
| Lev. Gap | | 0.90 | 0.84 | 0.52 | 0.51 |
| | | (0.33) | (0.29) | (0.13) | (0.13) |
| Size | -0.20 | -0.17 | -0.17 | -0.18 | -0.18 |
| | (0.05) | (0.04) | (0.04) | (0.04) | (0.04) |
| Be/Me | 0.16 | 0.13 | 0.13 | 0.18 | 0.18 |
| | (0.07) | (0.08) | (0.07) | (0.08) | (0.07) |
| Prof | 0.06 | 0.05 | 0.05 | 0.10 | 0.10 |
| | (0.01) | (0.02) | (0.02) | (0.02) | (0.02) |
| Inv | -0.27 | -0.28 | -0.26 | -0.28 | -0.26 |
| | (0.04) | (0.06) | (0.06) | (0.07) | (0.07) |
| $R^2$ | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| N. Obs | 1,140,792 | 880,338 | 880,338 | 761,192 | 761,192 |

**Table A4**

Robustness: Investment, Disinvestment, and Leverage Ratios

The table reports coefficient estimates of Fama-MacBeth cross-sectional regression of monthly stock returns on leverage variables and controls. "Neg. Inv." is an indicator variable equal to one for disinvesting firms. "Δ Lev. from Assets" measures the change in leverage resulting from variations in the denominator, keeping the debt stock constant. The sample includes all Compustat firms traded on NYSE, AMEX and NASDAQ between 1965 and 2020 and covered by the Center of Research in Security Prices (CRSP). Leverage targets and leverage gaps are the results of the rolling estimation procedure described in Section 3. Columns (1) to (4) report results based on leverage gaps estimated using the Flannery–Rangan approach ("Flannery-Rangan Target"). Columns (5) to (8) use leverage targets derived from the dynamic model, where model-based targets are projected onto a set of variables observed in the data ("Projection Target"). The resulting functional form is then used to estimate a multinomial logit model, as discussed in Section 5.2. Years from 1965 to 1979 are used as a "burn in" period. Annual accounting variables are matched to monthly returns from July 1980 to December 2020 following the standard procedure of Fama and French (1992). Newey-West standard errors with lag-length of 4 are in parentheses. $R^2$ and N. Obs denote the cross-sectional R-squared and the number of observations respectively. All variables are described in Appendix A.

| Dependent Variable: Monthly Stock Return | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Flannery-Rangan Target | | | | Projection Target | | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Leverage | 0.25 | 0.16 | 0.02 | 0.01 | 0.35 | 0.23 | 0.09 | 0.04 |
| | (0.32) | (0.32) | (0.31) | (0.31) | (0.31) | (0.31) | (0.30) | (0.30) |
| Lev. Gap | 0.81 | 0.74 | 0.83 | 0.79 | 0.57 | 0.54 | 0.52 | 0.50 |
| | (0.29) | (0.28) | (0.30) | (0.29) | (0.13) | (0.13) | (0.14) | (0.13) |
| Δ Lev. from Assets | 0.65 | | 0.18 | | 0.71 | | 0.33 | |
| | (0.19) | | (0.21) | | (0.20) | | (0.24) | |
| Neg. Inv. | | 0.21 | | 0.13 | | 0.21 | | 0.13 |
| | | (0.07) | | (0.08) | | (0.07) | | (0.07) |
| Size | -0.18 | -0.17 | -0.17 | -0.17 | -0.19 | -0.19 | -0.18 | -0.18 |
| | (0.04) | (0.04) | (0.04) | (0.04) | (0.04) | (0.04) | (0.04) | (0.04) |
| Be/Me | 0.09 | 0.10 | 0.14 | 0.13 | 0.10 | 0.11 | 0.17 | 0.17 |
| | (0.06) | (0.06) | (0.07) | (0.07) | (0.06) | (0.06) | (0.07) | (0.07) |
| Prof | | | 0.05 | 0.05 | | | 0.10 | 0.10 |
| | | | (0.02) | (0.02) | | | (0.02) | (0.02) |
| Inv | | | -0.19 | -0.23 | | | -0.14 | -0.22 |
| | | | (0.08) | (0.06) | | | (0.09) | (0.06) |
| $R^2$ | 0.02 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 | 0.03 |
| N. Obs | 880,067 | 880,452 | 879,953 | 880,338 | 760,970 | 761,294 | 760,868 | 761,192 |

## Table A5

### Robustness: Zero Leverage and Negative Net Leverage Firms

The table reports coefficient estimates of Fama-MacBeth cross-sectional regression of monthly stock returns on leverage variables and controls. The sample includes all Compustat firms traded on NYSE, AMEX and NASDAQ between 1965 and 2020 and covered by CRSP. Panel A adds industry × time fixed effects via within-group demeaning at the 2-digit and 3-digit SIC × month level and focuses on the subsample of zero-leverage and negative-net-leverage firms. Panel B focuses on firms with persistently zero book leverage or negative net leverage over the prior 3, 5, and 7 fiscal years. Newey-West standard errors (4 lags) in parentheses. $R^2$ denotes the cross-sectional R-squared.

**Panel A: Industry × Time Fixed Effects**

| | 2-Digit SIC × Time FE | | 3-Digit SIC × Time FE | |
|---|---|---|---|---|
| | FR Target (1) | Proj. Target (2) | FR Target (3) | Proj. Target (4) |
| Leverage | 1.05 | 1.47 | 0.83 | 1.08 |
| | (0.44) | (0.48) | (0.39) | (0.41) |
| Lev. Gap | 0.86 | 0.43 | 0.74 | 0.38 |
| | (0.51) | (0.18) | (0.50) | (0.19) |
| Size | -0.15 | -0.15 | -0.16 | -0.15 |
| | (0.04) | (0.04) | (0.04) | (0.05) |
| Be/Me | 0.13 | 0.16 | 0.15 | 0.19 |
| | (0.07) | (0.08) | (0.07) | (0.07) |
| $R^2$ | 0.02 | 0.03 | 0.02 | 0.03 |
| N. Obs | 332,321 | 216,495 | 332,321 | 216,495 |

**Panel B: Persistent Zero or Negative Leverage**

| | FR Target | | | Proj. Target | | |
|---|---|---|---|---|---|---|
| | 3 yr (1) | 5 yr (2) | 7 yr (3) | 3 yr (4) | 5 yr (5) | 7 yr (6) |
| Leverage | 1.60 | 1.52 | 1.80 | 1.85 | 1.40 | 1.94 |
| | (0.82) | (1.02) | (1.21) | (0.89) | (1.09) | (1.29) |
| Lev. Gap | 0.55 | 0.32 | -0.34 | 0.43 | 0.34 | 0.27 |
| | (0.58) | (0.60) | (0.76) | (0.19) | (0.24) | (0.29) |
| Size | -0.17 | -0.18 | -0.17 | -0.16 | -0.17 | -0.16 |
| | (0.05) | (0.05) | (0.05) | (0.05) | (0.05) | (0.05) |
| Be/Me | 0.07 | 0.07 | 0.08 | 0.08 | 0.07 | 0.09 |
| | (0.09) | (0.09) | (0.09) | (0.10) | (0.10) | (0.11) |
| $R^2$ | 0.03 | 0.03 | 0.04 | 0.04 | 0.04 | 0.05 |
| N. Obs | 258,336 | 215,656 | 172,120 | 152,760 | 119,643 | 91,140 |

## Table A6

### Amplifying and Dampening Effects of the Leverage Gap on Stock Returns

The table reports estimated coefficients from cross-sectional regressions of monthly stock returns on the overlevered-side and underlevered-side magnitudes of the leverage gap, defined as "$|\text{Gap}| \cdot \mathbf{1}_{OL}$" = $\max(\text{Gap}, 0)$ and "$|\text{Gap}| \cdot \mathbf{1}_{UL}$" = $\max(-\text{Gap}, 0)$ (both non-negative), together with market capitalization ("Size") and book-to-market equity ("Be/Me"). The two magnitude slopes isolate the *amplifying* effect on the overlevered side and the *dampening* effect on the underlevered side. Under the gap-premium hypothesis we expect the overlevered-side slope to be positive (more overlevered ⇒ higher returns) and the underlevered-side slope to be negative (more underlevered ⇒ lower returns). For each target we report two specifications: the odd-numbered columns exclude leverage, and the even-numbered columns add leverage ("Leverage") to the regression. Columns (1) and (2) report results from the simulated economy under the baseline calibration of Table 2 using the model-based real target (Section 4.2). Columns (3)–(4) and (5)–(6) report results on Compustat–CRSP data using the Flannery–Rangan and projection targets, respectively. Model coefficients are averages from 100 bootstrap simulations of 100 randomly drawn firms over 5000 time periods, with bootstrap standard errors in parentheses. Data coefficients are Fama–MacBeth estimates with Newey–West standard errors (4 lags) in parentheses. Returns are monthly and expressed in percentage; leverage and gap magnitudes are in levels; Size and Be/Me are in log levels. All variables are defined in Appendix A.

| Dependent Variable: Monthly Stock Return | | | | | | |
|---|---|---|---|---|---|---|
| | Model-Based Target | | Flannery-Rangan Target | | Projection Target | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Leverage | | 0.48 | | 0.24 | | 0.27 |
| | | (0.39) | | (0.33) | | (0.32) |
| $\|\text{Gap}\| \cdot \mathbf{1}_{OL}$ | 0.50 | 0.64 | 0.53 | 0.22 | 0.45 | 0.32 |
| | (0.40) | (0.41) | (0.58) | (0.49) | (0.66) | (0.68) |
| $\|\text{Gap}\| \cdot \mathbf{1}_{UL}$ | -2.26 | -1.83 | -1.42 | -1.31 | -0.58 | -0.57 |
| | (0.92) | (1.02) | (0.43) | (0.40) | (0.15) | (0.14) |
| Size | -0.19 | -0.11 | -0.18 | -0.18 | -0.19 | -0.20 |
| | (0.06) | (0.08) | (0.04) | (0.04) | (0.04) | (0.04) |
| Be/Me | 0.06 | 0.17 | 0.12 | 0.10 | 0.14 | 0.12 |
| | (0.04) | (0.11) | (0.07) | (0.06) | (0.07) | (0.06) |
| $R^2$ | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 |
| N. Obs | 487,492 | 487,492 | 880,452 | 880,452 | 761,294 | 761,294 |

## Table A7

### Empirical Patterns on Leverage and Returns — Equity Issuance Cost Counterfactuals

The table reports estimated coefficients from cross-sectional (Fama–MacBeth) regressions of stock returns on leverage ("Leverage"), leverage gaps ("Lev. Gap"), market capitalization ("Size"), and book-to-market equity ("Be/Me") in two counterfactual economies that perturb the equity issuance cost parameters of the baseline calibration of Table 2. The baseline values are the fixed equity flotation cost $\lambda_0 = 0.500$ and the proportional equity flotation cost $\lambda_1 = 0.0250$. The "Low Equity Issuance Costs" columns lower both parameters by 10% ($\lambda_0 = 0.450$, $\lambda_1 = 0.0225$); the "High Equity Issuance Costs" columns raise both parameters by 10% ($\lambda_0 = 0.550$, $\lambda_1 = 0.0275$). Leverage targets and gaps are computed using the model-based approach of Section 4.2. Within each block, Columns (1) and (4) regress returns on leverage, size, and book-to-market; Columns (2) and (5) replace leverage with the leverage gap; Columns (3) and (6) include both leverage and the leverage gap. All coefficients are averages from 100 simulations of 100 randomly drawn firms over 5000 time periods. Bootstrapped standard errors are reported in parentheses. Returns are monthly and expressed in percentage (multiplied by 100); leverage gaps, leverage targets, and leverage are in levels. Size and Book-to-Market are expressed in log levels. All variables are defined in Appendix A.

| Dependent Variable: Monthly Stock Return | | | | | | |
|---|---|---|---|---|---|---|
| | Low Equity Issuance Costs | | | High Equity Issuance Costs | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Leverage | 0.51 | | 0.49 | 0.54 | | 0.59 |
| | (0.30) | | (0.30) | (0.37) | | (0.36) |
| Lev. Gap | | 0.75 | 0.71 | | 0.87 | 0.95 |
| | | (0.31) | (0.31) | | (0.36) | (0.34) |
| Size | -0.08 | -0.16 | -0.09 | -0.15 | -0.24 | -0.15 |
| | (0.07) | (0.06) | (0.06) | (0.08) | (0.06) | (0.08) |
| Be/Me | 0.23 | 0.11 | 0.22 | 0.21 | 0.09 | 0.22 |
| | (0.08) | (0.03) | (0.08) | (0.10) | (0.04) | (0.10) |
| $R^2$ | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |

**Figure A2**

Example: Dynamics of Leverage Targets, Gaps and Returns

The figure presents a snapshot of the simulated time series for a sample firm exposed to a specific sequence of idiosyncratic productivity shocks $z_{i,t}$. Panel A plots the debt (left axis, solid line) and the capital (right axis, dashed line) choices. Panel B shows book leverage. Panel C plots the leverage target. Panel D reports cumulative stock returns over the period. Initially, the firm experiences a sequence of positive shocks (highest value for $z_{i,t}$ in the discretization), followed by a sequence of negative shocks (lowest value for $z_{i,t}$), starting at the time indicated by the vertical dashed line.